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Addition and Subtraction of Fractions

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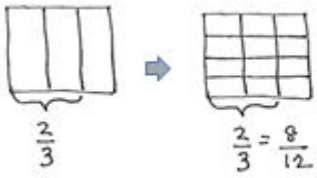
Grade 5 • Module 3

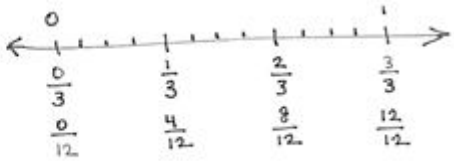
Addition and Subtraction of Fractions

OVERVIEW

In Module 3, students' understanding of addition and subtraction of fractions extends from earlier work with fraction equivalence and decimals. This module marks a significant shift away from the elementary grades' centrality of base ten units to the study and use of the full set of fractional units from Grade 5 forward, especially as applied to algebra.

In Topic A students revisit the foundational Grade 4 standards addressing equivalence. When equivalent, fractions represent the same amount of area of a rectangle, the same point on the number line. These equivalencies can also be represented symbolically.





$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Furthermore, equivalence is evidenced when adding fractions with the same denominator. The sum may be decomposed into parts (or re-composed into an equal sum). For example:

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$

$$\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$$

$$\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 1 + 1 + 1 = 3$$

$$\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1\frac{3}{5}$$

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$$\frac{7}{3} = \frac{6}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$$

This is also carrying forward work with decimal place value from Modules 1 and 2, confirming that like units can be composed and decomposed.

5 tenths + 7 tenths = 12 tenths = 1 and 2 tenths.
5 eighths + 7 eighths = 12 eighths = 1 and 4 eighths.

In Topic B, students move forward to see that fraction addition and subtraction is analogous to whole number addition and subtraction. Students add and subtract fractions with unlike denominators (5.NF.1) by replacing different fractional units with an equivalent fraction or like unit.

1 fourth + 2 thirds = 3 twelfths + 8 twelfths = 11 twelfths

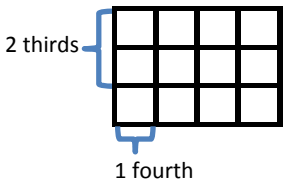
$$\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

This is not a new concept but certainly a new level of complexity. Students have added equivalent or like units since kindergarten, adding frogs to frogs, ones to ones, tens to tens, etc.

1 boy + 2 girls = 1 child + 2 children = 3 children

1 liter – 375 ml = 1000 ml – 375 ml = 625 ml

Throughout Topics A–D, a concrete to pictorial to abstract approach is used to convey this simple concept. Topic A uses paper strips and number line diagrams to clearly show equivalence. After a brief concrete introduction with folding paper, Topic B primarily uses the rectangular fractional model because it is useful for creating smaller like units via partitioning. (e.g., thirds and fourths are changed to twelfths to create equivent fractions as in the diagram below.) In Topic C, students move away from the pictorial altogether as they are empowered to write equations clarified by the model.



$$\frac{1}{4} + \frac{2}{3} = \left(\frac{1 \times 3}{4 \times 3}\right) + \left(\frac{2 \times 4}{3 \times 4}\right) = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

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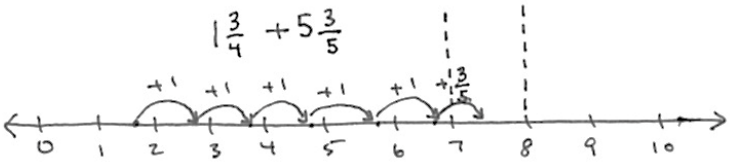
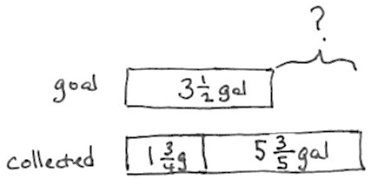
Topic C also uses the number line when adding and subtracting fractions greater than or equal to one so that students begin to see and manipulate fractions in relation to larger whole numbers and to each other. The number line takes fractions into the larger set of whole numbers, e.g., "Between what two whole numbers will the sum of 1 $^{2}/_{3}$ and 5 ¾ lie?"



___ < $1\frac{3}{4} + 5\frac{3}{5}$ < ___

This leads to understanding of and skill with solving more interesting problems, often embedded within multi-step word problems:

> Cristina and Matt's goal is to collect a total of 3 ½ gallons of sap from the maple trees. Cristina collected 1 ¾ gallons. Matt collected 5 $^{3}/_{5}$ gallons. By how much did they beat their goal?



$$1\frac{3}{4}\text{ gal} + 5\frac{3}{5}\text{ gal} - 3\frac{1}{2}\text{gal} = 3 + \left(\frac{3\times 5}{4\times 5}\right) + \left(\frac{3\times 4}{5\times 4}\right) - \left(\frac{1\times 10}{2\times 10}\right)$$

$$= 3 + \frac{15}{20} + \frac{12}{20} - \frac{10}{20} = 3\,\frac{17}{20}\text{ gal}$$

Cristina and Matt beat their goal by 3 $^{17}/_{20}$ gallons.

Word problems are part of every lesson. Students are encouraged to draw bar diagrams, which allow analysis of the same part/whole relationships they have worked with since Grade 1.

In Topic D, students strategize to solve multi-term problems and more intensely assess the reasonableness both of their solutions to word problems and their answers to fraction equations (5.NF.2).

> *"I know my answer makes sense because the total amount of sap they collected is going to be about 7 and a half gallons. Then, when we subtract 3 gallons, that is about 4 and a half. Then, 1 half less than that is about 4. 3 $^{17}/_{20}$ is just a little less than 4."*

The mid-module assessment follows Topic B. The end-of-module assessment follows Topic D.

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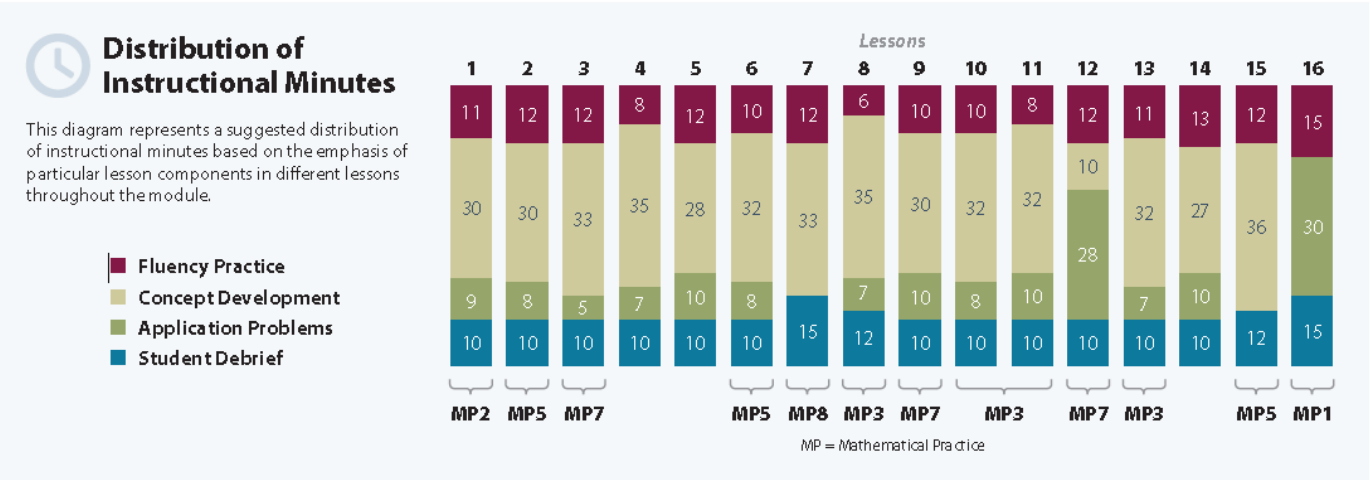

Focus Grade Level Standards

Use equivalent fractions as a strategy to add and subtract fractions.[1]

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.[2]

5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

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[1] *Examples in this module also include tenths and hundredths in fraction and decimal form.*
[2] *The balance of this cluster is taught in Modules 4 and 5.*

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Foundational Standards

4.NF.1 Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction *(n x a)/(n x b)* by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/*b*.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They

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reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5 **Use appropriate tools strategically**. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6 **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate

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accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP.7 **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Overview of Module Topics and Lesson Focus

Standards	Module Topics		Days
4.NF.1 4.NF.3c 4.NF.3d	A	**Equivalent Fractions** ▪ Lesson 1: Making Equivalent Fractions with the Number Line, Area Model, and with Numbers ▪ Lesson 2: Making Equivalent Fractions with Sums of Fractions with Like Denominators	2
5.NF.1 **5.NF.2**	B	**Fraction Addition and Subtraction: Making Like Units Pictorially** ▪ Lesson 3: Add Fractions with Unlike Units Using the Strategy of Creating Equivalent Fractions ▪ Lesson 4: Add Fractions with Sums Between One and Two ▪ Lesson 5: Subtract Fractions with Unlike Units Using the Strategy of Creating Equivalent Fractions ▪ Lesson 6: Subtract Fractions from Numbers Between One and Two ▪ Lesson 7: Solve Two-Step Word Problems	5

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		Mid-Module Assessment: Topics A and B (assessment ½ day, return ½ day, remediation or further applications 1 day)	3
5.NF.1 **5.NF.2**	C	**Fraction Addition and Subtraction: Making Like Units Numerically** ▪ Lesson 8: Add Fractions to and Subtract Fractions from Whole Numbers Using Equivalence and the Number Line as Strategies ▪ Lesson 9: Add Fractions Making Like Units Numerically ▪ Lesson 10: Add Fractions with Sums Greater than Two ▪ Lesson 11: Subtract Fractions Making Like Units Numerically ▪ Lesson 12: Subtract Fractions Greater Than or Equal to One	5
5.NF.1 **5.NF.2**	D	**Fraction Addition and Subtraction: Further Applications** ▪ Lesson 13: Use Fraction Benchmark Numbers to Assess Reasonableness of Addition and Subtraction Equations ▪ Lesson 14: Strategize to Solve Multi-Term Problems (e.g., $\frac{24}{5} + _____ + \frac{8}{7} = 9\frac{1}{3}$) ▪ Lesson 15: Solve Multi-Step Word Problems: Assess Reasonableness of Solutions Using Benchmark Numbers ▪ Lesson 16: Exploratory Lesson—Explore Part to Whole Relationships	4
		End-of-Module Assessment: Topics C and D (assessment ½ day, return ½ day, remediation or further applications 2 days)	3
Total Number of Instructional Days			**22**

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Fluency

New Fluency Topics Appearing in Module 3 Instruction

- Multiply any number by 1/10 and any positive power of 10
- All standard algorithms with whole numbers excluding long division with two-digit divisors

Familiar Fluency[3]

- Solving missing angle problems (4.MD.5-7)
- Application of area and perimeter formulas
- Conversions of measurements from a larger unit to a smaller unit
- Multiply and divide one-digit whole numbers using standard algorithms
- Generate equivalent fractions to halves, thirds, fourths, sixths, eighths, and tenths with denominators up to 40
- Generate fraction and decimal fraction equivalents of halves, fourths, fifths, and tenths as tenths and hundredths
- Factor any number up to 100 by a one-digit factor and another number

Terminology

New or Recently Introduced Terms

- Like denominators (e.g., 1/8 and 5/8)
- Unlike denominators (e.g., 1/8 and 1/7)
- Benchmark fraction (e.g., 1/2 is a benchmark fraction when comparing 1/3 and 3/5)

Familiar Terms and Symbols[4]

- Fraction (e.g., 3 fifths or 3/5)
- Equivalent fraction (e.g., 3/5 = 6/10)
- Denominator (denotes the fractional unit: fifths in 3 fifths, which is abbreviated to the 5 in 3/5)
- Numerator (denotes the count of fractional units: 3 in 3 fifths or 3 in 3/5)
- Fraction greater than or equal to one (e.g., 7/3, 3½, an abbreviation for 3 + ½)
- Fraction written in the largest possible unit (e.g., 3/6 = 1 x 3 / 2 x 3 = 1/2 or 1 three out of 2 threes = ½)
- Whole unit (e.g., any unit that is partitioned into smaller, equally-sized fractional units)
- Fractional unit (e.g., the fifth unit in 3 fifths denoted by the denominator 5 in 3/5)

[3] These are fluencies students have seen previously.
[4] These are terms and symbols students have seen previously.

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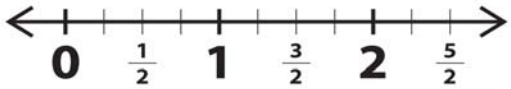
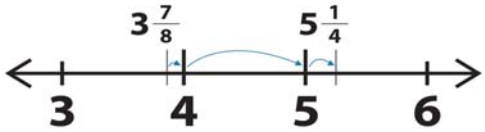
- Number sentence (e.g., “Three plus seven equals ten.” Usually written as “3 + 7 = 10.”)
- Meter, kilometer, centimeter, liter, kiloliter, gram, kilogram, feet, mile, yard, inch, gallon, quart, pint, cup, pound, ounce, hour, minute, second
- Tenth (1/10 or 0.1)
- Hundredth (1/100 or 0.01)
- “More than halfway” and “less than halfway.”
- “One tenth of” (e.g., 1/10 × 250)
- Between (e.g., 1/2 is between 1/3 and 3/5)
- < , > , =

Suggested Tools and Representations

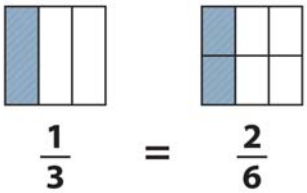
- Paper strips (for modeling equivalence)
- Number line (a variety of templates)
- Rectangular fraction model
- Fraction strips
- Bar diagrams



Example of a number line



Example of an “empty” number line



Example of a rectangular fraction model

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Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	5.NF.1, 5.NF.2, 5.NF.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	5.NF.1, 5.NF.2, 5.NF.3

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Scaffolds[5]

The scaffolds integrated into *A Story of Units* gives alternatives for how students access information, express and demonstrate their learning. Strategically-placed margin notes within each lesson elaborate and demonstrate the use of specific scaffolds at specific times. They address many needs presented by English Language Learners, Students with Disabilities, Students Performing above Grade Level and Students Performing below Grade Level, although many suggestions are applicable to more than one population. The charts that follow provide a general overview of the lesson-aligned scaffolds suggested in Module 3 lessons. They are organized by Universal Design for Learning (UDL) principles. To read more about the approach to differentiated instruction in *A Story of Units*, please refer to the document, 'How To Implement A Story of Units.'

[5] Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website, www.p12.nysed.gov/specialed/aim, for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

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Scaffolds for English Language Learners

UDL Principles	Scaffolds
Provide Multiple Means of Representation	▪ Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ▪ Couple teacher-talk with "math-they-can-see," such as models. ▪ Highlight or provide visuals for critical vocabulary in discussion. For example, demonstrate making 'a ten' by bundling 10 straws. ▪ Let students use models and gestures to calculate and explain. For example, a student searching to define a term may model with drawings or concrete objects or write the number sentence to match. ▪ Teach students how to ask questions (such as, "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters" such as: "I agree because…." "I noticed that…" "Your solution is different from/ the same as mine because…"
Provide Multiple Means of Action and Expression	▪ Provide a variety of ways to respond: Oral / Pair share / Concrete models, e.g., fingers Choral / Small group share / Pictorial models, e.g., ten-frame Personal white boards ▪ Support oral or written response with sentence frames, such as "_____ is ___ hundreds, __ tens, and ___ ones." ▪ Ask questions to probe what students mean as they attempt expression in a second language. ▪ Scaffold questioning to guide connections, analysis, and mastery. ▪ Group students by home language for arithmetic computation. ▪ Have students retell another student's idea to a partner. ▪ Use wait time.
Provide Multiple Means of Engagement	▪ Check for understanding frequently (e.g., 'show') to benefit those who may shy away from asking questions. ▪ Couple teacher-talk with illustrative gestures. ▪ Keep teacher-talk clear and concise. ▪ Point to visuals while speaking, using your hands to clearly indicate the image that corresponds to your words. ▪ Couple language with motion or signals for frequently used terms.

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Scaffolds for Students with Disabilities

In addition to the scaffolds below, familiarize yourself with and implement the specific modifications listed in your students' Individual Education Plans (IEPs) or Section 504 Accommodation Plans.

UDL Principles	Scaffolds
Provide Multiple Means of Representation	▪ Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ▪ Couple teacher-talk with "math-they-can-see," such as models. ▪ Highlight or provide visuals for critical vocabulary in discussion. For example, demonstrate making 'a ten' by bundling 10 straws. ▪ Let students use models and gestures to calculate and explain. For example, a student searching to define a term may model with drawings or concrete objects or write the number sentence to match. ▪ Teach students how to ask questions (such as, "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters" such as: "I agree because…." "I noticed that…" "Your solution is different from/ the same as mine because…"
Provide Multiple Means of Action and Expression	▪ Provide a variety of ways to respond: Oral / Pair share / Concrete models, e.g., fingers Choral / Small group share / Pictorial models, e.g., ten-frame Personal white boards ▪ Use visual signals (such as a clap, rather than "show") to elicit responses. ▪ Use think-pair-share to assess students' understanding. ▪ Support oral or written response with sentence frames, such as "_____ is ___ hundreds, __ tens, and ___ ones." ▪ Ask questions to probe what students mean as they attempt expression. ▪ Scaffold questioning to guide connections, analysis, and mastery. ▪ Have students retell another student's idea to a partner. ▪ Use wait time.
Provide Multiple Means of Engagement	▪ Keep teacher-talk clear and concise. ▪ Check for understanding frequently (e.g. 'show'). ▪ Teach in small chunks so students get practice with one step at a time. ▪ Use rhythms to help students (e.g., chant numbers in happy counting). ▪ Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.

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Scaffolds for Students Performing Above Grade Level

UDL Principles	Scaffolds
Provide Multiple Means of Representation	▪ Teach students how to ask questions (such as, "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters" such as: "I agree because…." "I noticed that…" "Your solution is different from/ the same as mine because…"
Provide Multiple Means of Action and Expression	▪ Encourage students to explain reasoning both orally and by modeling or writing. ▪ Extend exploration of math topics by means of games, puzzles, and brain teasers. ▪ Encourage students to notice and explore patterns in math content. Invite students to share their observations in discussion and writing. ▪ Foster curiosity about numbers and mathematical ideas. Facilitate exploration through open-ended activities or discussion. ▪ Offer extensions for early finishers. ▪ Adjust numbers in word problems or calculations to match levels.
Provide Multiple Means of Engagement	▪ Push student comprehension into higher levels of Bloom's Taxonomy with advancing questions. ▪ Ask, "Why?" and "What if?" questions. ▪ Get students up and moving, coupling fluency building with movement (e.g., skip-counting during transition times).

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Scaffolds for Students Performing Below Grade Level

UDL Principles	Scaffolds
Provide Multiple Means of Representation	▪ Clarify, compare, and make connections to math words in discussion, particularly during and after practice. ▪ Couple teacher-talk with "math-they-can-see," such as models. ▪ Highlight or provide visuals for critical vocabulary in discussion. For example, demonstrate making 'a ten' by bundling 10 straws. ▪ Let students use models and gestures to calculate and explain. For example, a student searching to define a term may model with drawings or concrete objects or write the number sentence to match. ▪ Teach students how to ask questions (such as, "Do you agree?" and "Why do you think so?") to extend "think-pair-share" conversations. Model and post conversation "starters" such as: "I agree because…." "I noticed that…" "Your solution is different from/ the same as mine because…"
Provide Multiple Means of Action and Expression	▪ Provide a variety of ways to respond: Oral / Pair share / Concrete models, e.g., fingers Choral / Small group share / Pictorial models, e.g., ten-frame Personal white boards ▪ Use think-pair-share to assess students' understanding. ▪ Support oral or written response with sentence frames, such as "_____ is ___ hundreds, __ tens, and ___ ones." ▪ Ask questions to probe what students mean as they attempt expression. ▪ Scaffold questioning to guide connections, analysis, and mastery. ▪ Adjust numbers in word problems or calculations to match levels. ▪ Adjust the amount of guided practice or use small groupings. ▪ Have students retell another student's idea to a partner. ▪ Use wait time.
Provide Multiple Means of Engagement	▪ Push student comprehension with advancing questions. ▪ Check for understanding frequently (e.g., 'show'). ▪ Couple teacher-talk with illustrative gestures. ▪ Keep teacher-talk clear and concise. ▪ Point to visuals while speaking, using your hands to clearly indicate the image that corresponds to your words. ▪ Teach in small chunks so students get practice with one step at a time. ▪ Use rhythms to help students (e.g., chant numbers in happy counting). ▪ Point to visuals and captions while speaking, using your hands to clearly indicate the image that corresponds to your words.

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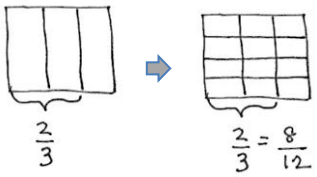
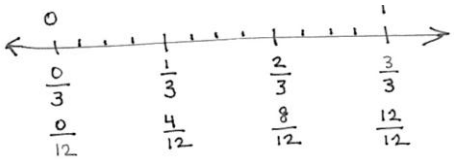
Topic A:
Equivalent Fractions

4.NF.1, 4.NF.3c, 4.NF.3d

Focus Standard:	4.NF.1	Extend understanding of fraction equivalence and ordering. 1. Explain why a fraction a/b is equivalent to a fraction *(n x a)/(n x b)* by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Instructional Days:	2	
Coherence -Links from:	G4–M5	Order and Operations with Fractions
-Links to:	G5–M4	Extensions and Applications of Multiplication and Division of Fractions and Decimal Fractions
	G6–M2	The Number System: Rational Numbers and Decimals

In Topic A students revisit the foundational Grade 4 standards addressing equivalence. When equivalent, fractions can be represented by the same amount of area of a rectangle, the same point on a number line. Areas are sub-divided. Lengths on the number line are divided into smaller equal lengths. On the number line below there are 4 x 3 parts of equal length. From both the area model and the number line it can be seen that 2 thirds is equivalent to 8 twelfths.





This equivalence can also be represented symbolically.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

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Furthermore, equivalence is evidenced when adding fractions with the same denominator. The sum may be decomposed into parts (or re-composed into an equal sum). For example:

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$

$$\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$$

$$\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 1 + 1 + 1 = 3$$

$$\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1\frac{3}{5}$$

$$\frac{7}{3} = \frac{6}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$$

CONCEPT CHART	A Teaching Sequence Towards Mastery of Equivalent Fractions
Concept 1:	**Making Equivalent Fractions with the Number Line, Area Model, and with Numbers (Lesson 1)**
Concept 2:	**Making Equivalent Fractions with Sums of Fractions with Like Denominators (Lesson 2)**

Mathematical Practices Brought to Life

In Lesson 1, students analyze what is happening to the units when an equivalent fraction is being made by changing larger units for smaller ones, honing their ability to "Look for and make use of structure." (MP.7) They study the area model to make generalizations, transferring that back onto the number line as they see the same process occurring there within the lengths.

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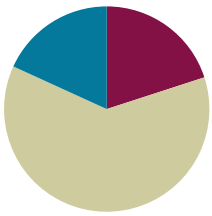
Lesson 1: Making Equivalent Fractions with the Number Line, Area Model and with Numbers

Suggested Lesson Structure

Fluency Practice	(11 minutes)
Application Problem	(9 minutes)
Concept Lesson	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (11 minutes)

- Sprint **4.OA.4** (9 minutes)
- Skip Counting by ¼ Hour **5.MD.1** (2 minutes)

Sprint (9 minutes)

Materials: (S) Write the Missing Factor Sprint (See directions for administration of sprints in the **G5-M3 Fluency Progressions.**)

Skip-Counting by ¼ Hour (2 minutes)

T: Let's count by ¼ hours. (Rhythmically point up until a change is desired. Show a closed hand and then point down. Continue, mixing it up.)

S: 1/4 hour, 2/4 hour, 3/4 hour, 1 hour (stop) 1 ¼ hour, 1 2/4 hour, 1 ¾ hour, 2 hours (stop).

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

Because students are counting units of time, a clock is a natural visual to incorporate. Gesture to points on the clock as students chant, and possibly use it to help signal the direction of the count.

Application (9 minutes)

15 kilograms of rice are separated equally into 4 containers. How many kilograms of rice are in each container? Express your answer as a decimal and as a fraction.

T: Let's read the problem together.

S: (Students read chorally.)

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T: Share with your partner: What do you see when you hear the story? What can you draw?

S: (Students share with partners.)

T: I'll give you one minute to draw.

T: Explain to your partner what your drawing shows.

T: (After a brief exchange.) What's the total weight of the rice?

S: 15 kilograms.

T: 15 kilograms are being split equally into how many containers?

S: 4 containers.

T: So the whole is being split into how many units?

S: 4 units.

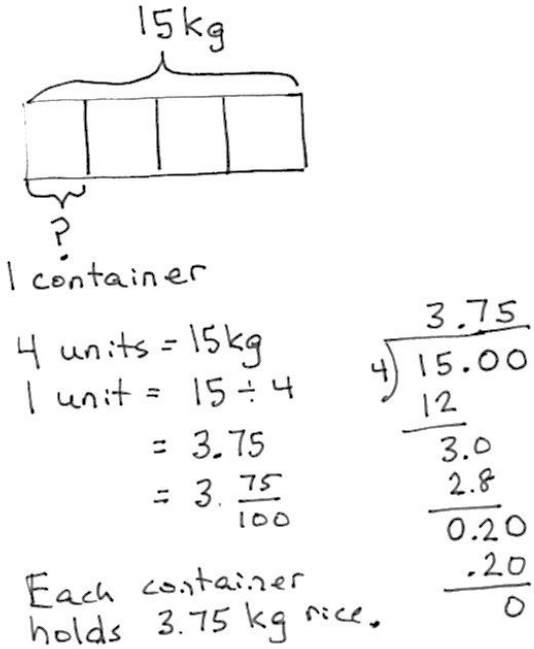
T: To find 1 container or 1 unit, we have to?

S: Divide.

T: Tell me the division sentence.

S: 15 ÷ 4.

T: Solve the problem on your personal white board. Write your answer both in decimal form and as a whole number and a decimal fraction. (Pause.) Show your board.



T: Turn and explain to your partner how you got the answer. 15 ÷ 4 = 3.75

T: (After students share.) Show the division equation with both answers.

S: 15 ÷ 4 = 3.75 = $3\frac{75}{100}$.

T: Another fraction equivalent to 75 hundredths is?

S: 3 fourths.

T: Also write your answer as a whole number and a fraction.

S: 15 ÷ 4 = 3.75 = $3\frac{75}{100}$= $3.\frac{3}{4}$.

T: So 3 and 3 fourths equals 3 and 75 hundredths.

T: Tell me your statement containing the answer.

S: Each container holds 3.75 kg or 3 ¾ kg of rice.

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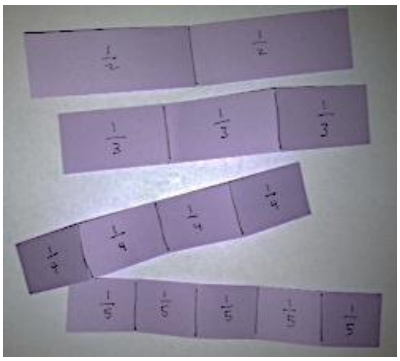
Concept Development (30 minutes)

Materials: (S) 4 Paper strips sized 4 ¼ x 1 per student (vertically cut an 8 ½" x 11" paper down the middle)

Problem 1



T: Take your paper strip. Hold it horizontally. Fold it vertically down the middle. How many equal parts do you have in the whole?

S: 2.

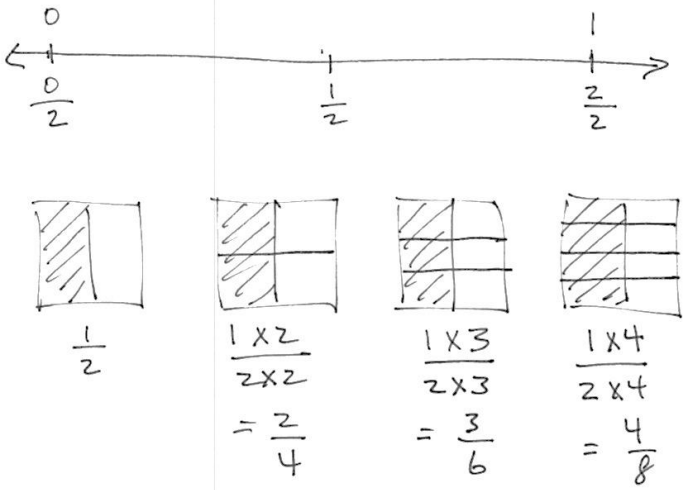
T: What fraction of the whole is 1 part?

S: 1 half.

T: Draw a line to show where you folded your paper and label each half ½, one out of 2 units.

T: As you did in fourth grade, take about 2 minutes to make paper strips to also show thirds, fourths, and fifths.

T: (After about 3 minutes to make the paper strips.) Draw a number line that is a little longer than your paper strip. Use your strip as a ruler to mark zero and 1 above the line, $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}$ below the line.



T: (After doing so.) Make about an inch by inch square beneath your line. This is representing the same 1 whole as the number line. For today, show half by vertically dividing the square. Shade 1 half on the left.

T: (After discussion.) Draw another square to the right of that one. Shade it in the same way to represent ½.

T: Partition it horizontally across the middle.

T: What fraction is shaded now?

S: $\frac{1}{2}$ or $\frac{2}{4}$.

T: (Record numerically referring to the picture) 1 group of 2 out of two groups of 2.

$$\frac{1}{2} = \frac{\textbf{1 group of two}}{\textbf{2 group of two}} \textbf{ or } \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

Sentence frames help students remember the linguistic and numerical patterns. As they gain confidence, flip it over so they do not become overly dependent.

$$__ = \frac{__ \text{ group of } __}{__ \text{ group of } __} \text{ or } \frac{__ \times __}{__ \times __} = __.$$

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T: Explain how we have represented the equivalent fractions to your part.

T: Show me $\frac{2}{4}$ on the number line. (Students show.) Yes, it is the exact same number as 1 half, the exact same point on the number line.

T: Work with your partner to draw another congruent square with 1 half shaded. This time partition it horizontally into 3 equal units (2 lines) and record the equivalent fraction as we did on the first example. If you finish early, continue the pattern.

NOTES ON SCAFFOLDING ELLS:

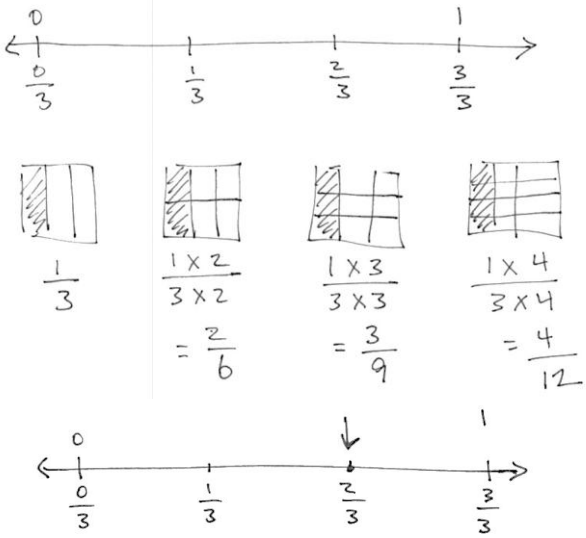
Because this lesson is so pictorial, it is perfect for ELLs. Support them making the connection of the words to the numbers, the numbers to the pictures, through the use of your gestures and hands. "One unit of two" (pause and point to the image and then to the numbers.

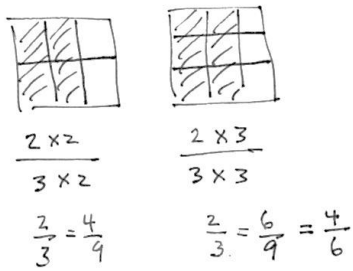
Problem 2

Make fractions equal to $\frac{1}{3}$.

This problem allows students to repeat the procedure with thirds, another benchmark fraction. As needed for your students, you might repeat the process thoroughly as outlined in problem 1. Work with a small group as others work independently, or let them try it with a partner. It is not necessary for all students to complete the same amount of work. Move on to problem 3 after about 4 minutes on problem 2.

Note in the drawing to the right that 6 ninths is shown to be equal to 4 sixths. Go back and make sure this point is clear with 2 sixths, 3 ninths and 4 twelfths.





Problem 3

Make fractions equal to $\frac{2}{3}$.

The next complexity is working with a non-unit fraction.

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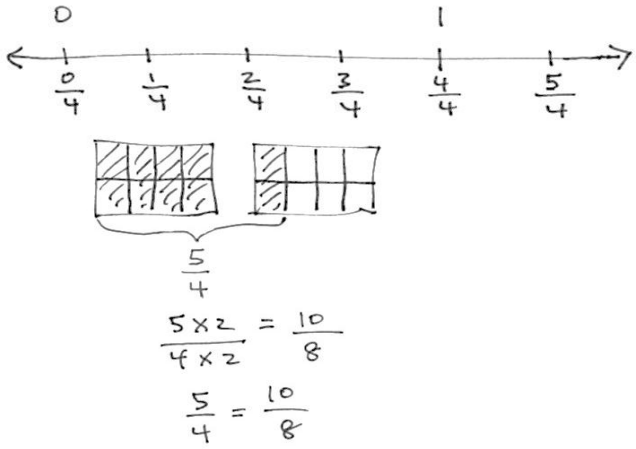
Problem 4

Make fractions equal to $\frac{5}{4}$.

The final complexity prior to working independently is to model a fraction greater than 1. The same exact process is used. Rectangles are used in the example just to break rigidity. This is not unique to squares!

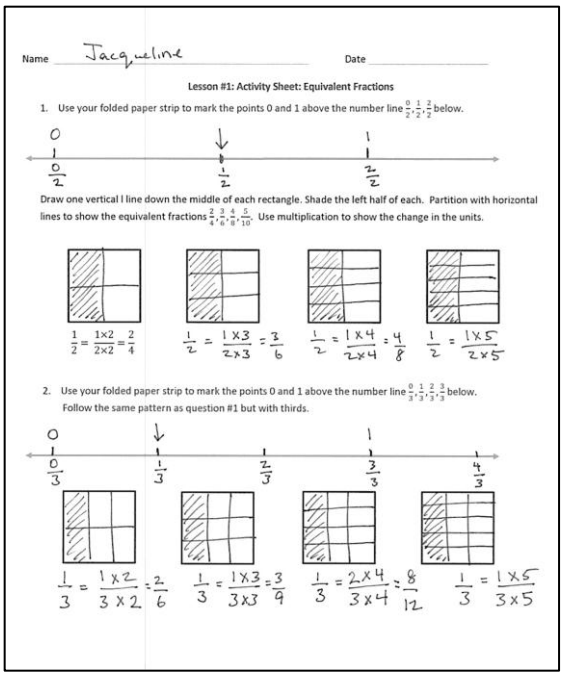


NOTES ON THE LESSON:

These number lines and "squares" are informal sketches, not precise. Avoid rulers and graph paper so that students get accustomed to realizing that these images are not intended to be perfect but symbolic. The lines represent straight lines. The thirds perfectly equal parts. The worksheet has squares drawn in to expedite the movement to the abstract number. It is highly preferable to start with hand drawn squares so that students do not get the mistaken impression that drawings have to be perfect to empower them. A mental schema is developing, not an attachment to the drawing.

Activity Worksheet (10 minutes)

Distribute the activity worksheet. Students work independently until there are 10 minutes remaining in the lesson.


Name Jacqueline Date

Lesson #1: Activity Sheet: Equivalent Fractions

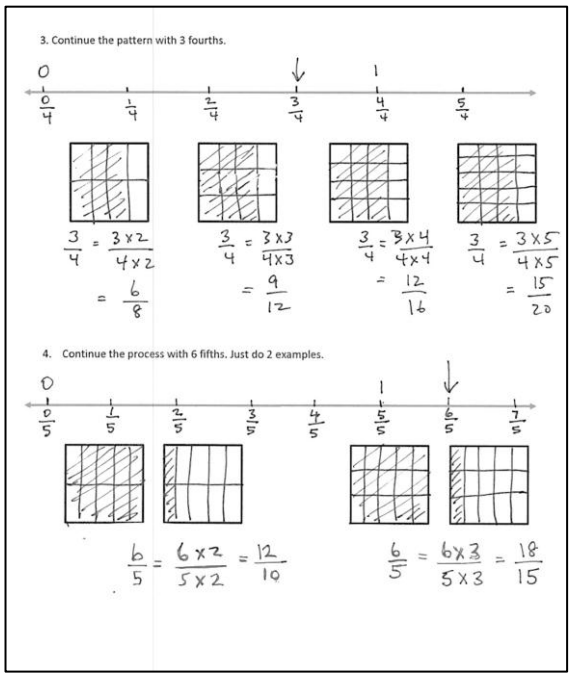
1. Use your folded paper strip to mark the points 0 and 1 above the number line $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}$ below.

Draw one vertical l line down the middle of each rectangle. Shade the left half of each. Partition with horizontal lines to show the equivalent fractions $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$. Use multiplication to show the change in the units.

$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ $\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ $\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$ $\frac{1}{2} = \frac{1 \times 5}{2 \times 5}$

2. Use your folded paper strip to mark the points 0 and 1 above the number line $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}$ below. Follow the same pattern as question #1 but with thirds.

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ $\frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9}$ $\frac{1}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ $\frac{1}{3} = \frac{1 \times 5}{3 \times 5}$


3. Continue the pattern with 3 fourths.

$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$ $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$ $\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

4. Continue the process with 6 fifths. Just do 2 examples.

$\frac{6}{5} = \frac{6 \times 2}{5 \times 2} = \frac{12}{10}$ $\frac{6}{5} = \frac{6 \times 3}{5 \times 3} = \frac{18}{15}$

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Student Debrief (10 minutes)

Materials: (S) Completed Worksheet

T: Looking at your worksheet, which fractions are equal to 1/3?

S: $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}$

T: Continue the pattern beyond those on the worksheet with your partner.

T: (After a moment.) Continue the pattern chorally.

S: $\frac{6}{18}, \frac{7}{21}, \frac{8}{24}, \frac{9}{27}, \frac{10}{30}$

T: Is $\frac{100}{300}$ equal to 1/3?

S: Yes.

T: How can we know if a fraction is equal to 1 third without drawing?

S: "When you multiply the numerator by 3, you get the denominator." → "When you divide the denominator by 3 you get the numerator." → "The total number of equal pieces is 3 times the number of selected equal pieces."

T: In the next minute, write as many other fractions as you can that are equal to 1 third and write them on your personal white board.

T: What do we know about all these fractions when we look at the number line?

S: They are the exact same point.

T: So there are an infinite number of fractions equivalent to 1/3?

S: Yes!

T: The fraction 1/3 is one number, just like the number two or three. It is not two numbers, just this one point on the number line.

More quickly repeat the process of generating equivalent fractions to 3 fourths and 5 sixths.

T: Discuss with your partner what is happening to the pieces, the units, when the numerator and denominator are getting larger.

MP.7

S: "The parts are getting smaller." → "The equal pieces are being replaced by smaller equal pieces but the area of the fraction is staying the same." → "The units are being partitioned into smaller equal units. The value of the fraction is exactly the same."

T: What would that look like, were we to see it on the number line?

S: The length would be divided into smaller and smaller parts.

T: So if I divide each of the lengths of 1 fourth into 3 smaller parts of equal length (use the fourths number line from earlier in the lesson), discuss with your partner what the new smaller unit will be. Compare it to the corresponding picture on your worksheet.

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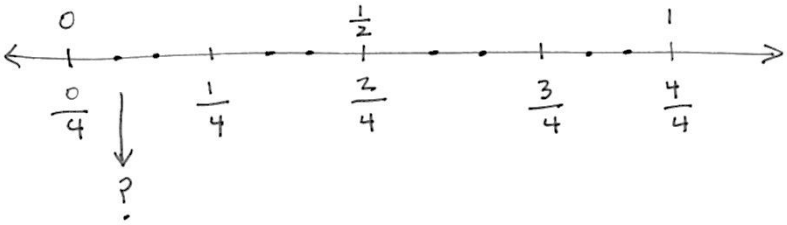
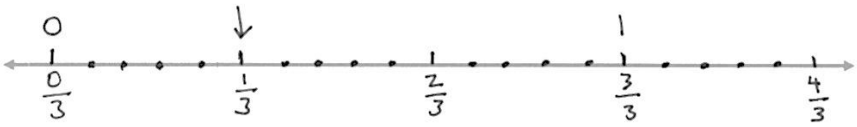
MP.7



T: On your worksheet, divide the lengths of 1 third into 5 equal smaller units. Think about what is happening to the units, to the length and to the name of the fraction. Close your lesson by discussing connections between the number line, the area model and the equivalent fractions with your partner.



Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A

Correct_____

Write the missing factor.

1	10 = 5 x ___		23	28 = 7 x___	
2	10 = 2 x ___		24	28 = 2 x 2 x ___	
3	8 = 4 x ___		25	28 = 2 x ___ x 2	
4	9 = 3 x ___		26	28 = ___ x 2 x 2	
5	6 = 2 x ___		27	36 = 3 x 3 x ___	
6	6 = 3 x ___		28	9 x 4 = 3 x 3 x ___	
7	12 = 6 x ___		29	9 x 4 = 6 x ___	
8	12 = 3 x ___		30	9 x 4 = 3 x 2 x ___	
9	12 = 4 x ___		31	8 x 6 = 4 x ___ x 2	
10	12 = 2 x 2 x ___		32	9 x 9 = 3 x ___ x 3	
11	12 = 3 x 2 x ___		33	8 x 8 = ___ x 8	
12	20 = 5 x 2 x ___		34	7 x 7 = ___ x 7	
13	20 = 5 x 2 x ___		35	8 x 3 = ___ x 6	
14	16 = 8 x ___		36	16 x 2 = ___ x 4	
15	16 = 4 x 2 x ___		37	2 x 18 = ___ x 9	
16	24 = 8 x ___		38	28 x 2 = ___ x 8	
17	24 = 4 x 2 x ___		39	24 x 3 = ___ x 9	
18	24 = 4 x ___ x 2		40	6 x 8 = ___ x 12	
19	24 = 3 x 2 x ___		41	27 x 3 = ___ x 9	
20	24 = 3 x ___ x 2		42	12 x 6 = ___ x 8	
21	6 x 4 = 8 x ___		43	54 x 2 = ___ x 12	
22	6 x 4 = 4 x 2 x ___		44	9 x 13 = ___ x 39	

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B Improvement_____# Correct_____

Write the missing factor.

1	6 = 2 x ___		23	28 = 4 x ___	
2	6 = 3 x ___		24	28 = 2 x 2 x ___	
3	9 = 3 x ___		25	28 = 2 x ___ x 2	
4	8 = 4 x ___		26	28 = ___ x 2 x 2	
5	10 = 5 x ___		27	36 = 2 x 2 x ___	
6	10 = 2 x ___		28	9 x 4 = 2 x 2 x ___	
7	20 = 10 x ___		29	9 x 4 = 6 x ___	
8	20 = 5 x 2 x ___		30	9 x 4 = 2 x 3 x + ___	
9	12 = 6 x ___		31	8 x 6 = 4 x ___ x 2	
10	12 = 3 x ___		32	8 x 8 = 4 x ___ x 2	
11	12 = 4 x ___		33	9 x 9 = ___ x 9	
12	12 = 2 x 2 x ___		34	6 x 6 = ___ x 6	
13	12 = 3 x 2 x ___		35	6 x 4 = ___x 8	
14	24 = 8 x ___		36	16 x 2 = ___ x 8	
15	24 = 4 x 2 x ___		37	2 x 18 = ___ x 4	
16	24 = 4 x ___ x 2		38	28 x 2 = ___ x 7	
17	24 = 3 x 2 x ___		39	24 x 3 = ___ x 8	
18	24 = 3 x ___ x 2		40	8 x 6 = ___ x 4	
19	16 = 8 x ___		41	12 x 6 = ___ x 9	
20	16 = 4 x 2 x ___		42	27 x 3 = ___ x 9	
21	8 x 2 = 4 x ___		43	54 x 2 = ___ x 9	
22	8 x 2 = 2 x 2 x ___		44	8 x 13 = ___ x 26	

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Name _____ Date _____

Equivalent Fractions

1. Use your folded paper strip to mark the points 0 and 1 above the number line $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}$ below.

Draw one vertical line down the middle of each rectangle, creating two parts. Shade the left half of each. Partition with horizontal lines to show the equivalent fractions $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$. Use multiplication to show the change in the units.

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

2. Use your folded paper strip to mark the points 0 and 1 above the number line $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}$ below. Follow the same pattern as question #1 but with thirds.

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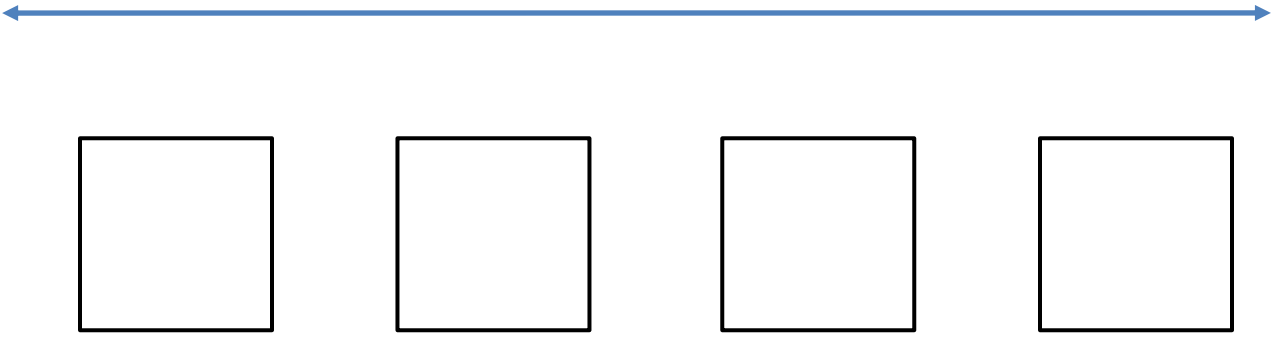
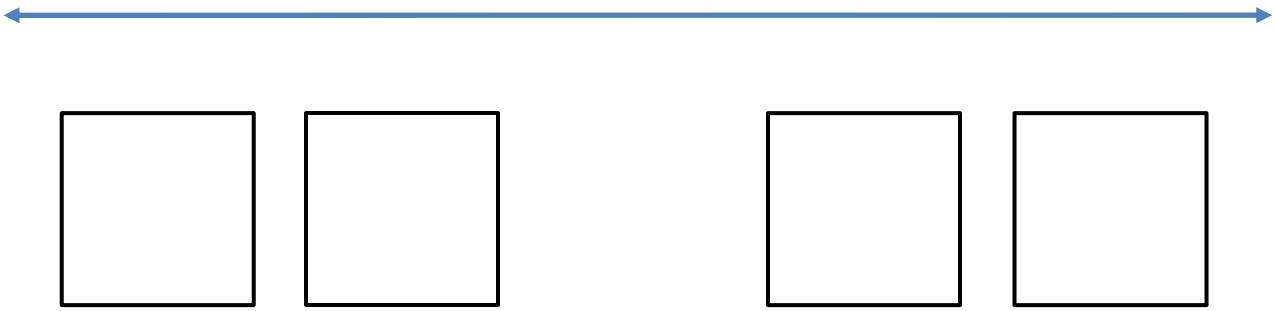
3. Continue the pattern with 3 fourths.



4. Continue the process with 6 fifths. Just do 2 examples.



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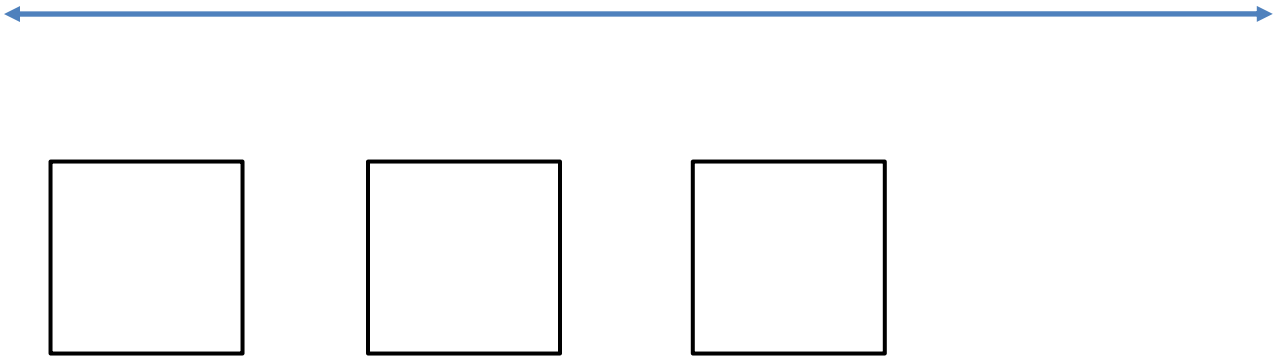
Name _____ Date _____

1. Estimate to mark the points 0 and 1 above the number line $\frac{0}{6}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}$ below. Use the squares below to represent fractions equivalent to 1 sixth using both arrays and equations.



$$\frac{1}{6} = \frac{1 \times 2}{6 \times 2} = \frac{3}{9}$$

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1. Use your folded paper strip to mark the points 0 and 1 above the number line $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}$ below.

Draw two vertical lines to break each rectangle into thirds. Shade the left third of each. Partition with horizontal lines to show equivalent fractions. Use multiplication to show the change in the units.

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

2. Use your folded paper strip to mark the points 0 and 1 above the number line $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ below. Follow the same pattern as question #1 but with fourths.

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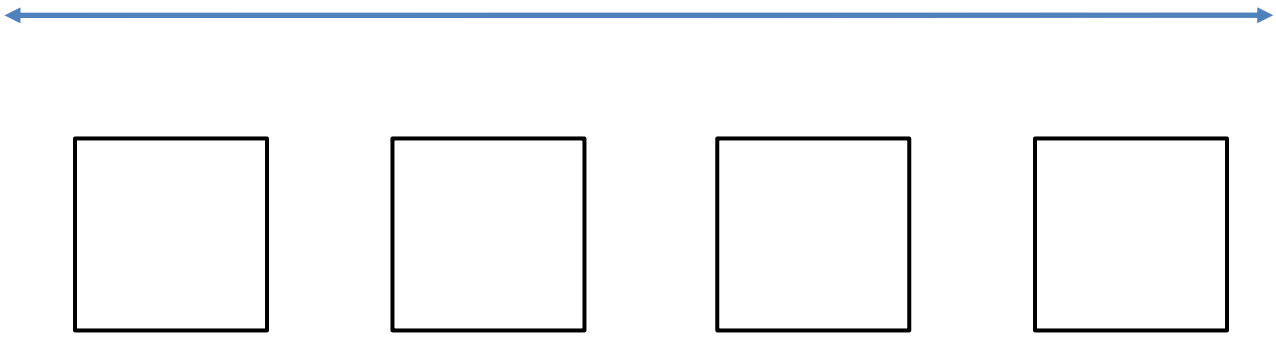
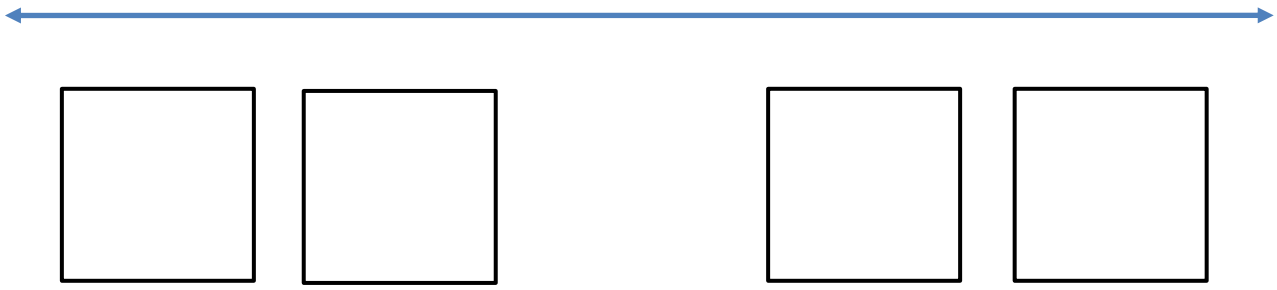
3. Continue the pattern with 4 fifths.



4. Continue the process with 9 eighths. Estimate to make the points on the number line. Just do 2 examples



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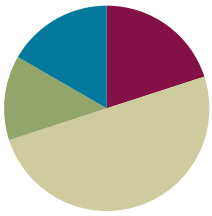
Lesson 2: Making Equivalent Fractions with Sums of Fractions with Like Denominators

Suggested Lesson Structure

- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Lesson (30 minutes)
- Student Debrief (10 minutes)

Total Time (60 minutes)



Fluency Practice (12 minutes)

- Equivalent Fractions **5.NF.1** (3 minutes)
- Sprint **4.NF.1** (9 minutes)

Equivalent Fractions (3 minutes)

T: (Write) $\frac{1}{2} =$

T: Say the fraction.

S: One half.

T: (Write) $\frac{1}{2} = \frac{\quad}{4}$

T: One half is how many fourths?

S: Two fourths.

Continue with possible sequence:

$\frac{1}{2} = \frac{\quad}{6}, \frac{1}{3} = \frac{\quad}{6}, \frac{2}{3} = \frac{\quad}{6}, \frac{2}{3} = \frac{\quad}{12}, \frac{3}{4} = \frac{\quad}{16}, \frac{3}{5} = \frac{\quad}{25}$

T: (Write) $\frac{1}{2} =$

T: Say the fraction.

S: One half.

T: (Write) $\frac{1}{2} = \frac{2}{\quad}$

NOTES ON SCAFFOLDING STUDENTS ABOVE AND BELOW GRADE LEVEL:

'Equivalent Fractions' is intentionally placed before the Sprint because it reviews the Sprint skill. Meet the needs of your students by adjusting the amount of time you spend on it. If you find that students struggle to complete Sprint A, you may want to do another minute or two of 'Equivalent Fractions' before moving them on to Sprint B.

NOTES ON SCAFFOLDING ELLS:

Adjusting number words and correctly pronouncing them as fractions ('fifths,' 'sixths,' etc.) may be challenging. If you have many ELLs, before starting you might quickly count together to practice enunciating word endings: hal**ves**, thir**ds**, four**ths**, fif**ths**, six**ths**, etc.

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T: One half or one part of two is the same as two parts of what unit?
S: Fourths.
Continue with possible sequence:

$$\frac{1}{2}=\frac{2}{__},\frac{1}{5}=\frac{2}{__},\frac{2}{5}=\frac{8}{__},\frac{2}{5}=\frac{8}{__},\frac{3}{4}=\frac{9}{__},\frac{4}{5}=\frac{16}{__}$$

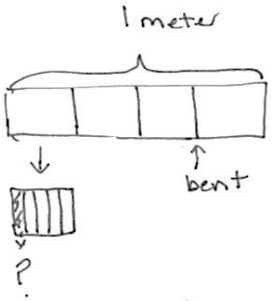
Sprint (9 minutes)

Materials: (S) Find the Missing Numerator or Denominator Sprint (See directions for administration of sprints in the **G5-M3 Fluency Progressions.**)

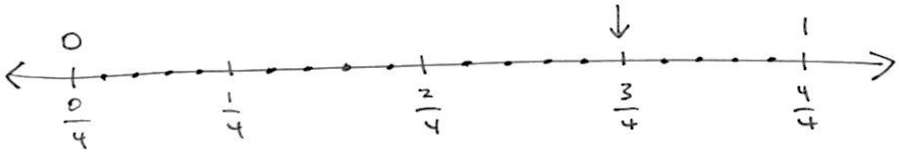
Application Problem (8 minutes)

Mr. Hopkins has a 1 meter wire he is using to make clocks. Each fourth meter is marked off with 5 smaller equal lengths. If Mr. Hopkins bends the wire at ¾ meter, what fraction of the marks is that?

T: (After the students have solved the problem, possibly using the RDW process independently or in partners.) Let's look at two of your solutions and compare them.



5 units = $\frac{1}{4}$
3 × 5 units = $\frac{3}{4}$
15 units = $\frac{3}{4}$
$\frac{15}{20} = \frac{3}{4}$



1 mark is $\frac{1}{20}$ m.
$\frac{3}{4}$ m is the same as $\frac{15}{20}$ m.

Mr. Hopkins bent the wire at $\frac{3}{4}$ m or at $\frac{15}{20}$ of the marks.

NOTES ON RDW:

Students have been using the RDW process since grade 1: Read, Draw, Write.

1. Read the problem.
2. Draw to represent the problem.
3. Write an equation(s) that either helps you to solve the problem or shows how you solved it.
4. Write a statement of the answer to the question.

Embedded within "D" are important reflective questions:

- What do I see?
- Can I draw something?
- What conclusions can I make from my drawing?

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T: When you look at these two solutions side by side what do you see? (You might use the following set of questions to help students compare the solutions as a whole class, or to encourage inter-partner communication as you circulate while they compare.)

- What did each of these students draw?
- What conclusions can you make from their drawings?
- How did they record their solutions numerically?
- How does the tape diagram relate to the number line?
- What does the tape diagram/number line clarify?
- What does the equation clarify?
- How could the statement with the number line be rephrased to answer the question?

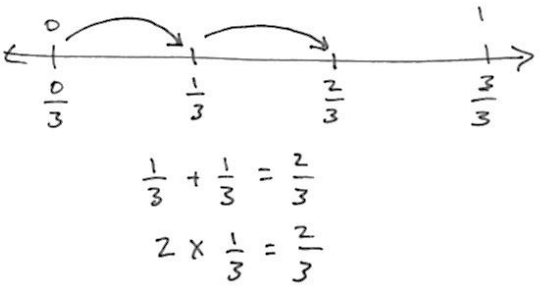
Concept Development (30 minutes)

Materials: (S) Blank paper

Problem 1

$$\frac{1}{3}+\frac{1}{3}$$

1 third + 1 third = 2 thirds



T: On a number line, mark the end points as zero and 1. Between zero and 1 estimate to make three parts of equal length and label them with their fractional value.

T: (After students work.) On your number line, show 1 third plus 1 third with arrows designating lengths. (Demonstrate and then pause as students work).

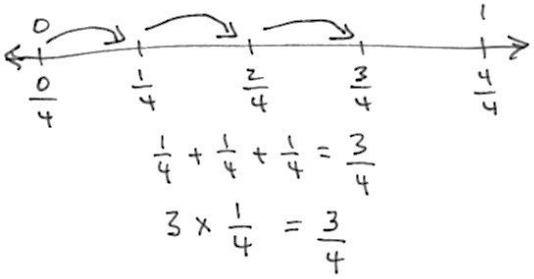
T: The answer is?

S: 2 thirds.

T: Talk to your partner. Express this as a multiplication equation and as an addition sentence.

S: $\frac{1}{3}+\frac{1}{3}=2\times\frac{1}{3}=\frac{2}{3}$

T: Following the same pattern of adding unit fractions by joining lengths, show 3 fourths on a number line.



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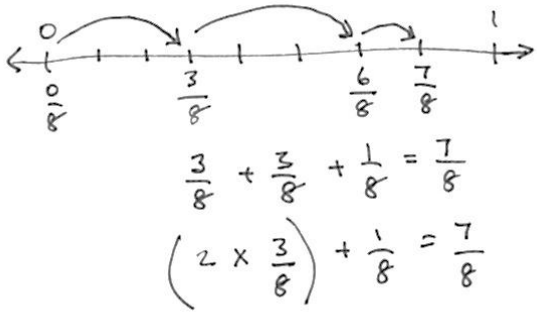
Problem 2

$$\frac{3}{8}+\frac{3}{8}+\frac{1}{8}$$

3 eighths + 3 eighths + 1 eighth



T: On a number line, again mark the end points as zero and one. Between zero and one, estimate to make 8 parts of equal length. This time only label what is necessary to show 3 eighths.

T: (After students work.) Represent 3 eighths + 3 eighths + 1 eighth on your number line.

T: The answer is?

S: 7 eighths.

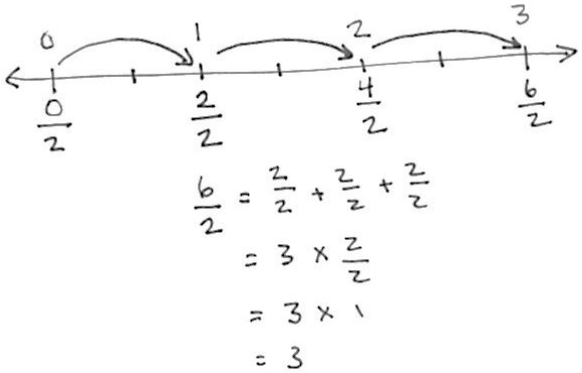
T: Talk to your partner. Express this as a multiplication equation and as an addition equation.

S: $\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=\left(2\times\frac{3}{8}\right)+\frac{1}{8}=\frac{7}{8}$

Problem 3

$$\frac{6}{2}=\frac{2}{2}+\frac{2}{2}+\frac{2}{2}=1+1+1=3$$

6 halves = 3 x 2 halves = 3 ones = 3



T: On a number line, mark the end points as 0 halves and 6 halves below the number line. Estimate to make 6 parts of equal length. This time only label 2 halves.

T: (After students work.) Record the whole number equivalents above the line.

T: Represent 3 x 2 halves on your number line.

T: (After students have worked) The answer is?

S: 6 halves or 3.

T: 3. What is the unit?

S: 3 ones.

T: Talk to your partner. Express this as an addition equation and as an multiplication equation.

S: $\frac{6}{2}=\frac{2}{2}+\frac{2}{2}+\frac{2}{2}=3\times\frac{2}{2}=3$

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Problem 4

$$\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1\frac{3}{5}$$

8 fifths = 5 fifths + 3 fifths = 1 and 3 fifths

T: Use a number line. Mark the end points as 0 fifths and 10 fifths below it. Estimate and give a value to the halfway point.

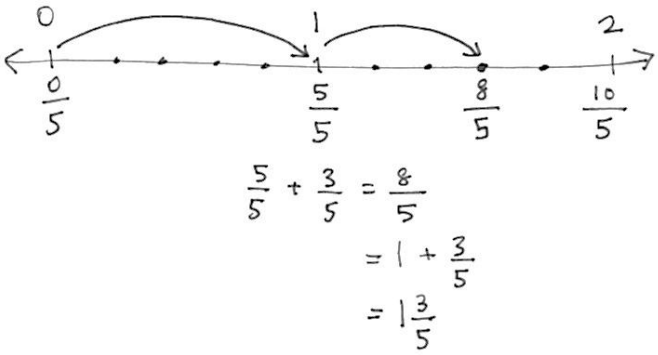
T: What will be the value of the halfway point?

S: 5 fifths.

T: Make 10 parts of equal length from 0 fifths to 10 fifths.

T: (After students work.) Record the whole number equivalents above the line.

T: (After students work.) Label 8 fifths on your number line.



T: Show 8 fifths as the sum of 5 fifths and 3 fifths on your number line.

S: (After students work.)

T: Talk to your partner. Express this as an addition equation in two ways: as the sum of fifths and as the sum of a whole number and fifths.

T: (After students work.) Another way of expressing 1 plus 3 fifths is?

S: 1 and 3 fifths.

S: $\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 3 \times \frac{2}{2} = 3$

T: 8 fifths is between what 2 whole numbers?

S: 1 and 2.

Problem 5:

$$\frac{7}{3} = \frac{6}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$$

7 thirds = 6 thirds + 1 third = 2 and 1 third.

T: Use a number line. Mark the end points as 0 thirds and 9 thirds below the number line. Divide the whole length into three equal smaller lengths and mark their values using thirds. Work with a partner.

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T: (After students work). What are the values of those points?

S: 3 thirds and 6 thirds.

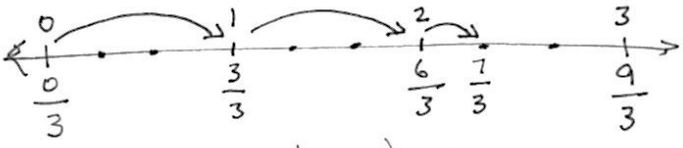
T: Mark the whole number equivalents above the line.

T: (After students work.) Divide each of those whole number lengths into three smaller lengths. Mark the number 7 thirds.

T: (After students work.) Show 7 thirds as two units of 3 thirds and one more third on your number line and in an equation. Work together if you get stuck.

T: (After working and dialogue) 7 thirds is between what two whole numbers?

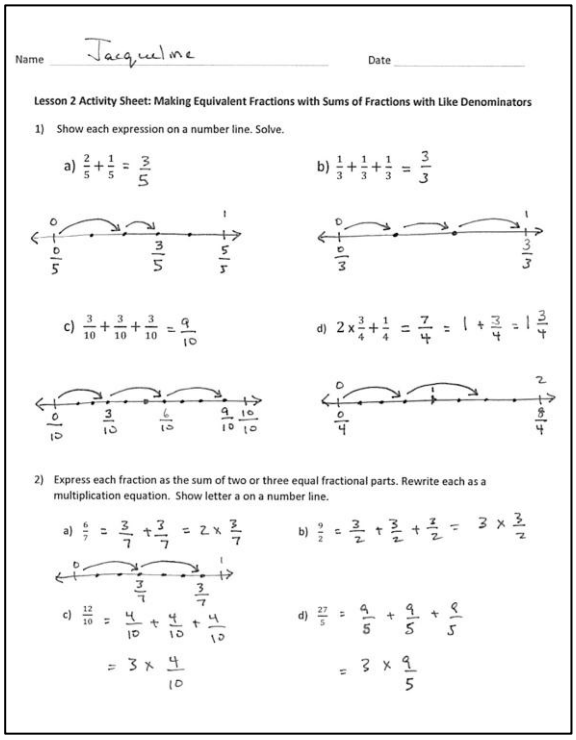
S: 2 and 3.



$$\frac{7}{3} = \left(2 \times \frac{3}{3}\right) + \frac{1}{3}$$
$$= \frac{6}{3} + \frac{1}{3}$$
$$= 2 + \frac{1}{3}$$
$$= 2\frac{1}{3}$$

Activity Worksheet (10 minutes)

Materials: (S) Worksheet



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Lesson 2 Activity Sheet: Making Equivalent Fractions with Sums of Fractions with Like Denominators

1) Show each expression on a number line. Solve.

a) $\frac{2}{5}+\frac{1}{5} = \frac{3}{5}$

b) $\frac{1}{3}+\frac{1}{3}+\frac{1}{3} = \frac{3}{3}$

c) $\frac{3}{10}+\frac{3}{10}+\frac{3}{10} = \frac{9}{10}$

d) $2 \times \frac{3}{4}+\frac{1}{4} = \frac{7}{4} = 1 + \frac{3}{4} = 1\frac{3}{4}$

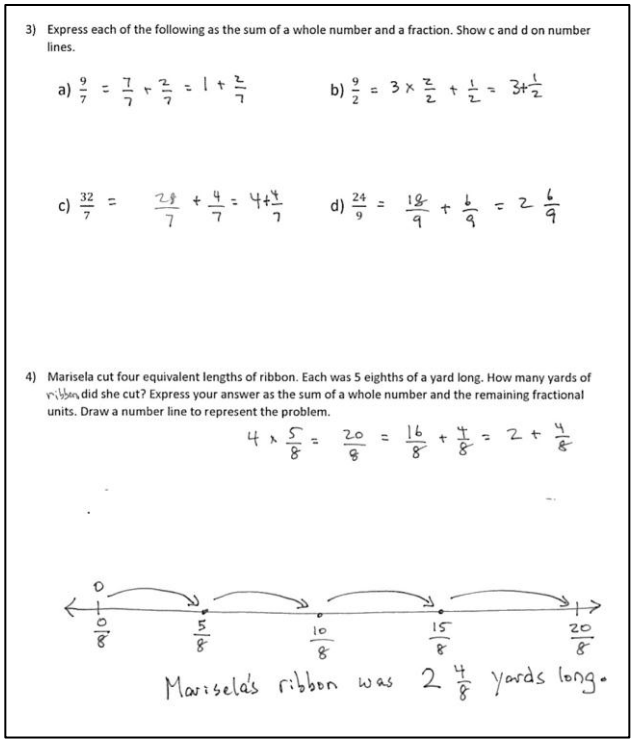
2) Express each fraction as the sum of two or three equal fractional parts. Rewrite each as a multiplication equation. Show letter a on a number line.

a) $\frac{6}{7} = \frac{3}{7} + \frac{3}{7} = 2 \times \frac{3}{7}$

b) $\frac{9}{2} = \frac{3}{2} + \frac{3}{2} + \frac{3}{2} = 3 \times \frac{3}{2}$

c) $\frac{12}{10} = \frac{4}{10} + \frac{4}{10} + \frac{4}{10} = 3 \times \frac{4}{10}$

d) $\frac{27}{5} = \frac{9}{5} + \frac{9}{5} + \frac{9}{5} = 3 \times \frac{9}{5}$



3) Express each of the following as the sum of a whole number and a fraction. Show c and d on number lines.

a) $\frac{9}{7} = \frac{7}{7} + \frac{2}{7} = 1 + \frac{2}{7}$

b) $\frac{9}{2} = 3 \times \frac{2}{2} + \frac{1}{2} = 3 + \frac{1}{2}$

c) $\frac{32}{7} = \frac{28}{7} + \frac{4}{7} = 4 + \frac{4}{7}$

d) $\frac{24}{9} = \frac{18}{9} + \frac{6}{9} = 2\frac{6}{9}$

4) Marisela cut four equivalent lengths of ribbon. Each was 5 eighths of a yard long. How many yards of ribbon did she cut? Express your answer as the sum of a whole number and the remaining fractional units. Draw a number line to represent the problem.

$4 \times \frac{5}{8} = \frac{20}{8} = \frac{16}{8} + \frac{4}{8} = 2 + \frac{4}{8}$

Marisela's ribbon was $2\frac{4}{8}$ yards long.

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Student Debrief (10 minutes)

Materials: (S) Completed Worksheet

T: Come to the debrief and bring your worksheet. Compare your work to your neighbor's. On which problems do you have different answers? Discuss your differences. Both may be correct.

T: (After about 3 minutes.)

T: What is a way to express 3/7 as a sum?

S: 1 sevenths + 1 seventh + 1 seventh.

T: Another way?

S: 2 sevenths + 1 seventh.

T: These are equivalent forms of 3 sevenths.

T: On your worksheet find and talk to your partner about different equivalent forms of your numbers.

S: "6 sevenths could be expressed as 3 sevenths + 3 sevenths or as 3 times 2 sevenths." → "9 sevenths can be expressed as 1 + 2 sevenths." → "7 fourths can be expressed as 2 times 3 fourths + 1 fourth." → "1 and 3 fourths can be expressed as 7 fourths." → "32 sevenths can be expressed as 28 sevenths + 4 sevenths or 4 and 4 sevenths."

T: I'm hearing you express these numbers in many equivalent forms. Why do you think I chose to use the tool of the number line in this lesson? Talk this over with your partner. If you were the teacher of this lesson, why might you use the number line?

T: (After students discuss.) When we were studying decimal place value, we saw that 9 tenths + 3 tenths is equal to 12 tenths or 1 + 2 tenths or 1 and 2 tenths.

T: Once more, please review the solution and number line you made for question 4 about Marisela's ribbon. Discuss the equivalence of 20 eighths and 2 and 4 eighths as it relates to the number line.

T: (After students talk.) Discuss the relationship of the equivalence of these sums.

9 tenths + 3 tenths = 12 tenths = 1 + 2 tenths = 1 $\frac{1}{10}$.

9 elevenths + 3 elevenths = 12 elevenths = 1 + 1 eleventh = 1 $\frac{1}{11}$.

T: (After students talk.) Yes, our place value system is another example of equivalence.

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A # Correct _____

Find the missing numerator or denominator.

1	$\frac{1}{2} = \frac{\ }{4}$		23	$\frac{1}{3} = \frac{\ }{12}$	
2	$\frac{1}{5} = \frac{2}{\ }$		24	$\frac{2}{3} = \frac{\ }{12}$	
3	$\frac{2}{5} = \frac{\ }{10}$		25	$\frac{8}{12} = \frac{\ }{3}$	
4	$\frac{3}{5} = \frac{\ }{10}$		26	$\frac{12}{16} = \frac{3}{\ }$	
5	$\frac{4}{5} = \frac{\ }{10}$		27	$\frac{3}{5} = \frac{\ }{25}$	
6	$\frac{1}{3} = \frac{2}{\ }$		28	$\frac{4}{5} = \frac{28}{\ }$	
7	$\frac{2}{3} = \frac{\ }{6}$		29	$\frac{18}{24} = \frac{3}{\ }$	
8	$\frac{1}{3} = \frac{3}{\ }$		30	$\frac{24}{30} = \frac{\ }{5}$	
9	$\frac{2}{3} = \frac{\ }{9}$		31	$\frac{5}{6} = \frac{35}{\ }$	
10	$\frac{1}{4} = \frac{\ }{8}$		32	$\frac{56}{63} = \frac{\ }{9}$	
11	$\frac{3}{4} = \frac{\ }{8}$		33	$\frac{64}{72} = \frac{8}{\ }$	
12	$\frac{1}{4} = \frac{3}{\ }$		34	$\frac{5}{8} = \frac{\ }{64}$	
13	$\frac{3}{4} = \frac{9}{\ }$		35	$\frac{5}{6} = \frac{45}{\ }$	
14	$\frac{2}{4} = \frac{\ }{2}$		36	$\frac{45}{81} = \frac{\ }{9}$	
15	$\frac{2}{6} = \frac{1}{\ }$		37	$\frac{6}{7} = \frac{48}{\ }$	
16	$\frac{2}{10} = \frac{1}{\ }$		38	$\frac{36}{81} = \frac{\ }{9}$	
17	$\frac{4}{10} = \frac{\ }{5}$		39	$\frac{8}{56} = \frac{1}{\ }$	
18	$\frac{8}{10} = \frac{\ }{5}$		40	$\frac{35}{63} = \frac{5}{\ }$	
19	$\frac{3}{9} = \frac{\ }{3}$		41	$\frac{1}{6} = \frac{12}{\ }$	
20	$\frac{6}{9} = \frac{\ }{3}$		42	$\frac{3}{7} = \frac{36}{\ }$	
21	$\frac{3}{12} = \frac{1}{\ }$		43	$\frac{48}{60} = \frac{4}{\ }$	
22	$\frac{9}{12} = \frac{\ }{4}$		44	$\frac{72}{84} = \frac{\ }{7}$	

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B Improvement _____ # Correct _____

Find the missing numerator or denominator.

1	$\frac{1}{5} = \frac{2}{\ }$		23	$\frac{1}{3} = \frac{4}{\ }$	
2	$\frac{2}{5} = \frac{\ }{10}$		24	$\frac{2}{3} = \frac{8}{\ }$	
3	$\frac{3}{5} = \frac{\ }{10}$		25	$\frac{8}{12} = \frac{2}{\ }$	
4	$\frac{4}{5} = \frac{\ }{10}$		26	$\frac{12}{16} = \frac{\ }{4}$	
5	$\frac{1}{2} = \frac{2}{\ }$		27	$\frac{3}{5} = \frac{15}{\ }$	
6	$\frac{1}{3} = \frac{\ }{6}$		28	$\frac{4}{5} = \frac{\ }{35}$	
7	$\frac{2}{3} = \frac{4}{\ }$		29	$\frac{18}{24} = \frac{\ }{4}$	
8	$\frac{1}{3} = \frac{\ }{9}$		30	$\frac{24}{30} = \frac{4}{\ }$	
9	$\frac{2}{3} = \frac{6}{\ }$		31	$\frac{5}{6} = \frac{\ }{42}$	
10	$\frac{1}{4} = \frac{2}{\ }$		32	$\frac{56}{63} = \frac{8}{\ }$	
11	$\frac{3}{4} = \frac{6}{\ }$		33	$\frac{64}{72} = \frac{\ }{9}$	
12	$\frac{1}{4} = \frac{\ }{12}$		34	$\frac{5}{8} = \frac{40}{\ }$	
13	$\frac{3}{4} = \frac{\ }{12}$		35	$\frac{5}{6} = \frac{\ }{54}$	
14	$\frac{2}{4} = \frac{1}{\ }$		36	$\frac{45}{81} = \frac{5}{\ }$	
15	$\frac{2}{6} = \frac{\ }{3}$		37	$\frac{6}{7} = \frac{\ }{56}$	
16	$\frac{2}{10} = \frac{\ }{5}$		38	$\frac{36}{81} = \frac{4}{\ }$	
17	$\frac{4}{10} = \frac{2}{\ }$		39	$\frac{8}{56} = \frac{\ }{7}$	
18	$\frac{8}{10} = \frac{4}{\ }$		40	$\frac{35}{63} = \frac{\ }{9}$	
19	$\frac{3}{9} = \frac{1}{\ }$		41	$\frac{1}{6} = \frac{\ }{72}$	
20	$\frac{6}{9} = \frac{2}{\ }$		42	$\frac{3}{7} = \frac{\ }{84}$	
21	$\frac{1}{4} = \frac{\ }{12}$		43	$\frac{48}{60} = \frac{\ }{5}$	
22	$\frac{9}{12} = \frac{3}{\ }$		44	$\frac{72}{84} = \frac{6}{\ }$	

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Name _____ Date _____

1) Show each expression on a number line. Solve.

a) $\frac{2}{5} + \frac{1}{5}$

b) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

c) $\frac{3}{10} + \frac{3}{10} + \frac{3}{10}$

d) $2 \times \frac{3}{4} + \frac{1}{4}$

2) Express each fraction as the sum of two or three equal fractional parts. Rewrite each as a multiplication equation. Show letter a) on a number line.

a) $\frac{6}{7}$

b) $\frac{9}{2}$

c) $\frac{12}{10}$

d) $\frac{27}{5}$

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3) Express each of the following as the sum of a whole number and a fraction. Show c) and d) on number lines.

a) $\frac{9}{7}$

b) $\frac{9}{2}$

c) $\frac{32}{7}$

d) $\frac{24}{9}$

4) Marisela cut four equivalent lengths of ribbon. Each was 5 eighths of a yard long. How many yards of fabric did she cut? Express your answer as the sum of a whole number and the remaining fractional units. Draw a number line to represent the problem.

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1) Show each expression on a number line. Solve.

a) $\frac{5}{5} + \frac{2}{5}$

b) $\frac{6}{3} + \frac{2}{3}$

2) Express each fraction as the sum of two or three equal fractional parts. Rewrite each as a multiplication equation. Show letter b) on a number line.

a) $\frac{6}{9}$

b) $\frac{15}{4}$

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1) Show each expression on a number line. Solve.

a) $\frac{4}{9}+\frac{1}{9}$

b) $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

c) $\frac{2}{7}+\frac{2}{7}+\frac{2}{7}$

d) $2 \times \frac{3}{5}+\frac{1}{5}$

2) Express each fraction as the sum of two or three equal fractional parts. Rewrite each as a multiplication equation. Show letter a on a number line.

a) $\frac{6}{11}$

b) $\frac{9}{4}$

c) $\frac{12}{8}$

d) $\frac{27}{10}$

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3) Express each of the following as the sum of a whole number and a fraction. Show c) and d) on number lines.

a) $\frac{9}{5}$

b) $\frac{7}{2}$

c) $\frac{25}{7}$

d) $\frac{21}{9}$

4) Natalie sawed five boards of equal length to make a stool. Each was 9 tenths of a meter long. How many meters of board did she saw? Express your answer as the sum of a whole number and the remaining fractional units. Draw a number line to represent the problem.

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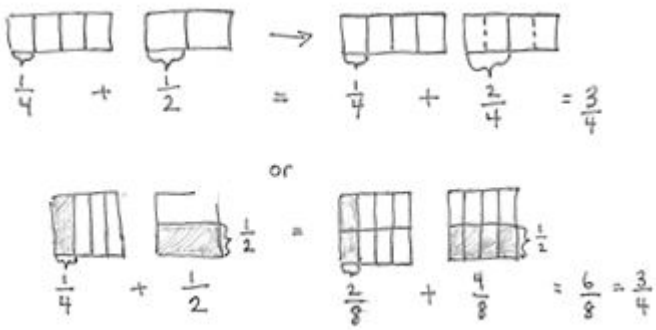
Topic B:

Fraction Addition and Subtraction: Making Like Units Pictorially

5.NF.1, 5.NF.2

Focus Standard:	5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
	5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
Instructional Days:	5	
Coherence -Links from:	G4–M5	Order and Operations with Fractions
-Links to:	G5–M4	Extensions and Applications of Multiplication and Division of Fractions
	G6–M2	The Number System: Rational Numbers and Decimals

In Module 3, students use the familiar rectangular fraction model to add and subtract fractions with unlike denominators. In Grade 4, they also use this model when adding fractions where one denominator is the multiple of the other.



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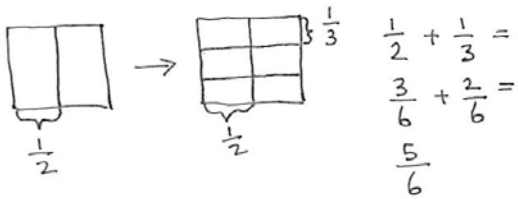
Now in Grade 5 they make like units with all addends or both minuend and subtrahend. First, they draw a wide rectangle and partition it with vertical lines as they would a bar diagram (also known as tape diagram), representing the first fraction with a bracket. They then partition the same rectangle again to show the second fraction.



This method requires that they see 3 units as equal to 1 half and 2 units as equal to one third. If your students need extra work with the rectangular fraction model, carefully study the lessons in Grade 4 Module 5 for scaffolding ideas. Students use the same systematic drawing for addition and subtraction. Omitting "shading" in Grade 5 encourages analysis about the manipulation and conversion of the units. In this manner, students are prepared to generalize with understanding to multiply the numerator and denominator by the same number. The topic closes with a lesson devoted to solving two-step word problems.

CONCEPT CHART	A Teaching Sequence Towards Mastery of Adding and Subtracting Fractions with Unlike Denominators
Concept 1:	**Add Fractions with Unlike Units Using the Strategy of Creating Equivalent Fractions (Lesson 3)**
Concept 2:	**Add Fractions with Sums Between One and Two (Lesson 4)**
Concept 3:	**Subtract Fractions with Unlike Units Using the Strategy of Creating Equivalent Fractions (Lesson 5)**
Concept 4:	**Subtract Fractions From Numbers Between One and Two (Lesson 6)**
Concept 5:	**Solve Two-Step Word Problems (Lesson 7)**

Mathematical Practices Brought to Life

In Lesson 3, students "Look for and Make Use of Structure" (MP.7) as they reason in the debrief about fractions in relationship to each other, to one whole, and to one half. In Lesson 7, students "Construct viable arguments and critique the reasoning of others" as they discuss the best representation for modeling a word problem while analyzing two different solution strategies.

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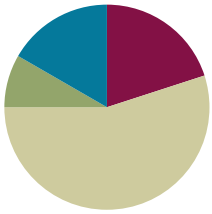
Lesson 3: Add Fractions with Unlike Units Using the Strategy of Creating Equivalent Fractions

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problems	(5 minutes)
Concept Development	(33 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Equivalent Fractions Sprint **5.NF.1** (10 minutes)
- Adding Like Fractions **5.NF.1** (1 minute)
- Fractions as Division **5.NF.3** (1 minute)

Sprint (10 minutes)

Materials: (S) Equivalent Fractions Sprint (The directions for administration of Sprints are in the **G5-M3 Fluency Progressions.**)

Adding Like Fractions (1 minute)

T: Let's add fractions mentally. Say answers as whole numbers when possible.

T: One third plus one third equals?

S: Two thirds.

T: One fourth plus one fourth equals?

S: Two fourths.

T: 1/5 + 2/5 =

S: 3/5.

T: 3/7 + 4/7 =

S: 1.

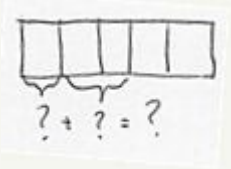
T: 1/4 + 1/3 + 3/4 + 2/3 =

S: 2.

Continue and adjust to meet student needs. Use a variety of fraction combinations.

NOTES ON SCAFFOLDING ELLS

Rather than name the fraction, draw it and ask students to write the corresponding equation on personal boards. Use brackets to indicate the addends.



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Fractions as Division (1 minute)

Materials: (S) Personal white boards

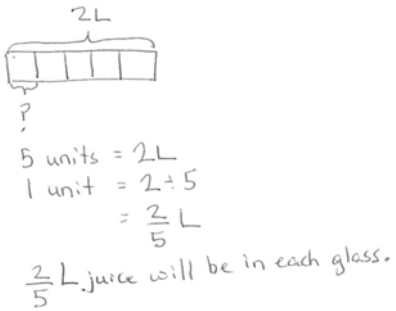
T: When I show a fraction, you write it as a division statement.
T: (Write) 3/4
S: (Write) 3 ÷ 4 = 3/4.
T: (Write) 5/2
S: (Write) 5 ÷ 2 = 5/2.

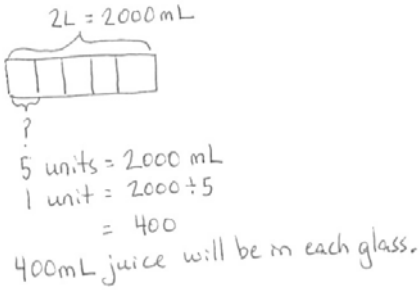
Continue with fractions that are less than and greater than one. Possible sequence: 1/3, 7/4, 5/8, 9/5, 3/10, 13/6.

Application Problems (5 minutes)

Alex squeezed 2 liters of juice for breakfast. If he pours the juice equally into 5 glasses, how many liters of juice will be in each glass? (Bonus: How many milliliters are in each glass?)

T: Let's read the problem together.
S: (Students read chorally.)
T: What is our whole?
S: 2 liters.
T: How many parts are we breaking 2 liters into?
S: 5.
T: Say your division sentence.
S: 2 liters divided by 5 equals 2/5 liter.
T: Is that less or more than one whole liter? How do you know? Tell your partner.
S: "Less than a whole because 5 ÷ 5 is 1. 2 is less than 5 so you are definitely going to get less than 1." → "I agree because if you share 2 things with 5 people, each one is going to get a part. There isn't enough for each person to get one whole." → "Less than a whole because the numerator is less than the denominator."
T: Was anyone able to do the bonus question? How many milliliters are in 2 liters?
S: 2000.
T: What is 2000 divided by 5?
S: 400.





NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

Students performing above grade level enjoy the challenge of a bonus problem. If time permits have one of the students draw the bonus problem model on the board and share his/her solution with the class.

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T: Say a sentence for how many mL are in each glass.

S: 400 mL of juice will be in each glass.

Concept Development (33 minutes)

Materials: (T) White board (S) Personal white boards, enough 4 1/2" x 4 1/2" papers for each student to have at least 2 depending on how you decide to do the "folding" prior to drawing the rectangular array model

T: Let's think back on what we learned about adding in third grade. What is 3 adults plus 4 adults?

T: (Write) 3 adults + 4 adults

S: 7 adults.

T: We can add 1 fourth plus 1 fourth because the units are the same.

1 fourth + 1 fourth = 2 fourths

$$\frac{1}{4}+\frac{1}{4}=$$

T: What is 5 children plus 8 adults? (Write "5 children + 8 adults")

S: We can't add children and adults.

T: Why is that? Talk to your partner about that.

S: (Students share.)

T: I heard Michael tell his partner that children and adults are not the same unit. We would need to have like units before we added. What do children and adults have in common?

S: They are people.

T: Let's add people, not children and adults. Say the addition sentence with people.

S: 5 people + 8 people = 13 people.

T: 5 children + 8 adults = 5 people + 8 people = 13 people

T: We could also add 5 ones plus 8 ones, which equals?

S: 13 ones.

Problem 1

T: Can I add 1 half plus 1 fourth? Discuss with your partner.

$$\frac{1}{2}+\frac{1}{4}=$$

T: (Circulate and listen for clear reasoning.)

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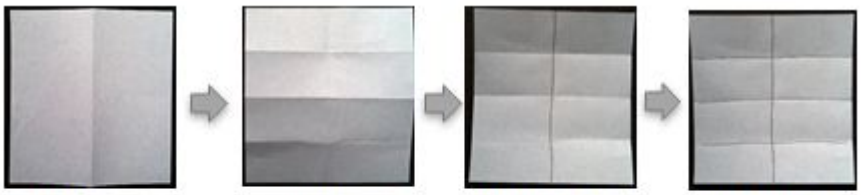
T: Pedro, could you share your thoughts?

S: I cannot add 1 half plus 1 fourth until the units are the same. We need to find like units.

T: Let's first do that by folding paper. (Lead students through the process of folding illustrated below.)



Fold 4 1/2" x 4 1/2" paper vertically in half. Clarify the 2 units.

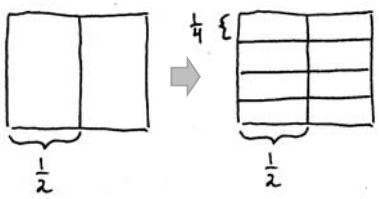
Fold the same paper horizontally in fourths. Clarify the 4 units.

Draw a line at the fold of the half. Clarify that 1/2 is also 4/8.

Draw a line at the folds of the fourths. Clarify that 1/4 is also 2/8.

T: Now let's do the same process by drawing.

T: (Draw a rectangle model for students.) When I make 1 whole into smaller units of 1/2 each, how many units will I have?



S: 2 units.

T: (Partition the rectangle into 2 units.) One half tells me to select how many of the 2 units?

S: One.

T: Let's label our unit with . Now we will partition this same /rectangle horizontally into 4 rows, or 4 units, to show fourths.

T: One fourth tells me to select how many of the 4 units?

S: One.

T: We bracket the first row to show 1/4 of the rectangle. How many total units does this rectangle now have?

S: 8.

T: What is the fractional value of one unit?

S: 1 eighth.

T: 1 eighth is our like unit. We have converted 1/2 into eighths. How many eighths are equal to 1 half? (Point to the 4 boxes bracketed by 1/2.)

S: 4 eighths.

T: How many eighths are equal to 1/4? (Point to the row bracketed as 1/4.)

S: 2 eighths.

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL

Folding paper is a concrete strategy that helps build conceptual understanding. This helps ease the hardest part of using a rectangular fraction model: recognizing the original fractions once the horizontal lines are drawn. Help students see ¼ = 2/8 and 1/2 = 4/8 by pointing and showing:

1/4 = 1/8 + 1/8.

1/2 = 1/8 + 1/8 + 1/8 + 1/8.

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T: Say the addition sentence now using eighths as our common denominator.

S: 4 eighths + 2 eighths = 6 eighths.

T: We can make larger units within 6/8. Tell your partner how you might do that.

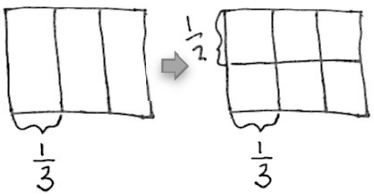
S: "Two can be divided into 6 and 8. 6 ÷ 2 = 3 and 8 ÷ 2 = 4. 3/4." → "We can make larger units of 2 each. 3 twos out of 4 twos. That's 3 out of 4 or 3 fourths." → "6/8 is partitioned into 6 out of 8 smaller units. It can be made into 3 out of 4 larger, equal pieces by grouping in 2's."

$$\frac{1}{2}+\frac{1}{4}=\frac{4}{8}+\frac{2}{8}=\frac{6}{8}=\frac{3}{4}$$

1 half + 1 fourth = 4 eighths + 2 eighths = 6 eighths = 3 fourths

Problem 2

$$\frac{1}{3}+\frac{1}{2}$$



In problem 2 you can have students fold a paper again to transition into drawing, or start directly with drawing. This is a simple problem involving two unit fractions, like problem 1. The primary purpose is to reinforce understanding of what is happening to the units within a very simple context. Problem 3 moves on to address a unit fraction plus a non-unit fraction.

T: Do our units get bigger or smaller when we create like units? Talk to your partner.

S: "There are more units." → "The units get smaller because it is the same amount of space but more parts." → "The units are definitely getting smaller. There are more, yes. But, they are smaller." → "It's like the 5 units double but not in size, in number." → "1 unit will become 2 units." → "Or we can also think of it as 1 unit will become 5 units. That's what is happening to the half."

T: Let's draw a diagram to help solve the problem and see if you are right.

T: Did the half become 3 smaller units and each third become 2 smaller units?

S: Yes!

T: Tell me the addition sentence.

S: 2 sixths + 3 sixths = 5 sixths.

$$\frac{1}{3}-\frac{1}{2}=\frac{2}{6}-\frac{3}{6}=\frac{3}{6}$$

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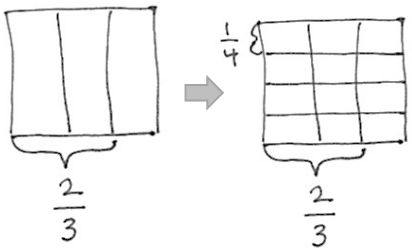
Problem 3

$$\frac{2}{3}+\frac{1}{4}=$$



T: When we partition a rectangle into thirds, how many units will we have in all? (Draw and partition as you would a bar diagram.)

S: 3.

T: (Partition thirds vertically.) How many of those units are we selecting?

S: 2.

T: (Bracket 2 thirds.) To show 1 fourth, how many units will we draw?

S: 4.

T: (Partition fourths horizontally.)

T: How many total units does this rectangle now have?

S: 12.

T: What is the fractional value of 1 unit?

S: 1 twelfth.

T: How many twelfths are equal to 2 thirds? (Motion up and down to clarify the 2 columns bracketed equivalent to 2/3).

S: 8 twelfths.

T: How many twelfths are equal to 1/4? (Motion back and forth to clarify out the row equivalent to 1/4.)

S: 3 twelfths.

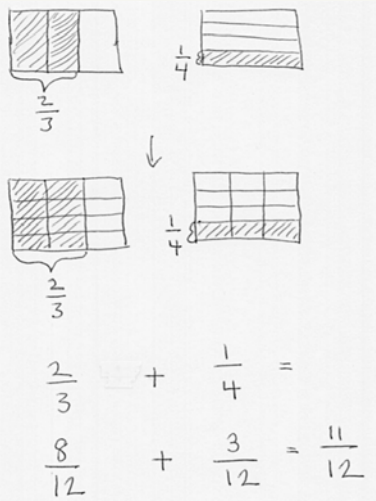
T: Say the addition sentence now using twelfths as our like denominator.

S: 8 twelfths plus 3 twelfths equals 11 twelfths.

$$\frac{2}{3}+\frac{1}{4}=\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$$

NOTES ON SCAFFOLDING MODEL DRAWING FOR STUDENTS BELOW GRADE LEVEL:

Many students may find recognizing the second fraction to be challenging. For example, most students are very comfortable with a rectangle subdivided with vertical lines into equal parts and can tell you the value of those parts. However, when drawing in the second addend, the "cross-hatching" can be disorienting. It may be wise to back up to the fourth grade strategy of drawing two separate rectangles, one for each addend, and shading the parts.



T: Read with me. (2 thirds + 1 fourth = 8 twelfths + 3 twelfths = 11 twelfths.)

T: With your partner, review the process we used to solve 2/3 +1/4 step by step. Partner A goes first, then partner B. Use your drawing to help you.

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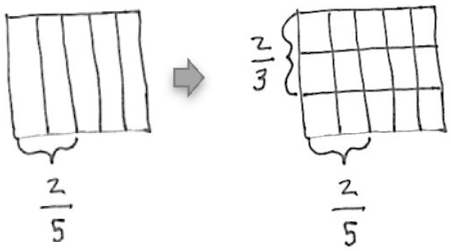
Problem 4

$$\frac{2}{5}+\frac{2}{3}$$

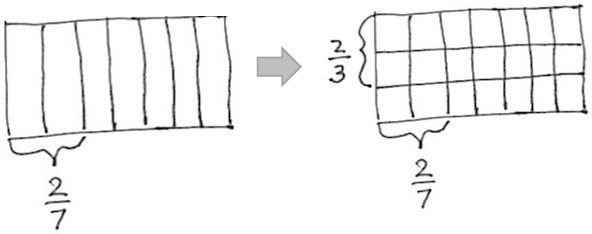


This problem adds the complexity of finding the sum of two non-unit fractions, both with the numerator of 2. Working with fractions with common numerators invites healthy reflection on the size of fifths as compared to thirds. Students can reason that while there are the same number of units (2), thirds are larger than fifths because the whole is broken into 3 parts instead of 5 parts. Therefore there is more in each part. In addition, it can be reasoned that 2 thirds is larger than 2 fifths because when fifteenths are used for both, the number of units in 2 thirds (10) is more than the number used in 2 fifths (6). This problem also presents an opportunity to remind students about the importance of attending to precision (MP.6). When comparing fractions, care is taken to talk about the same whole amount as demonstrated by the rectangle. Such attention to precision also leads students to understand that 2 thirds cup is not larger than 2/5 gallon.

Problem 5

$$\frac{2}{7}+\frac{2}{3}=$$

$$\frac{2}{7}+\frac{2}{3}=\frac{6}{21}+\frac{14}{21}=\frac{20}{21}$$



2 sevenths + 2 thirds = 6 twenty-oneths + 14 twenty-oneths = 20 twenty-oneth

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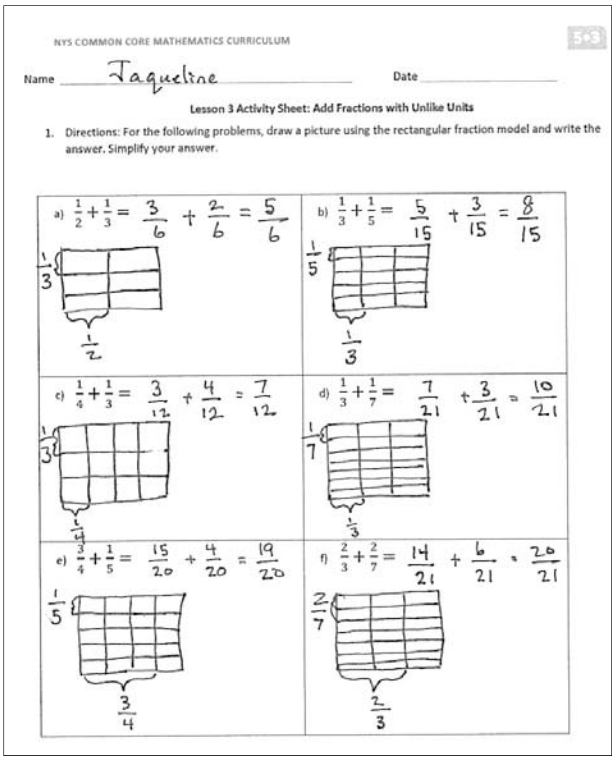
Activity Worksheet

Students complete the activity worksheet.

(Page 1 is pictured below. Page 2 includes word problems.)



NYS COMMON CORE MATHEMATICS CURRICULUM 5•3

Name Jaqueline Date

Lesson 3 Activity Sheet: Add Fractions with Unlike Units

1. Directions: For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

a) $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

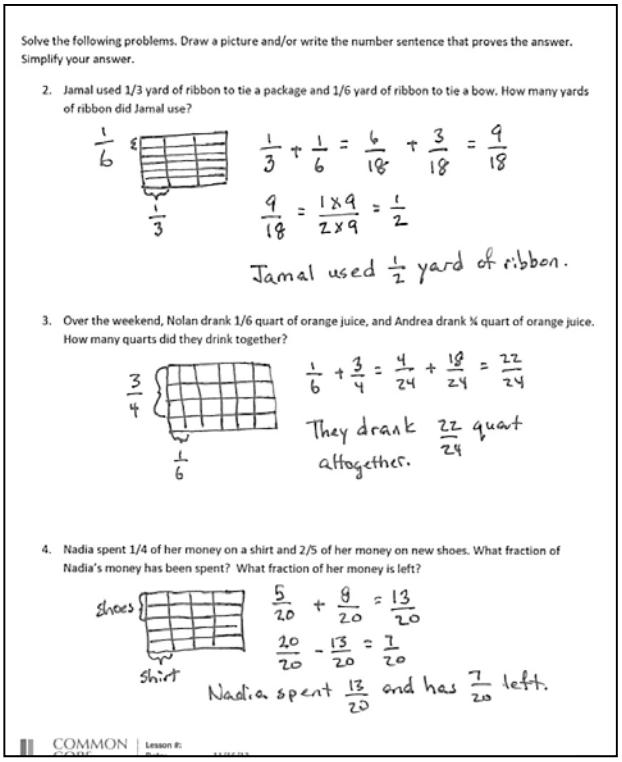
b) $\frac{1}{3}+\frac{1}{5}=\frac{5}{15}+\frac{3}{15}=\frac{8}{15}$

c) $\frac{1}{4}+\frac{1}{3}=\frac{3}{12}+\frac{4}{12}=\frac{7}{12}$

d) $\frac{1}{3}+\frac{1}{7}=\frac{7}{21}+\frac{3}{21}=\frac{10}{21}$

e) $\frac{3}{4}+\frac{1}{5}=\frac{15}{20}+\frac{4}{20}=\frac{19}{20}$

f) $\frac{2}{3}+\frac{2}{7}=\frac{14}{21}+\frac{6}{21}=\frac{20}{21}$



Solve the following problems. Draw a picture and/or write the number sentence that proves the answer. Simplify your answer.

2. Jamal used 1/3 yard of ribbon to tie a package and 1/6 yard of ribbon to tie a bow. How many yards of ribbon did Jamal use?

$\frac{1}{3}+\frac{1}{6}=\frac{6}{18}+\frac{3}{18}=\frac{9}{18}$

$\frac{9}{18}=\frac{1\times 9}{2\times 9}=\frac{1}{2}$

Jamal used $\frac{1}{2}$ yard of ribbon.

3. Over the weekend, Nolan drank 1/6 quart of orange juice, and Andrea drank ¾ quart of orange juice. How many quarts did they drink together?

$\frac{1}{6}+\frac{3}{4}=\frac{4}{24}+\frac{18}{24}=\frac{22}{24}$

They drank $\frac{22}{24}$ quart altogether.

4. Nadia spent 1/4 of her money on a shirt and 2/5 of her money on new shoes. What fraction of Nadia's money has been spent? What fraction of her money is left?

$\frac{5}{20}+\frac{8}{20}=\frac{13}{20}$

$\frac{20}{20}-\frac{13}{20}=\frac{7}{20}$

Nadia spent $\frac{13}{20}$ and has $\frac{7}{20}$ left.

Student Debrief (10 minutes)

T: Go over the answers to your problems for 1 minute with a partner. Don't change your work.

S: (Students work together.)

T: Now let's correct together. I will say the addition problem, you say the answer. Problem a) 1 half plus 1 third is?

S: 5 sixths.

Continue. Then give students about 2 minutes to correct their errors as shown below.

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T: Analyze the following problems. How are they related?

(a) and (b)

(a) and (c)

(b) and (d)

(d) and (f)

S: (Allow for student conversation.)

T: Steven noticed something about problems a) and b). Please share.

MP.7

S: The answer to b) is smaller than a) since you are adding only 1/5 to 1/2. Both answers are less than 1 but a) is much closer to 1. b) is really close to 1/2 because 8/16 would be 1/2.

T: Kara, can you share what you noticed about letters d) and f)?

S: I noticed that both problems used thirds and sevenths. But the numerators in d) were 1 and the numerators in f) were 2. Since the numerators doubled, the answer doubled from 10 twenty-oneths to 20 twenty-oneths.

T: I am glad to hear you are able to point out relationships between different problems.

T: Share with your partner about what you learned how to do today.

S: (Students share)

T: (Help students name the objective: We learned how to add fractions that have unlike units using a rectangular fraction model to create like units.)

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A # Correct _____

Find the missing numerator or denominator.

1	$\frac{1}{2} = \frac{\quad}{4}$		23	$\frac{1}{3} = \frac{\quad}{12}$	
2	$\frac{1}{5} = \frac{2}{\quad}$		24	$\frac{2}{3} = \frac{\quad}{12}$	
3	$\frac{2}{5} = \frac{\quad}{10}$		25	$\frac{8}{12} = \frac{\quad}{3}$	
4	$\frac{3}{5} = \frac{\quad}{10}$		26	$\frac{12}{16} = \frac{3}{\quad}$	
5	$\frac{4}{5} = \frac{\quad}{10}$		27.	$\frac{3}{5} = \frac{\quad}{25}$	
6	$\frac{1}{3} = \frac{2}{\quad}$		28	$\frac{4}{5} = \frac{28}{\quad}$	
7	$\frac{2}{3} = \frac{\quad}{6}$		29	$\frac{18}{24} = \frac{3}{\quad}$	
8	$\frac{1}{3} = \frac{3}{\quad}$		30	$\frac{24}{30} = \frac{\quad}{5}$	
9	$\frac{2}{3} = \frac{\quad}{9}$		31	$\frac{5}{6} = \frac{35}{\quad}$	
10	$\frac{1}{4} = \frac{\quad}{8}$		32	$\frac{56}{63} = \frac{\quad}{9}$	
11	$\frac{3}{4} = \frac{\quad}{8}$		33	$\frac{64}{72} = \frac{8}{\quad}$	
12	$\frac{1}{4} = \frac{3}{\quad}$		34	$\frac{5}{8} = \frac{\quad}{64}$	
13	$\frac{3}{4} = \frac{9}{\quad}$		35	$\frac{5}{6} = \frac{45}{\quad}$	
14	$\frac{2}{4} = \frac{\quad}{2}$		36	$\frac{45}{81} = \frac{\quad}{9}$	
15	$\frac{2}{6} = \frac{1}{\quad}$		37	$\frac{6}{7} = \frac{48}{\quad}$	
16	$\frac{2}{10} = \frac{1}{\quad}$		38	$\frac{36}{81} = \frac{\quad}{9}$	
17	$\frac{4}{10} = \frac{\quad}{5}$		39	$\frac{8}{56} = \frac{1}{\quad}$	
18	$\frac{8}{10} = \frac{\quad}{5}$		40	$\frac{35}{63} = \frac{5}{\quad}$	
19	$\frac{3}{9} = \frac{\quad}{3}$		41	$\frac{1}{6} = \frac{12}{\quad}$	
20	$\frac{6}{9} = \frac{\quad}{3}$		42	$\frac{3}{7} = \frac{36}{\quad}$	
21	$\frac{3}{12} = \frac{1}{\quad}$		43	$\frac{48}{60} = \frac{4}{\quad}$	
22	$\frac{9}{12} = \frac{\quad}{4}$		44	$\frac{72}{84} = \frac{\quad}{7}$	

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B Improvement _____ # Correct _____

Find the missing numerator or denominator.

1	$\frac{1}{5} = \frac{2}{_}$		23	$\frac{1}{3} = \frac{4}{_}$	
2	$\frac{2}{5} = \frac{_}{10}$		24	$\frac{2}{3} = \frac{8}{_}$	
3	$\frac{3}{5} = \frac{_}{10}$		25	$\frac{8}{12} = \frac{2}{_}$	
4	$\frac{4}{5} = \frac{_}{10}$		26	$\frac{12}{16} = \frac{_}{4}$	
5	$\frac{1}{2} = \frac{2}{_}$		27	$\frac{3}{5} = \frac{15}{_}$	
6	$\frac{1}{3} = \frac{_}{6}$		28	$\frac{4}{5} = \frac{_}{35}$	
7	$\frac{2}{3} = \frac{4}{_}$		29	$\frac{18}{24} = \frac{_}{4}$	
8	$\frac{1}{3} = \frac{_}{9}$		30	$\frac{24}{30} = \frac{4}{_}$	
9	$\frac{2}{3} = \frac{6}{_}$		31	$\frac{5}{6} = \frac{_}{42}$	
10	$\frac{1}{4} = \frac{2}{_}$		32	$\frac{56}{63} = \frac{8}{_}$	
11	$\frac{3}{4} = \frac{6}{_}$		33	$\frac{64}{72} = \frac{_}{9}$	
12	$\frac{1}{4} = \frac{_}{12}$		34	$\frac{5}{8} = \frac{40}{_}$	
13	$\frac{3}{4} = \frac{_}{12}$		35	$\frac{5}{6} = \frac{_}{54}$	
14	$\frac{2}{4} = \frac{1}{_}$		36	$\frac{45}{81} = \frac{5}{_}$	
15	$\frac{2}{6} = \frac{_}{3}$		37	$\frac{6}{7} = \frac{_}{56}$	
16	$\frac{2}{10} = \frac{_}{5}$		38	$\frac{36}{81} = \frac{4}{_}$	
17	$\frac{4}{10} = \frac{2}{_}$		39	$\frac{8}{56} = \frac{_}{7}$	
18	$\frac{8}{10} = \frac{4}{_}$		40	$\frac{35}{63} = \frac{_}{9}$	
19	$\frac{3}{9} = \frac{1}{_}$		41	$\frac{1}{6} = \frac{_}{72}$	
20	$\frac{6}{9} = \frac{2}{_}$		42	$\frac{3}{7} = \frac{_}{84}$	
21	$\frac{1}{4} = \frac{_}{12}$		43	$\frac{48}{60} = \frac{_}{5}$	
22	$\frac{9}{12} = \frac{3}{_}$		44	$\frac{72}{84} = \frac{6}{_}$	

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Name _____ Date _____

1. For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

a) $\frac{1}{2} + \frac{1}{3} =$

b) $\frac{1}{3} + \frac{1}{5} =$

c) $\frac{1}{4} + \frac{1}{3} =$

d) $\frac{1}{3} + \frac{1}{7} =$

e) $\frac{3}{4} + \frac{1}{5} =$

f) $\frac{2}{3} + \frac{2}{7} =$

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Solve the following problems. Draw a picture and/or write the number sentence that proves the answer. Simplify your answer.

2. Jamal used 1/3 yard of ribbon to tie a package and 1/6 yard of ribbon to tie a bow. How many yards of ribbon did Jamal use?

3. Over the weekend, Nolan drank 1/6 quart of orange juice, and Andrea drank 3/4 quart of orange juice. How many quarts did they drink together?

4. Nadia spent 1/4 of her money on a shirt and 2/5 of her money on new shoes. What fraction of Nadia's money has been spent? What fraction of her money is left?

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Name _____ Date _____

Solve by drawing the rectangular fraction model.

1. $\frac{1}{2} + \frac{1}{5} =$

2. In one hour, Ed used 2/5 of the time to complete his homework and 1/4 of the time to check his email. How much time did he spend completing homework and checking email? Write your answer as a fraction. (Bonus: write the answer in minutes.)

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Name _____ Date _____

1. For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

a) $\frac{1}{4} + \frac{1}{3} =$

b) $\frac{1}{4} + \frac{1}{5} =$

c) $\frac{1}{4} + \frac{1}{6} =$

d) $\frac{1}{5} + \frac{1}{9} =$

e) $\frac{1}{4} + \frac{2}{5} =$

f) $\frac{3}{5} + \frac{3}{7} =$

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Solve the following problems. Draw a picture and/or write the number sentence that proves the answer.

2. Rajesh jogged 3/4 mile, and then walked 1/6 mile to cool down. How far did he travel?

3. Cynthia completed 2/3 of the items on her to-do list in the morning, and finished 1/8 of the items during her lunch break. How much of her to-do list is finished by the end of her lunch break? (Bonus: How much of her to-do list does she still have to do after lunch?)

4. Sam read 2/5 of her book over the weekend, and 1/6 of it on Monday. What fraction of the book has she read? What fraction of the book is left?

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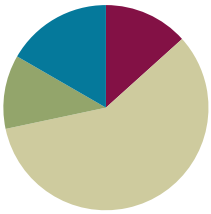
Lesson 4: Add Fractions with Sums Between One and Two

Suggested Lesson Structure

Fluency Practice	(8 minutes)
Application Problems	(7 minutes)
Concept Development	(35 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (8 minutes)

- Adding Fractions to Make One Whole **4.NF.3a** (4 minutes)
- Division as Fractions **5.NF.3** (4 minutes)

Adding Fractions to Make One Whole (4 minutes)

T: I will name a fraction. You say a fraction with the same denominator so that together our fractions add up to 1 whole. For example, if I say 1 third, you say 2 thirds.

1/3 + 2/3 = 3/3 or 1 whole. Say your answer at the signal.

T: 1 fourth? (Signal)

S: 3 fourths.

T: 1 fifth? (Signal)

S: 4 fifths.

T: 2 tenths? (Signal)

S: 8 tenths.

Continue with possible sequence: 1/3, 3/5, 1/2, 5/10, 6/7, 3/8.

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

Depending on your group, consider supporting students visually. You may want to make "fraction cards" that show circles divided into fourths, fifths, tenths, etc. As you name each fraction, flash the corresponding card. ELLs will have a visual support to accompany language, and students performing below grade level can see how many more to make one whole.

Division as Fractions (4 minutes)

Materials: (S) Personal white boards and markers

T: I will say a division sentence. You write it as a fraction. At my signal, show your board and say your fraction. (Write 3 ÷ 2.)

S: (Students show 3/2.) 3 halves.

T: (Write 2 ÷ 3.)

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S: (Students show 2/3.) 2 thirds.

T: 3 ÷ 4.

S: 3 fourths.

T: 6 ÷ 4.

S: 6 fourths.

Continue with possible sequence: 4 ÷ 5, 7 ÷ 2, 10 ÷ 3, 6 ÷ 8.

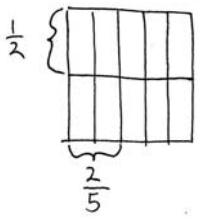
NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

Ask students to simplify fractions, making ones with fraction units when applicable.

Application Problem (7 minutes)

Leslie has 1 liter of milk in her fridge to drink today. She drank 2/5 liter of milk for breakfast and 1/2 liter of milk for dinner. How many liters did Leslie drink during breakfast and dinner?

(Bonus: How much milk does Leslie have left over to go with her dessert, a brownie? Give your answer as a fraction of liters and as a decimal.)



$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

Leslie drank $\frac{9}{10}$ L, or 0.9 L.

$1 - \frac{9}{10} = \frac{1}{10}$

She had $\frac{1}{10}$ L left, or 0.1 L.

T: Let's read the problem together.

S: (Students read chorally.)

T: What is our whole?

S: 1 liter.

T: Tell your partner how you might solve this problem.

S: (Allow for student conversations. Listen closely to select a student to diagram this problem.)

T: I see that Joe has a great model to help us solve this problem. Joe, please come draw your picture for us on the board. (Joe draws. Meanwhile ask students to support his drawing. For example, ask, "Why did Joe separate his rectangle into 5 parts?" Allow for student responses while Joe draws.)

T: Thank you Joe. Let's say an addition sentence that represents this word problem.

S: 2 fifths plus 1 half.

T: Why can't we add these two fractions?

S: "They are different." → "They have different denominators." → "The units are different. We must find a like unit between fifths and halves."→"We can use equal fractions to add them – the fractions will look different, but they will still be the same amount."

T: Joe found like units from his drawing. How many units are inside his rectangle?

S: 10.

NOTES ON SCAFFOLDING ELLS:

It can be helpful to ELLs to have others provide oral language to describe the models they draw. If appropriate, select an ELL to make the drawing for the class to maximize this benefit.

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T: That means we will use 10 as our denominator to solve this problem. Say your addition sentence now using tenths.

S: 4 tenths plus 5 tenths equals 9 tenths.

T: Good. Please say a sentence about how much milk Leslie drank for breakfast and dinner to your partner.

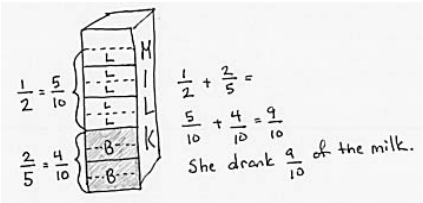
S: Leslie drank 9/10 liter of milk today for breakfast and dinner.

T: With your words, how would you write 9 tenths as a decimal?

S: Zero point nine.

T: Great. Now we need to solve the last bonus question: How much milk will Leslie have available for dessert? Tell your partner how you solved this.

S: (Possible student response) "I know Leslie drank 9/10 L of milk so far. I know she has 1 whole liter, which is also 10 tenths. 9 tenths plus 1 tenth equals 10 tenths, so Leslie has 1 tenth liter of milk for her brownie."



Concept Development (35 minutes)

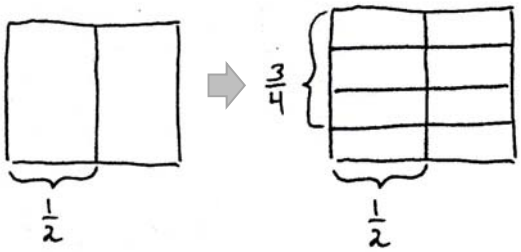
Problem 1

T: (Write or project)

$$\frac{1}{3}+\frac{1}{4}$$

T: When you see this problem, can you estimate the answer? Will it be more or less than 1? Talk with your partner about it.

S: The answer is less than one because 1/3 and 1/4 are both less than 1/2. So if two fractions that are each less than 1/2 are added together, they will add up to a fraction less than one whole.



T: Now look at this problem. Estimate the answer.

T: (Project)

$$\frac{1}{2}+\frac{3}{4}$$

S: (Students discuss)

T: I overheard Camden say the answer will be more than one whole. Can you explain why?

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S 3/4 is more than half and it's added to 1 half, we will have an answer more than 1 whole.

T: What stops us from simply adding?

S: The units do not match.

T: (Draw the rectangular model for students.)

T: How many parts do I need to draw for 1 half?

S: 2.

T: (Partition the rectangle into 2 units.) How many parts should I labelto show one-half?

S: 1.

T: Just like yesterday, we label our picture with 1/2. Now we will partition this same rectangle horizontally into how many rows to show fourths?

S: 4.

T: How many rows do we use to represent 3 fourths?

S: 3.

T: We bracket 3 fourths of this rectangle. What is the fractional value of one unit now?

S: 1 eighth.

T: Eighths will be our like unit. We can convert 1/2 into eighths. How many eighths are in 1 half? (Point to the 4 boxes bracketed by 1/2.)

S: 4 eighths.

T: How many eighths are in 3/4? (Point out the 6 boxes bracketed by 3/4.)

S: 6 eighths.

T: Say the addition sentence now using eighths as our common denominator.

S: 4 eighths plus 6 eighths equals 10 eighths.

$$\frac{1}{2}+\frac{3}{4}=\frac{4}{8}+\frac{6}{8}=\frac{10}{8}$$

1 half + 3 fourths = 4 eighths + 6 eighths = 10 eighths

T: Good. What is unusual about our answer 10 eighths? Tell your partner.

S: The answer has the numerator larger than the denominator. We can write it as a mixed number instead.

T: How many eighths make 1 whole?

S: 8 eighths.

T: 8 eighths plus what equals 10 eighths?

S: 2 eighths.

T: Did anyone use another unit to express your answer?

S: "I used fourths. I know that eighths are half as large as fourths. So 2 eighths is the same amount as 1 fourth."→

T: Can you share your answer with us?

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S: 1 and 1 fourth.

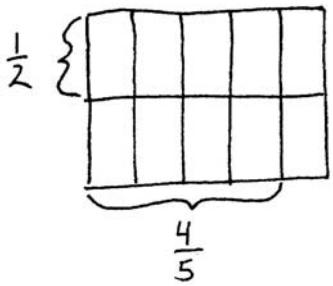
$\frac{8}{8}+\frac{2}{8}=1+\frac{2}{8}=1\frac{2}{8}\,or\,1\frac{1}{4}$ T:

Let's try another.

Problem 2

T: (Project)

$\frac{4}{5}+\frac{1}{2}=$



T: (Give students time to draw a model. Proceed with questioning as before to arrive at the following.)

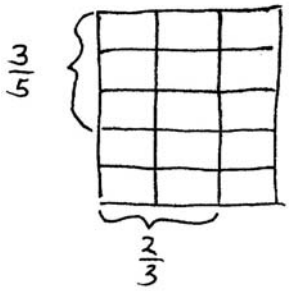
T: Share with your partner how to change 13 tenths into a mixed number.

S: 10 tenths plus 3 tenths equals 13 tenths. 10 tenths makes a whole and 3 tenths is left over. The answer is 1 and 3/10.

Problem 3

T: Let's try another. This time, both numerators are greater than one, so make sure your brackets are clear. Draw your model only. Put your pencil down when you are finished drawing the model. (Allow 1-2 minutes to draw. When all pencils are down, continue.)

$\frac{2}{3}+\frac{3}{5}=$



T: Discuss with your partner what you bracketed and why. I will be circulating to check for your understanding.

Allow 1 minute for students to discuss. Walk around and observe drawings and conversations. Then proceed with questioning as before to arrive at the following.

$$\frac{2}{3}+\frac{3}{5}=\frac{10}{15}+\frac{9}{15}=\frac{19}{15}=\frac{15}{15}+\frac{4}{15}=1+\frac{4}{15}=1\frac{4}{15}$$

T: What can we do to make our answer easier to understand?

S: Write it as a mixed number.

T: Do that now individually. (Allow 1 minute to work.) Compare your work with your partner. What is our final answer of 2 thirds plus 3 fifths?

S: 1 and 4 fifteenths.

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Problem 4

T: For our last problem today I want you to solve it on your own first. Draw a model and use a number sentence. Once everyone is finished we will check your work.

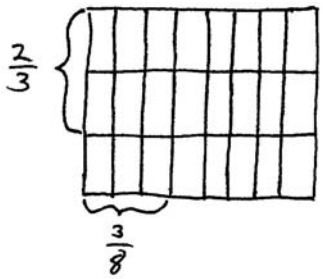
$$\frac{3}{8}+\frac{2}{3}=$$

T: (Allow time for students to work, about 3 minutes.) When finding like units of eighths and thirds, what unit did you find?

S: 24.

T: Say your addition sentence using twenty-fourths.

S: 9 twenty-fourths plus 16 twenty-fourths equals 25 twenty-fourths.



$$\frac{3}{8}+\frac{2}{3}=\frac{9}{24}+\frac{16}{24}=\frac{25}{24}$$

T: Change 25/24 to a mixed number. Jerry can you share how to do that?

S: 25 twenty-fourths = 24 twenty-fourths + 1 twenty-fourth.

$$\frac{25}{24}=\frac{24}{24}+\frac{1}{24}$$

T: What does 24 twenty-fourths make, everyone?

S: 1 whole.

T: Say the final answer to 3 eighths plus 2 thirds.

S: 1 and 1 twenty-fourth.

$$\frac{25}{24}=\frac{24}{24}+\frac{1}{24}=1\frac{1}{24}$$

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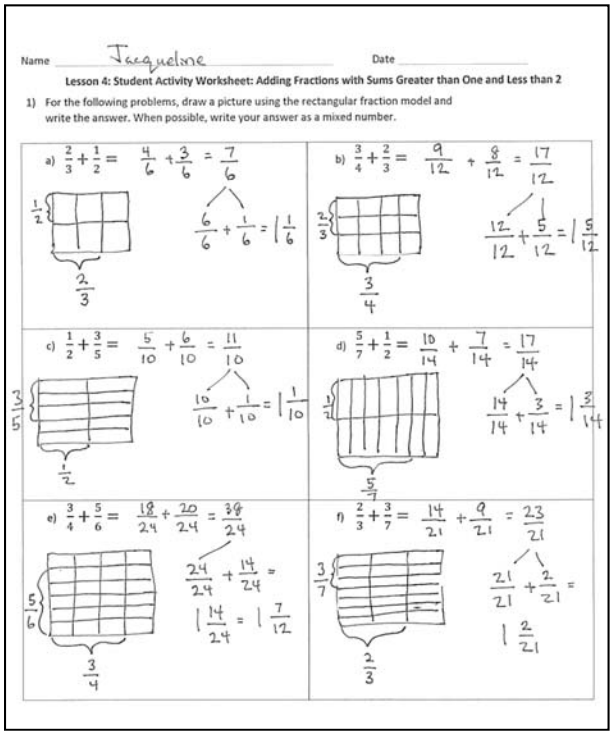
Activity Worksheet (10 minutes)

Materials: (S) Worksheet


Name Jacqueline Date

Lesson 4: Student Activity Worksheet: Adding Fractions with Sums Greater than One and Less than 2

1) For the following problems, draw a picture using the rectangular fraction model and write the answer. When possible, write your answer as a mixed number.

a) $\frac{2}{3}+\frac{1}{2}=\frac{4}{6}+\frac{3}{6}=\frac{7}{6}$ $\frac{6}{6}+\frac{1}{6}=1\frac{1}{6}$

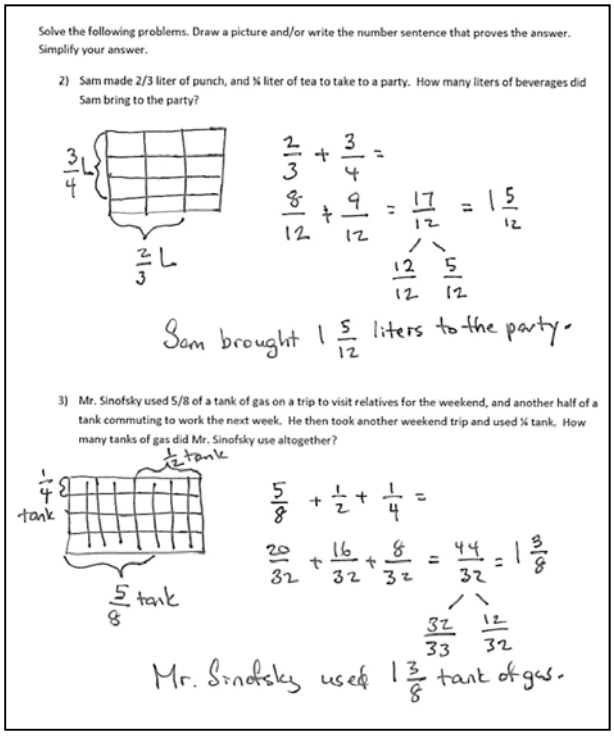
b) $\frac{3}{4}+\frac{2}{3}=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}$ $\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$

c) $\frac{1}{2}+\frac{3}{5}=\frac{5}{10}+\frac{6}{10}=\frac{11}{10}$ $\frac{10}{10}+\frac{1}{10}=1\frac{1}{10}$

d) $\frac{5}{7}+\frac{1}{2}=\frac{10}{14}+\frac{7}{14}=\frac{17}{14}$ $\frac{14}{14}+\frac{3}{14}=1\frac{3}{14}$

e) $\frac{3}{4}+\frac{5}{6}=\frac{18}{24}+\frac{20}{24}=\frac{38}{24}$ $\frac{24}{24}+\frac{14}{24}=1\frac{14}{24}=1\frac{7}{12}$

f) $\frac{2}{3}+\frac{3}{7}=\frac{14}{21}+\frac{9}{21}=\frac{23}{21}$ $\frac{21}{21}+\frac{2}{21}=1\frac{2}{21}$


Solve the following problems. Draw a picture and/or write the number sentence that proves the answer. Simplify your answer.

2) Sam made 2/3 liter of punch, and ¾ liter of tea to take to a party. How many liters of beverages did Sam bring to the party?

$\frac{2}{3}+\frac{3}{4}=$

$\frac{8}{12}+\frac{9}{12}=\frac{17}{12}=1\frac{5}{12}$

Sam brought $1\frac{5}{12}$ liters to the party.

3) Mr. Sinofsky used 5/8 of a tank of gas on a trip to visit relatives for the weekend, and another half of a tank commuting to work the next week. He then took another weekend trip and used ¼ tank. How many tanks of gas did Mr. Sinofsky use altogether?

$\frac{5}{8}+\frac{1}{2}+\frac{1}{4}=$

$\frac{20}{32}+\frac{16}{32}+\frac{8}{32}=\frac{44}{32}=1\frac{3}{8}$

Mr. Sinofsky used $1\frac{3}{8}$ tank of gas.

Student Debrief (10 minutes)

T: Have your worksheet ready to correct. I will say the addition sentence. You say the answer as a mixed number. Problem a) 2 thirds plus 1 half?

S: 1 and 1 sixth.

Continue giving answers to the entire worksheet.

T: I am going to give you 2 minutes to talk to your partner about any relationships you noticed on today's worksheet. Be specific. I will be circulating to hear your conversations.

Allow for students to discuss. Then proceed with a similar conversation to the one below.

T: Ryan, I heard you talking about problems 'a' and 'c'. Can you share what you found with the class?

S: I saw that both problems used 1 half. So I compared the second fraction, and saw that they used 2/3 in problem 'a' and 3/5 in problem 'b'. I remember from comparing fractions last year that 2/3 is greater than 3/5. It is really close. 2/3 is 10/15 and 3/5 is 9/15 and so the answers for 'a' and 'c' also show that 'a' is greater than 'c' because 'a' adds 2/3.

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T: Thank you Ryan. Can someone else share please?

S: I noticed that every single fraction on this worksheet is greater than or equal to one half. That means when I add two fractions that are greater than one half together, my answer will be greater than one. That also means that I will have to change my answer to a mixed number.

T: Thank you. Now I will give you 1 minute to look at Jacqueline's work. What tool did she use to convert her fractions greater than 1 to mixed numbers?

S: Number bonds!

T: Turn and talk to your neighbor briefly about what you observe about her use of number bonds and how that compared with your conversion method.

T: What tool did you use to convert your fractions into like units?

S: The rectangle model.

T: (After students share.) How does this work today relate to our work yesterday?

S: "Again, we took larger units and broke them into smaller equal units to find like denominators." → "Yesterday all our answers were less than 1 whole. Today we realized we could use the model when the sum is greater than 1." → "Our model doesn't show the sum of the units, it just shows us the number of units that we need to use to add." → "Yeah, that meant we didn't have to draw a whole other rectangle." → "I get it better today than yesterday. Now I really see what is happening."

T: Show me your learning on your exit ticket!

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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Name _____ Date _____

1) For the following problems, draw a picture using the rectangular fraction model and write the answer. When possible, write your answer as a mixed number.

a) $\frac{2}{3} + \frac{1}{2} =$

b) $\frac{3}{4} + \frac{2}{3} =$

c) $\frac{1}{2} + \frac{3}{5} =$

d) $\frac{5}{7} + \frac{1}{2} =$

e) $\frac{3}{4} + \frac{5}{6} =$

f) $\frac{2}{3} + \frac{3}{7} =$

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Solve the following problems. Draw a picture and/or write the number sentence that proves the answer. Simplify your answer.

2) Penny used 2/5 lb of flour to bake a vanilla cake. She used another 3/4 lb of flour to bake a chocolate cake. How much flour did she use altogether?

3) Carlos wants to practice piano 2 hours each day. He practices piano for 3/4 hour before school and 7/10 hour when he gets home. How many hours has Carlos practiced piano? How much longer does he need to practice before going to bed in order to meet his goal?

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Name _____ Date _____

Draw a model to help solve the following problems. Write your answer as a mixed number.

1. $\frac{5}{6}+\frac{1}{4}=$

2. Patrick drank 3/4 liter of water Monday before going jogging. He drank 4/5 liter of water after his jog. How much water did Patrick drink altogether? Write your answer as a mixed number.

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Name _____ Date _____

1) Directions: For the following problems, draw a picture using the rectangular fraction model and write the answer. When possible, write your answer as a mixed number.

a) $\frac{3}{4} + \frac{1}{3} =$

b) $\frac{3}{4} + \frac{2}{3} =$

c) $\frac{1}{3} + \frac{3}{5} =$

d) $\frac{5}{6} + \frac{1}{2} =$

e) $\frac{2}{3} + \frac{5}{6} =$

f) $\frac{4}{3} + \frac{4}{7} =$

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Solve the following problems. Draw a picture and/or write the number sentence that proves the answer. Simplify your answer.

2) Sam made 2/3 liter of punc, and 3/4 liter of tea to take to a party. How many liters of beverages did Sam bring to the party?

3) Mr. Sinofsky used 5/8 of a tank of gas on a trip to visit relatives for the weekend and another half of a tank commuting to work the next week. He then took another weekend trip and used 1/4 tank of gas. How many tanks of gas did Mr. Sinofsky use altogether?

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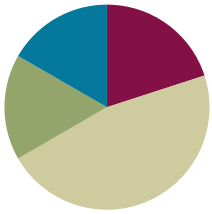
Lesson 5: Subtract Fractions With Unlike Units Using the Strategy of Creating Equivalent Fractions

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problems	(10 minutes)
Concept Development	(28 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Sprint **4.NF.3a** (12 minutes)

Sprint (12 minutes)

Materials: (S) Subtracting Fractions From a Whole Sprint (The directions for administration of Sprints are in the **G5-M3 Fluency Progressions.**)

NOTES ON SCAFFOLDING STUDENTS BELOW GRADE LEVEL:

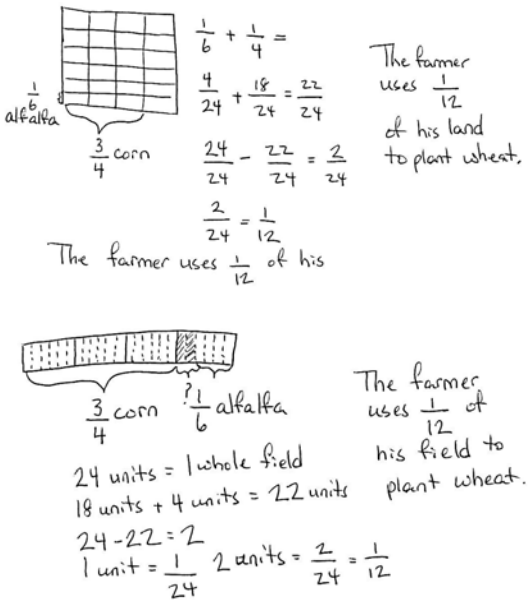
Between correcting Sprint A and giving Sprint B, have students share their strategies for quickly solving the problems. This very brief discussion may help some students catch on to a more efficient approach for Sprint B.

Application Problem (10 minutes)

A farmer uses 3/4 of his field to plant corn, 1/6 of his field to plant beans , and the rest to plant wheat. What fraction of his crop is used for wheat?

You might at times simply remind the students of their RDW process in order to solve a problem independently.What is desired is that students will internalize the simple set of questions as well as the systematic approach of read, draw, write an equation and write a statement:

- What do I see?
- What can I draw?
- What conclusions can I make from my drawing?



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Concept Development (28 minutes)

Materials: (S) Personal white boards

(Write '3 boys – 1 girl = ___')

T: Talk to your partner about the answer. (Pause) Share your thoughts, please.

S: "3 boys – 1 girl, you can't do it. You don't have any girls." → "2 students if you rename them as students."

T: (Write the following.)

3 students – 1 student = 2 students

1 half – 1 third =

T: How is this problem the same as the one before? Turn and talk to your partner.

S: "The units are not the same." → "We have to change the units to be able to say an answer."

NOTES ON SCAFFOLDING ELLS:

If this problem is acted out, it can clarify confusion about units. Students will see that we can rename the group 'students' to encompass everyone and have like units.

Repeat the process with Problem 1 using pattern blocks. Pattern blocks are perfect since the traditionally yellow trapezoid is 1/2, the blue rhombus 1/3 and the green triangles 1/6.

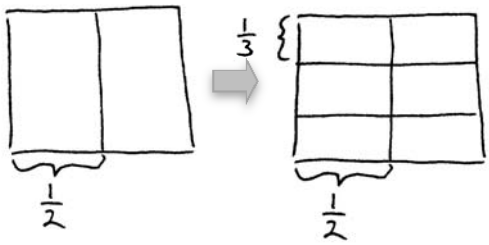
Problem 1

T: We'll need to change both units. (Write the following.)

$$\frac{1}{2}-\frac{1}{3}=$$

T: I draw one rectangle, partition it into 2 equal units. Then, I'll write 1 half below one part to make it easier to see what 1 half is after I change the units.

T: I make thirds with horizontal lines and write 1 third next to it. (Make the new units by drawing thirds horizontally.)



T: How many new units do we have?

S: 6 units.

T: 1 half is how many sixths?

S: 1 half is 3 sixths.

T: 1 third is how many sixths?

S: 1 third is 2 sixths.

$$\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}$$

T: Say the subtraction sentence and answer with like units.

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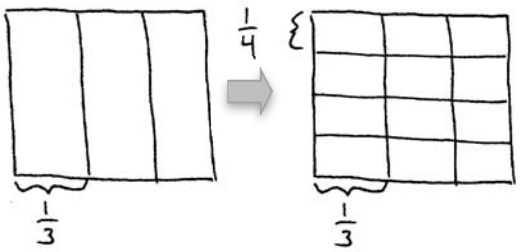
S: 3 sixths - 2 sixths = 1 sixth.

T: With unlike units?

S: 1 half - 1 third = 1 sixth.

Problem 2



This next problem presents only the additional complexity of more units.

$$\frac{1}{3}-\frac{1}{4}=$$

T: Subtract 1/4 from 1/3 and then talk to your partner about your process.

S: "To create like units we can do exactly as we did when added or when subtracting 1/2 - 1/3, make smaller units." → "First we draw parts vertically just like when we did the bar diagram. Then we partition horizontally." → "The only thing we have to remember is that we are subtracting the units, not adding."

T: (After students share.) What is our new smaller unit?

S: Twelfths.

T: 1 third is?

S: 4 twelfths.

T: 1 fourth is?

S: 3 twelfths.

$$\frac{1}{3}-\frac{1}{4}=\frac{4}{12}-\frac{3}{12}$$

T: Say the subtraction sentence and answer with like units.

S: 4 twelfths - 3 twelfths = 1 twelfth.

T: With unlike units?

S: 1 third - 1 fourth = 1 twelfth.

Problem 3

This is simple subtraction of unit fractions, just like Problem 2. Have those who finish Problem 2 quickly solve this one independently and then compare and explain their solution to a partner.

$$\frac{1}{2}-\frac{1}{5}$$

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

Additional problems like #3 allow you to work with those who need more support. If your students have a wide ability range, prepare additional problems that challenge but stay within the topic of instruction.

For example, make a list of problems subtracting consecutive denominators.

1/5 – 1/6

1/6 – 1/7

1/7 – 1/8 etc.

Students performing above grade level can look for patterns. What is happening to the answers?

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T: What do you notice about all three of our first problems?

S: "All the fractions have a numerator of 1."→ "The denominator of the whole amount is smaller than of the part we are subtracting." → "It's like that because when the denominator is smaller, the fraction is bigger." → "Yeah and we aren't doing negative numbers until sixth grade." → "The first two problems had a numerator of 1 in the difference, too."

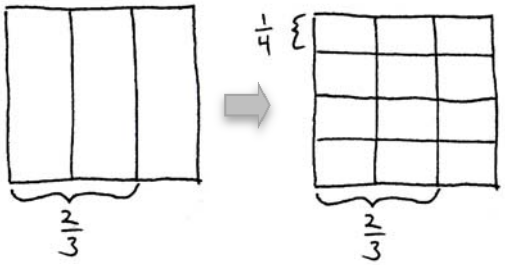
T: I chose those problems for exactly that reason. Fractions with a numerator of 1 are called unit fractions and are generally easier to manipulate. Let's try this next problem subtracting from a non-unit fraction.

Problem 4

$$\frac{2}{3}-\frac{1}{4}=$$

$$\frac{2}{3}-\frac{1}{4}=\frac{8}{12}-\frac{3}{12}$$

T: Explain to your partner the difference in solving a problem when there is a non-unit fraction such as 2/3 rather than 1/3.



Problem 5

$$\frac{1}{2}-\frac{2}{7}=$$

T: What is different about this next problem?

S: It has a non-unit fraction being subtracted.

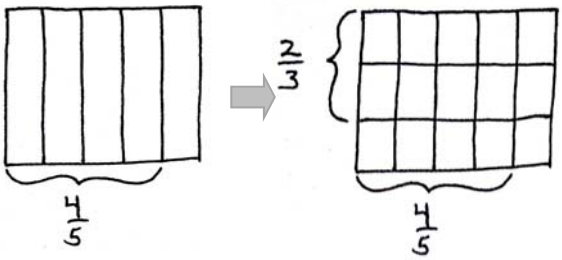
T: Very observant. Be careful when subtracting so that you take away the correct amount of units.

Problem 6

Here students encounter both a whole and subtracted part, which are non-unit fractions.

$$\frac{4}{5}-\frac{2}{3}=$$

$$\frac{4}{5}-\frac{2}{3}=\frac{12}{15}-\frac{10}{15}$$



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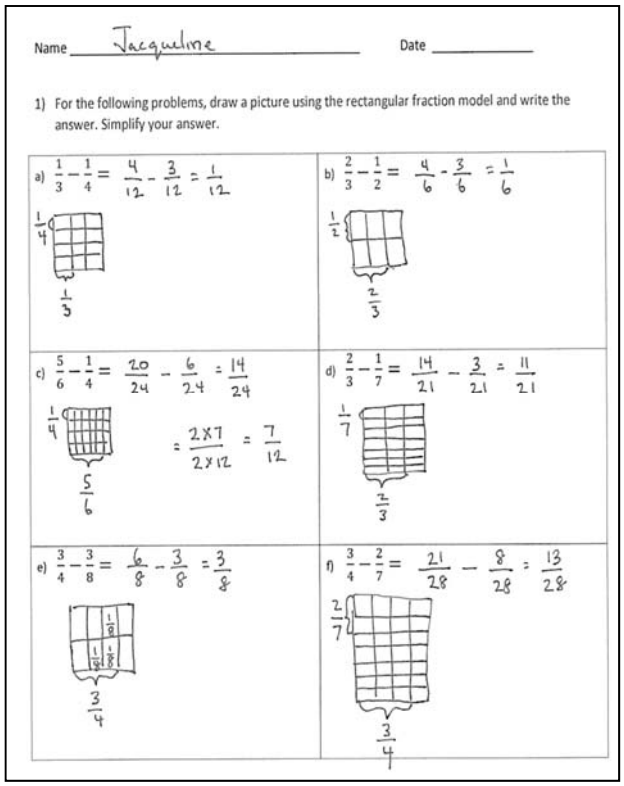
T: Turn to your partner and review the difference in labeling when you have a 2 non-unit fraction such as 2/3 rather than 1/3.

S: "We have to label two rows if we want to show 2/3." → "Yeah, nothing really changes, we just bracket more parts."

Activity Worksheet (10 minutes)

Materials: (S) Worksheet, pencil, and paper


Name Jacqueline Date

1) For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

a) $\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$

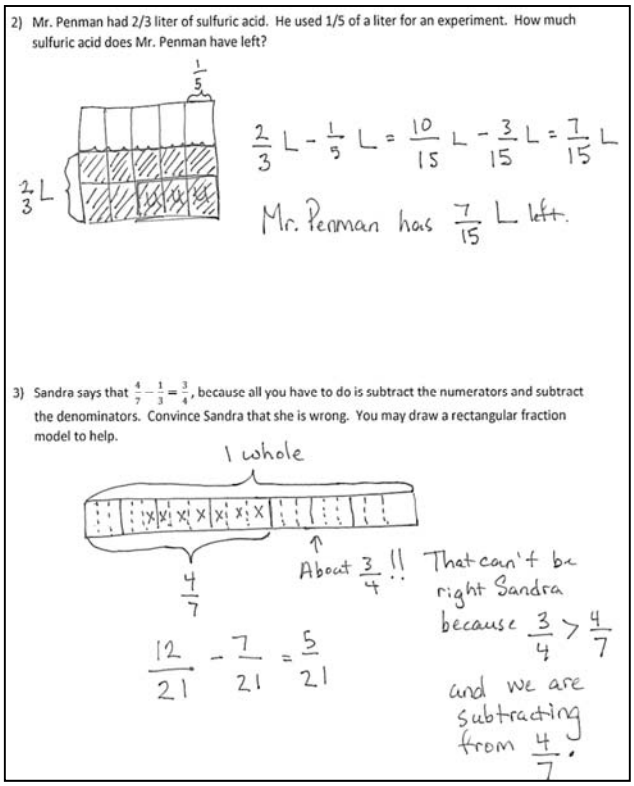
b) $\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$

c) $\frac{5}{6} - \frac{1}{4} = \frac{20}{24} - \frac{6}{24} = \frac{14}{24} = \frac{2 \times 7}{2 \times 12} = \frac{7}{12}$

d) $\frac{2}{3} - \frac{1}{7} = \frac{14}{21} - \frac{3}{21} = \frac{11}{21}$

e) $\frac{3}{4} - \frac{3}{8} = \frac{6}{8} - \frac{3}{8} = \frac{3}{8}$

f) $\frac{3}{4} - \frac{2}{7} = \frac{21}{28} - \frac{8}{28} = \frac{13}{28}$


2) Mr. Penman had 2/3 liter of sulfuric acid. He used 1/5 of a liter for an experiment. How much sulfuric acid does Mr. Penman have left?

$\frac{2}{3}$ L − $\frac{1}{5}$ L = $\frac{10}{15}$ L − $\frac{3}{15}$ L = $\frac{7}{15}$ L

Mr. Penman has $\frac{7}{15}$ L left.

3) Sandra says that $\frac{4}{7} - \frac{1}{3} = \frac{3}{4}$, because all you have to do is subtract the numerators and subtract the denominators. Convince Sandra that she is wrong. You may draw a rectangular fraction model to help.

1 whole

About $\frac{3}{4}$!! That can't be right Sandra because $\frac{3}{4} > \frac{4}{7}$ and we are subtracting from $\frac{4}{7}$.

$\frac{12}{21} - \frac{7}{21} = \frac{5}{21}$

Student Debrief (10 minutes)

T: Bring your activity worksheet to the Debrief. Take 1 minute to check your answers on numbers 1 and 2 with your partner. Do not change your answers, however. If you have a different answer, try to figure out why.

S: (Students work together for 1 minute.)

T: (Circulate. Look for common errors to guide your questioning during the next phase of the Debrief.)

T: I'll read the answers to numbers 1 and 2 now.

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T: Review and correct your mistakes for two minutes. If you had no errors, please raise your hand. I will assign you to support a peer.

T: Compare with your partner. How do these problems relate to each other?

a and b.

b and d.

e and f.

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

Meet with a small group while other students do the Debrief activities independently.

Suggestions for facilitating the Debrief:

- Circulate and ask the following questions.
- Post the questions and have student leaders lead small group discussions.
- Have students write about one relationship in their math journal.
- Have students do a pair/share.
- Meet with a small group of ELLs or students below grade level while others do one of the above.
- Debrief the whole class after partner sharing.

T: What do you notice about a and b?

S: "2/3 is double 1/3." → "1/2 is double ¼ and 1/6 is double 1/12."

T: What do you notice about b and d?

S: "Both problems start with 2/3." → "2/3 is the whole in both, but in one problem you are taking away 1/2 changed for 3 units." → "When you are subtracting 3/21 you are taking away 3 much smaller units." → "That means the answer to 'b' is bigger." → "1/6 is less than 11/21." → "Yeah, 11/21 is a little more than a half. Half of 21 is 10.5. Eleven is greater than that." → "1/6 is closer to zero."

T: What do you notice about e and f?

S: "Both problems start with 3/4." "But in one you are taking away 3/8 and the other you are taking away 2/7." → "3/8 is half of 3/4." → "Yeah, double 3/8 is 3/4." → "13/28 is 1/14 away from a half but 3/8 is 1/8 less than a half. So 13/28 is a bigger answer so 2/7 is less than 3/8."

T: Share your strategies on the word problems.

T: (After students share briefly.) If you were going to design a worksheet for this lesson, what would you have done differently? Would you have included as many unit fractions? More word problems?

S: (Students share.)

Exit Ticket

After the student debrief, instruct students to complete the exit ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the exit ticket. You may read the questions aloud to the students.

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A # Correct _____

Subtract. Give each answer in its simplest form.

1	$4 - \frac{1}{2} =$		23	$3 - \frac{1}{8} =$	
2	$3 - \frac{1}{2} =$		24	$3 - \frac{3}{8} =$	
3	$2 - \frac{1}{2} =$		25	$3 - \frac{5}{8} =$	
4	$1 - \frac{1}{2} =$		26	$3 - \frac{7}{8} =$	
5	$1 - \frac{1}{3} =$		27	$2 - \frac{7}{8} =$	
6	$2 - \frac{1}{3} =$		28	$4 - \frac{1}{7} =$	
7	$4 - \frac{1}{3} =$		29	$3 - \frac{6}{7} =$	
8	$4 - \frac{2}{3} =$		30	$2 - \frac{3}{7} =$	
9	$2 - \frac{2}{3} =$		31	$4 - \frac{4}{7} =$	
10	$2 - \frac{1}{4} =$		32	$3 - \frac{5}{7} =$	
11	$2 - \frac{3}{4} =$		33	$4 - \frac{3}{4} =$	
12	$3 - \frac{3}{4} =$		34	$2 - \frac{5}{8} =$	
13	$3 - \frac{1}{4} =$		35	$3 - \frac{3}{10} =$	
14	$4 - \frac{3}{4} =$		36	$4 - \frac{2}{5} =$	
15	$2 - \frac{1}{10} =$		37	$4 - \frac{3}{7} =$	
16	$3 - \frac{9}{10} =$		38	$3 - \frac{7}{10} =$	
17	$2 - \frac{7}{10} =$		39	$3 - \frac{5}{10} =$	
18	$4 - \frac{3}{10} =$		40	$4 - \frac{2}{8} =$	
19	$3 - \frac{1}{5} =$		41	$2 - \frac{9}{12} =$	
20	$3 - \frac{2}{5} =$		42	$4 - \frac{2}{12} =$	
21	$3 - \frac{4}{5} =$		43	$3 - \frac{2}{6} =$	
22	$3 - \frac{3}{5} =$		44	$2 - \frac{8}{12} =$	

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B Improvement _____ # Correct _____

Subtract. Give each answer in its simplest form.

1	$1 - \frac{1}{2} =$		23	$2 - \frac{1}{8} =$	
2	$2 - \frac{1}{2} =$		24	$2 - \frac{3}{8} =$	
3	$3 - \frac{1}{2} =$		25	$2 - \frac{5}{8} =$	
4	$4 - \frac{1}{2} =$		26	$2 - \frac{7}{8} =$	
5	$1 - \frac{1}{4} =$		27	$4 - \frac{7}{8} =$	
6	$2 - \frac{1}{4} =$		28	$3 - \frac{1}{7} =$	
7	$4 - \frac{1}{4} =$		29	$2 - \frac{6}{7} =$	
8	$4 - \frac{3}{4} =$		30	$4 - \frac{3}{7} =$	
9	$2 - \frac{3}{4} =$		31	$3 - \frac{4}{7} =$	
10	$2 - \frac{1}{3} =$		32	$2 - \frac{5}{7} =$	
11	$2 - \frac{2}{3} =$		33	$3 - \frac{3}{4} =$	
12	$3 - \frac{2}{3} =$		34	$4 - \frac{5}{8} =$	
13	$3 - \frac{1}{3} =$		35	$2 - \frac{3}{10} =$	
14	$4 - \frac{2}{3} =$		36	$3 - \frac{2}{5} =$	
15	$3 - \frac{1}{10} =$		37	$3 - \frac{3}{7} =$	
16	$2 - \frac{9}{10} =$		38	$2 - \frac{7}{10} =$	
17	$4 - \frac{7}{10} =$		39	$2 - \frac{5}{10} =$	
18	$3 - \frac{3}{10} =$		40	$3 - \frac{6}{8} =$	
19	$2 - \frac{1}{5} =$		41	$4 - \frac{3}{12} =$	
20	$2 - \frac{2}{5} =$		42	$3 - \frac{10}{12} =$	
21	$2 - \frac{4}{5} =$		43	$2 - \frac{4}{6} =$	
22	$3 - \frac{3}{5} =$		44	$4 - \frac{4}{12} =$	

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Name _____ Date _____

1) For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

a) $\frac{1}{3} - \frac{1}{4} =$

b) $\frac{2}{3} - \frac{1}{2} =$

c) $\frac{5}{6} - \frac{1}{4} =$

d) $\frac{2}{3} - \frac{1}{7} =$

e) $\frac{3}{4} - \frac{3}{8} =$

f) $\frac{3}{4} - \frac{2}{7} =$

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2) Mr. Penman had 2/3 liter of salt water. He used 1/5 of a liter for an experiment. How much salt waterdoes Mr. Penman have left?

3) Sandra says that $\frac{4}{7}$ - $\frac{1}{3}$ = $\frac{3}{4}$ because all you have to do is subtract the numerators and subtract the denominators. Convince Sandra that she is wrong. You may draw a rectangular fraction model to help.

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Name _____ Date _____

Directions: Draw a model, write a subtraction sentence with like units, and circle your answer for each subtraction problem.

1. $\frac{1}{2} - \frac{1}{7} =$

2. $\frac{3}{5} - \frac{1}{2} =$

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Name _____ Date _____

1) The picture shows 3/4 of the square shaded. Use the picture to show how to create equal fractions with like units that would allow you to subtract 1/3, and then find the difference.

$\frac{3}{4} - \frac{1}{3} =$

2) Find the difference. Use a rectangular fraction model to show how to convert to fractions with common denominators.

a. $\frac{5}{6} - \frac{1}{3} =$

b. $\frac{2}{3} - \frac{1}{2} =$

c. $\frac{5}{6} - \frac{1}{4} =$

d. $\frac{4}{5} - \frac{1}{2} =$

e. $\frac{2}{3} - \frac{2}{5} =$

f. $\frac{5}{7} - \frac{2}{3} =$

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3) Robin used 1/4 pound of butter to make a cake. Afterward she had 5/8 of a pound left. How much butter did she have at first?

4) Katrina needs 3/5 kilogram of flour for a recipe. Her mother has 3/7 kilogram in her pantry. Is this enough flour to make the recipe If not, how much more will she need?

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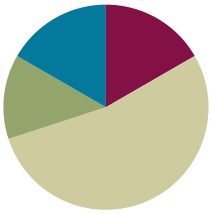
Lesson 6: Subtract Fractions from Numbers Between One and Two

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Application Problems	(8 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (10 minutes)

- Name the Fraction to Complete the Whole **4.NF.3b** (4 minutes)
- Taking from the Whole **5.NF.7** (3 minutes)
- Fraction Units to Ones and Fractions **5.NF.7** (3 minutes)

Name the Fraction to Complete the Whole (4 minutes)

T: I'll say a fraction, you say the missing part to make one whole. Ready?

T: 1/2.

S: 1/2.

T: 4/5.

S: 1/5.

T: 1/7.

S: 6/7.

T: 4/9.

S: 5/9.

T: 18/20.

S: 2/20.

T: 147/150.

S: 3/150.

T: Share your strategy for making one whole with a partner.

T: With your partner, take turns giving each other problems to solve. You have one minute.

NOTES ON STUDENTS ABOVE AND BELOW GRADE LEVEL:

When students begin to quiz each other, group them in "level alike" pairs.

Below Grade Level Performers: Provide a bar diagram template in personal boards so that students can quickly draw each fraction and see the missing part.

Above Grade Level Performers: Give them ½ as a target number. Their partner can give them any fraction less than one. They tell how much to add or subtract to get to one half, e.g. 3/7 → add 1/14, 9/10 → subtract 4/10.

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Taking from the Whole (3 minutes)

Materials: (S) Personal white boards and markers

T: I'll say a subtraction equation. You say the answer. 1 – 1 half.
S: 1 half.
T: 1 – 1 third.
S: 2 thirds.
T: 1 – 2 thirds.
S: 1 third.
T: 1 – 2 fifths.
S: 3 fifths.
T: 1 – 4 fifths.
S: 1 fifth.

Continue with possible sequence:

$$1-\frac{1}{3},\ 1-\frac{3}{4},\ 1-\frac{3}{7},\ 1-\frac{5}{9},\ 1-\frac{5}{10}$$

NOTES ON SCAFFOLDING STUDENTS PERFORMING BELOW GRADE LEVEL:

If students struggle to answer chorally, write the subtraction equations on the board and have them answer on personal white boards.

Fraction Units to Ones and Fractions (3 minutes)

Materials: (S) Personal white board and markers

T: I'll say a fraction, you say it as ones and fraction units. Three halves.
S: One and one half.
T: Five halves.
S: Two and one half.
T: Seven halves.
S: Three and one half.
T: Eleven halves.
S: Five and one half.

Continue with possible sequence:

$$\frac{4}{3}, \frac{5}{3}, \frac{10}{3}, \frac{22}{3} \text{ and } \frac{5}{4}, \frac{7}{4}, \frac{11}{4}, \frac{39}{4}.$$

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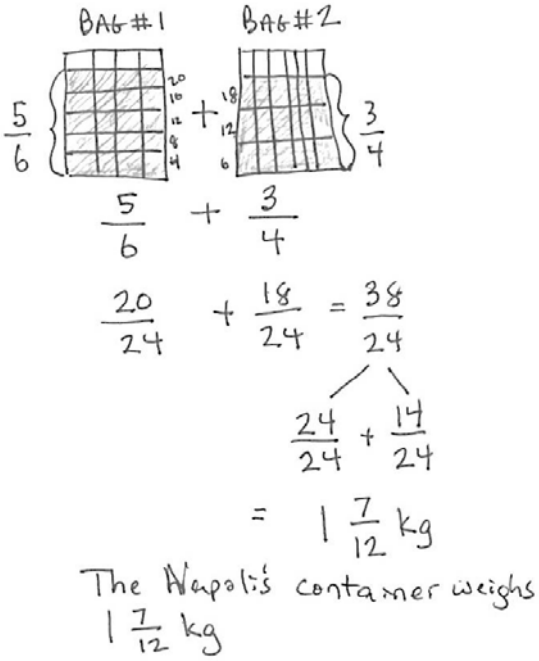
Application Problem (8 minutes)

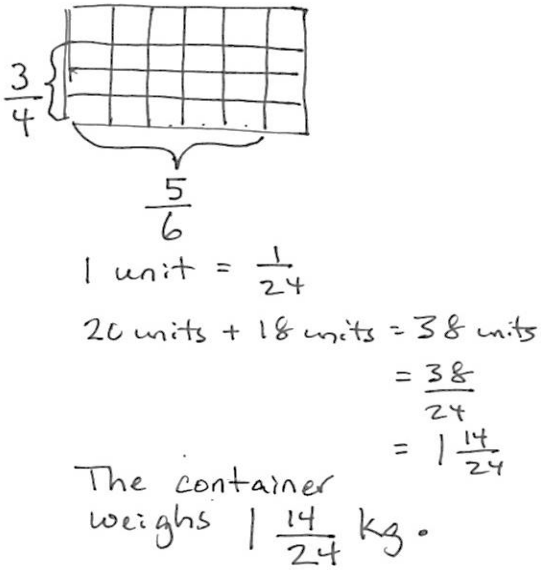
The Napoli family combined two bags of dry cat food in a plastic container. One bag had 5/6 kg. The other bag had 3/4 kg. What was the total weight of the container after the bags were combined?

T: Use your RDW process to solve the problem independently. Use your questions to support you in your work. What do you see? Can you draw something? What conclusions can you make from your drawing?

T: We will analyze two solution strategies in four minutes.

After four minutes, lead students through a brief comparison of a more concrete strategy like the one below on the left and the more abstract strategy below on the right. Be sure students realize that both answers, 1 7/12 and 1 14/24, are correct.





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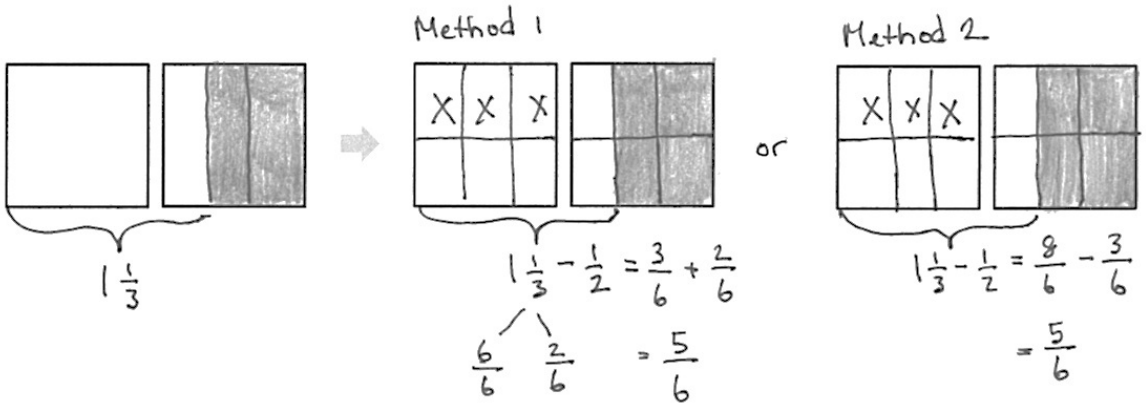
Concept Development (32 minutes)

Pictorial (20 minutes)

Materials: (S) Personal white boards, template if you choose to use it

Problem 1

$$1\frac{1}{3}-\frac{1}{2}=$$



T: Read the subtraction expression.

S: 1 and 1 third – 1 half.

T: How many thirds is 1 and 1 third?

S: 4 thirds.

T: (Draw) What should we do now? Turn and talk to your partner.

S: Make like units.

T: How many new smaller units are in each whole?

S: 6 units.

T: 4 thirds is how many sixths?

S: 8 sixths.

T: 1 half is how many sixths?

S: 3 sixths.

T: Looking at my drawing, how would you subtract 3 sixths or a half? Discuss this with your partner.

S: "You can take the half from the whole and then add back the third." → "Then you are adding to subtract?" → "Yes, you are adding the part you had left after you take away." → "It makes it easier because we know really well how to subtract any fraction from a one whole." → "Yeah but it's just easier for me to take the 3 sixths from the 8 sixths." → "For me, it's easier to take it from the whole and add back the rest."

MP.5

T: It's like subtracting 80 from 130. It's easier for me to take 80 from 100 and add 20 and 30.

S: Can we do it whatever way?

T: Of course. Choose the way that is easiest for you.

NOTES ON SCAFFOLDING ELLS:

Have students draw each step along with you on personal white boards, so that they match your language with the model and the steps of the process. At key moments have them orally label the parts of the model to practice using language.

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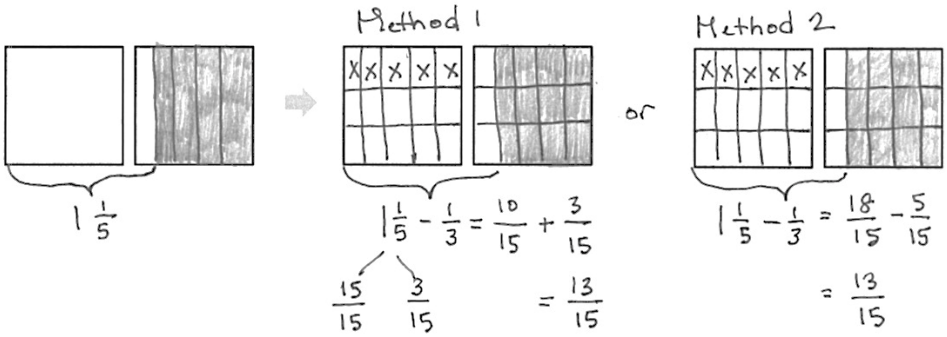
T: Let's call the different solution strategies "method 1" and "method 2." If you use method 1, it's good to record it with a number bond.

S: 8 sixths – 3 sixths = 5 sixths. 1 and 1 third – 1 half = 5 sixths.

Problem 2

$$1\frac{1}{5}-\frac{1}{3}=$$



T: I draw one rectangle to show 1 and a second rectangle to show 1 fifth.

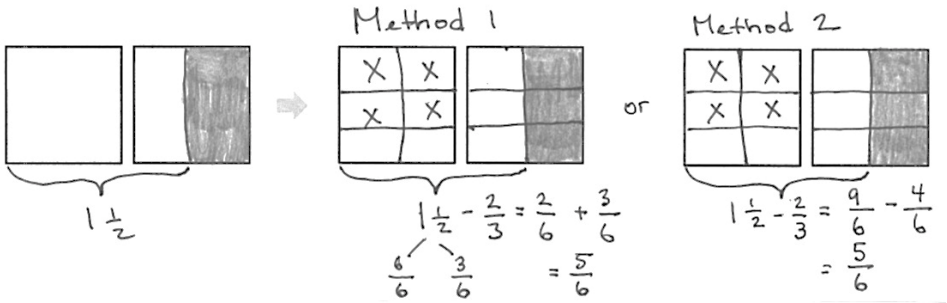
T: Do I have thirds to take away from the 6 fifths?

S: No.

T: Explain to your partner how to solve this problem. Use both methods.

Problem 3

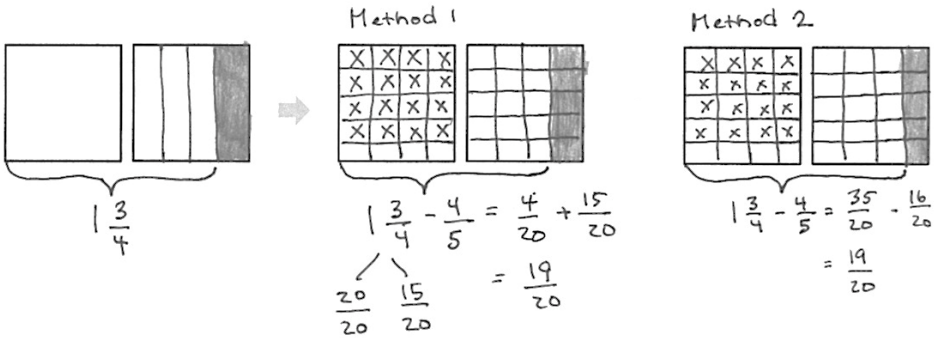
$$1\frac{1}{2}-\frac{2}{3}=$$



The only complexity added on here is that of the subtraction of a non-unit fraction.

Problem 4

$$1\frac{3}{4}-\frac{4}{5}=$$



In this problem, the new complexity is the choice of two non-unit fractions.

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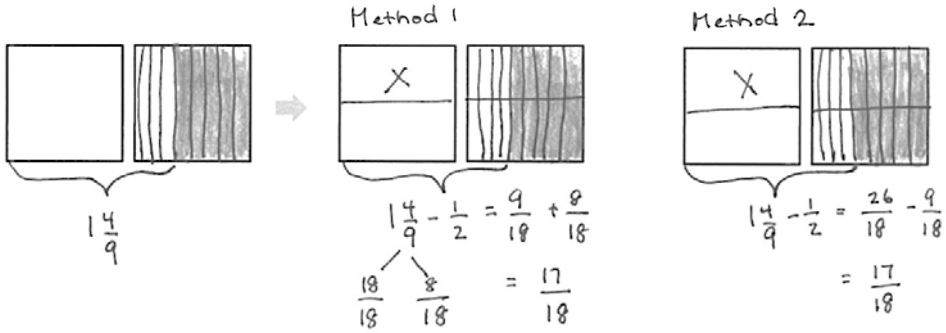
Problem 5

$$1\frac{4}{9}-\frac{1}{2}=$$

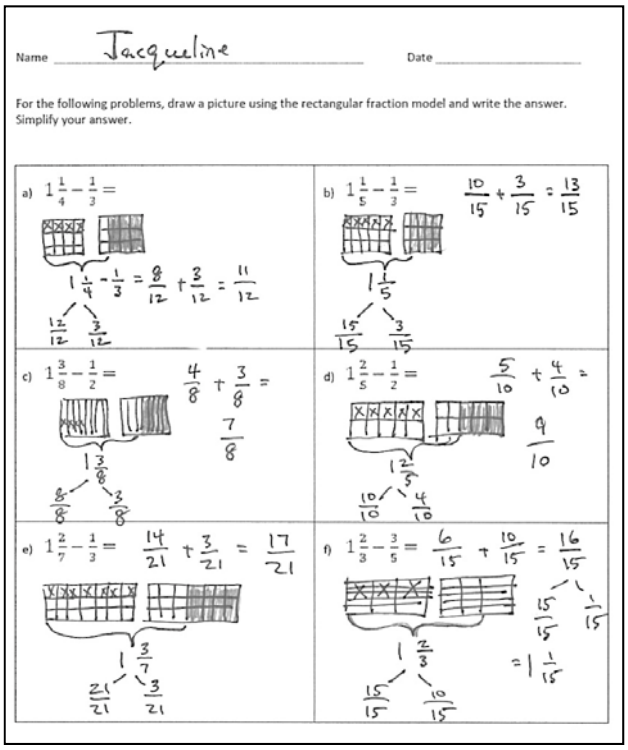


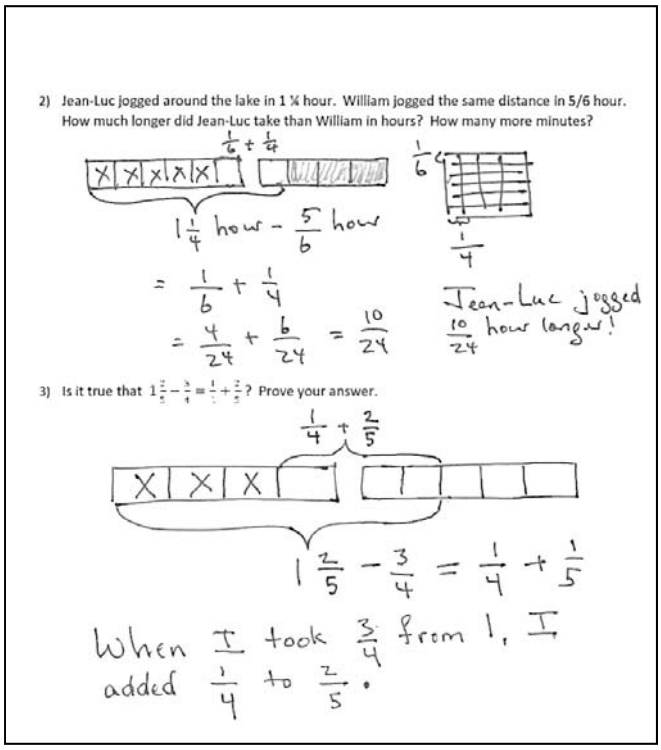
T: I really don't want to have to draw all those ninths, so I'm going to do a shortcut for now just to remind myself of what to do.

Activity Worksheet (12 minutes)

Materials: (S) Worksheet

T: Take 12 minutes to work on these problems.





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Student Debrief (10 minutes)

T: Take one minute to compare your work with a partner's. (Circulate and look for common errors or student work to use instructionally.)

T: I'll read the answers to numbers 1 and 2 now.

T: (Students correct their work for about 2 minutes.) If you had no errors, I will assign you to support a peer.

T: Compare these problems with a partner:

a and b

c and d

e and f

S: "I remember that 1/3 – 1/4 is 1/12 so then 11/4 – 1/3 is 1/12 less than 1." → "It's the same with 1 1/3 – 1/5. It's 2/15 and so the answer is 2/15 less than 1." → "You could use the same strategy on all of them."

T: Jacqueline, can you explain your solution to question 2?

S: I realized that the problem was really easy, just subtraction. I could take 5/6 from 1 and add it to 1/4. 1/6 and 1/4 is easy because it's just unit fractions 4/24 and 6/24. So the answer is 10 twenty fourths.

T: Did anyone solve it differently?

S: Yes. I just converted the fractions to like units and subtracted so it was 24 twenty-fourths and 6 twenty-fourths. 30 twenty-fourths – 20 twenty-fourths = 10 twenty-fourths.

NOTES ON SCAFFOLDING ELLS:

As students share out various strategies, use a modified 'text representation' activity. Have the rest of the class demonstrate the ideas their peers orally express on personal white boards.

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A

Correct_____

Express as an improper fraction.

1	$1\frac{1}{5}=$	/	23	$2\frac{7}{10}=$	/
2	$2\frac{1}{5}=$	/	24	$4\frac{9}{10}=$	/
3	$3\frac{1}{5}=$	/	25	$1\frac{1}{8}=$	/
4	$4\frac{1}{5}=$	/	26	$1\frac{5}{6}=$	/
5	$1\frac{1}{4}=$	/	27	$4\frac{5}{6}=$	/
6	$1\frac{3}{4}=$	/	28	$4\frac{5}{8}=$	/
7	$1\frac{2}{5}=$	/	29	$1\frac{5}{8}=$	/
8	$1\frac{3}{5}=$	/	30	$2\frac{3}{8}=$	/
9	$1\frac{4}{5}=$	/	31	$3\frac{3}{10}=$	/
10	$2\frac{4}{5}=$	/	32	$4\frac{7}{10}=$	/
11	$3\frac{4}{5}=$	/	33	$4\frac{4}{5}=$	/
12	$2\frac{1}{4}=$	/	34	$4\frac{1}{8}=$	/
13	$2\frac{3}{4}=$	/	35	$4\frac{3}{8}=$	/
14	$3\frac{1}{4}=$	/	36	$4\frac{7}{8}=$	/
15	$3\frac{3}{4}=$	/	37	$1\frac{5}{12}=$	/
16	$4\frac{1}{3}=$	/	38	$1\frac{7}{12}=$	/
17	$4\frac{2}{3}=$	/	39	$2\frac{1}{12}=$	/
18	$2\frac{3}{5}=$	/	40	$3\frac{1}{12}=$	/
19	$3\frac{3}{5}=$	/	41	$2\frac{7}{12}=$	/
20	$4\frac{3}{5}=$	/	42	$3\frac{5}{12}=$	/
21	$2\frac{1}{6}=$	/	43	$3\frac{11}{12}=$	/
22	$3\frac{1}{8}=$	/	44	$4\frac{7}{12}=$	/

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B # Correct_____

Express as an improper fraction.

1	$1\frac{1}{2}=$	/	23	$2\frac{3}{10}=$	/
2	$2\frac{1}{2}=$	/	24	$3\frac{1}{10}=$	/
3	$3\frac{1}{2}=$	/	25	$1\frac{1}{6}=$	/
4	$4\frac{1}{2}=$	/	26	$1\frac{3}{8}=$	/
5	$1\frac{1}{3}=$	/	27	$3\frac{5}{6}=$	/
6	$1\frac{2}{3}=$	/	28	$3\frac{5}{8}=$	/
7	$1\frac{3}{10}=$	/	29	$2\frac{5}{8}=$	/
8	$1\frac{7}{10}=$	/	30	$1\frac{7}{8}=$	/
9	$1\frac{9}{10}=$	/	31	$4\frac{3}{10}=$	/
10	$2\frac{9}{10}=$	/	32	$3\frac{7}{10}=$	/
11	$3\frac{9}{10}=$	/	33	$2\frac{5}{6}=$	/
12	$2\frac{1}{3}=$	/	34	$2\frac{7}{8}=$	/
13	$2\frac{2}{3}=$	/	35	$3\frac{7}{8}=$	/
14	$3\frac{1}{3}=$	/	36	$4\frac{1}{6}=$	/
15	$3\frac{2}{3}=$	/	37	$1\frac{1}{12}=$	/
16	$4\frac{1}{4}=$	/	38	$1\frac{11}{12}=$	/
17	$4\frac{3}{4}=$	/	39	$4\frac{1}{12}=$	/
18	$2\frac{2}{5}=$	/	40	$2\frac{5}{12}=$	/
19	$3\frac{2}{5}=$	/	41	$2\frac{11}{12}=$	/
20	$4\frac{2}{5}=$	/	42	$3\frac{7}{12}=$	/
21	$3\frac{1}{6}=$	/	43	$4\frac{5}{12}=$	/
22	$2\frac{1}{8}=$	/	44	$4\frac{11}{12}=$	/

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Name _____ Date _____

For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

a) $1\frac{1}{4} - \frac{1}{3} =$

b) $1\frac{1}{5} - \frac{1}{3} =$

c) $1\frac{3}{8} - \frac{1}{2} =$

d) $1\frac{2}{5} - \frac{1}{2} =$

e) $1\frac{2}{7} - \frac{1}{3} =$

f) $1\frac{2}{3} - \frac{3}{5} =$

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2) Jean-Luc jogged around the lake in 1 1/4 hour. William jogged the same distance in 5/6 hour. How much longer did Jean-Luc take than William in hours? How many more minutes?

3) Is it true that $1\frac{2}{5} - \frac{3}{4} = \frac{1}{4} + \frac{2}{5}$? Prove your answer.

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Name _____ Date _____

For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

1. $1\frac{1}{5}-\frac{1}{2}=$

2. $1\frac{1}{3}-\frac{5}{6}=$

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Name _____ Date _____

1) Find the difference. Use a rectangular fraction model to show how to convert to fractions with common denominators.

a) $1 - \frac{5}{6} =$

b) $\frac{3}{2} - \frac{5}{6} =$

c) $\frac{4}{3} - \frac{5}{7} =$

d) $1\frac{1}{8} - \frac{3}{5} =$

e) $1\frac{2}{5} - \frac{3}{4} =$

f) $1\frac{5}{6} - \frac{7}{8} =$

g) $1\frac{2}{7} - \frac{3}{4} =$

h) $1\frac{3}{12} - \frac{2}{3} =$

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2) Sam had 1 1/2 m of rope. He cut off 5/8 m and used it for a project. How much rope does Sam have left?

3) Jackson had 1 3/8 kg of fertilizer. He used some to fertilize a flower bed andhe only had 2/3 kg left. How much fertilizer was used in the flower bed?

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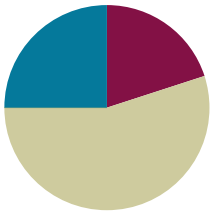
Lesson 7: Solve Two-Step Word Problems

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(33 minutes)
Student Debrief	(15 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Sprint **4.NF.2** (12 minutes)

Sprint (12 minutes)

Materials: (S) Circle the Equivalent Fraction Sprint (See directions for administration of Sprints in the **G5-M3 Fluency Progressions.**)

Concept Development (33 minutes)

Materials: (S) Worksheet, Personal white board

Problem 1

George weeded 1/5 of the garden and Tiffany weeded some, too. When they were finished, 2/3 of the garden still needed to be weeded. What fraction of the garden did Tiffany weed?

T: Let's read the problem together.
S: (Students read chorally.)
T: Share with your partner: What do you see when you hear the story? What can you draw?
S: (Students share with partners.)
T: I'll give you one minute to draw.
T: (After a brief exchange.) What fraction of the garden did Tiffany weed? Is it a part or the whole?
S: Part.
T: Do we know the whole?
S: Yes.

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

You might strategically pair ELLs and students performing below grade level. For example, a student performing below grade level who is accustomed to translating for a parent may blossom when asked to translate for a newcomer. The relationship may become mutually beneficial if the newcomer shows strong ability and can help his or her partner with math concepts.

Also, help engagement by putting student names in them, acting them out and making small pictures to support comprehension.

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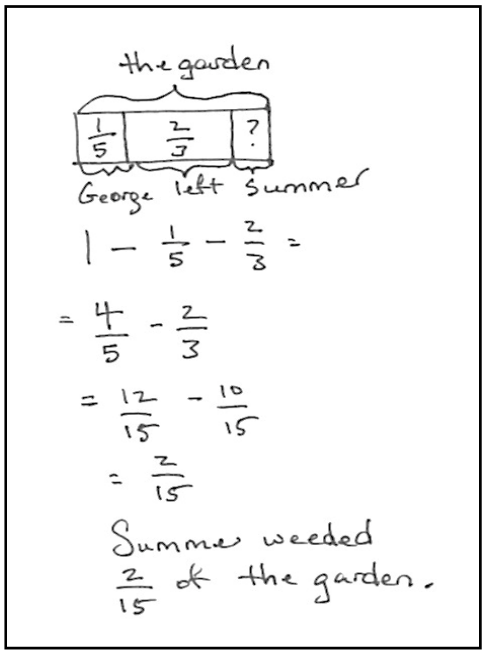

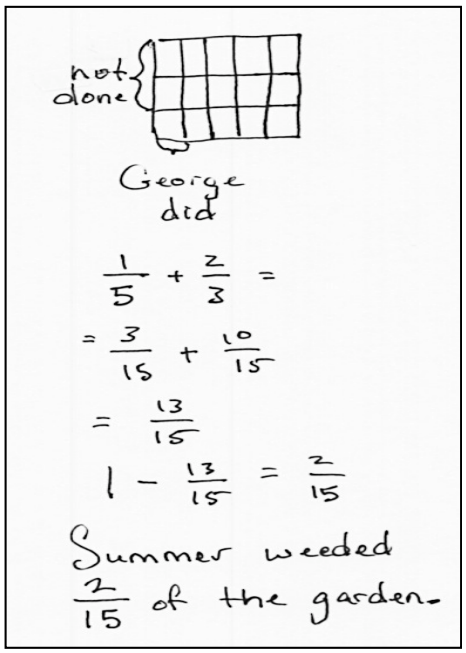

T: What is it?

S: 1.

T: From the whole, we separate 1/5 for George, an unknown amount for Summer, and have a leftover part of 2/3.

T: How do you solve for Summer's part? Turn and share.

S: (Students share.)

T: Solve the problem on your personal white board. (Pause.) Show your board.

T: Turn and explain to your partner how you got the answer.

T: (After students share.) Jason, please share?

S: After I drew the bar diagram, I just subtracted the part George weeded and the part that was left from the whole.

T: Barbara, please share?

S: My way to solve this problem is to add up the 2 parts to create a bigger part, then subtract from the whole.

S: Summer weeded 2/15 of the garden.

T: Barbara and Jason have presented their solving methods which came right from their drawings. With your partner, analyze their drawings. How are they the same and how are they different?

S: (Students share.)

T: Are they both correct?

S: Yes?

T: How do you know?

S: "They both make sense." → "They both got the right answer." → "They both showed the same relationships but in different ways."

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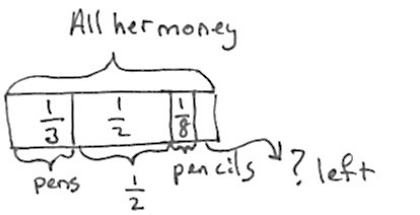
Problem 2

Jing spent 1/4 of her money on a pack of pens, 1/2 of her money on a pack of markers, and 1/8 of her money on a pack of pencils.

What fraction of Jing's money has been spent? What fraction of her money is left?

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

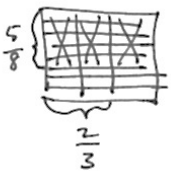
If students finish quickly, have them re-work problems using a different strategy. Then during the problem debrief, take a moment to have them share which strategy they prefer and why.



$1 - \frac{1}{3} - \frac{1}{2} - \frac{1}{8}$

$= \frac{2}{3} - \frac{1}{2} - \frac{1}{8}$

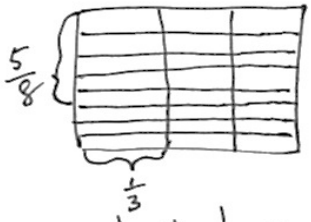
$= \frac{2}{3} - \frac{5}{8}$



$= \frac{16}{24} - \frac{15}{24}$

$= \frac{1}{24}$

Jing had $\frac{1}{24}$ of her money left.



$\frac{1}{3} + \frac{1}{2} + \frac{1}{8}$

$= \frac{1}{3} + \frac{5}{8}$

$= \frac{8}{24} + \frac{15}{24}$

$= \frac{23}{24}$

$1 - \frac{23}{24} = \frac{1}{24}$

Jing had $\frac{1}{24}$ of her money left.

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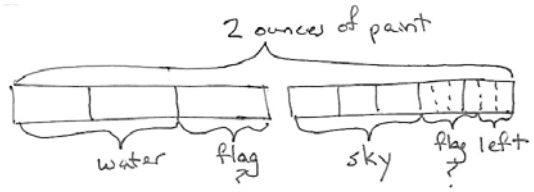
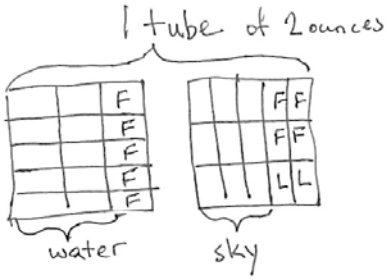
Problem 3

Shelby bought a 2 ounce tube of blue paint. She used 2/3 ounce to paint the water, 3/5 ounce to paint the sky, and some to paint a flag. After that she has 2/15 ounce left. How much paint did Shelby use to paint her flag?



$\frac{1}{3} + \frac{4}{15} = \frac{5}{15} + \frac{4}{15} = \frac{9}{15}$

Shelby used $\frac{9}{15}$ ounce to paint the flag.



$\frac{10}{15} + \frac{9}{15} + \frac{2}{15}$

$= \frac{21}{15} = 1\frac{6}{15}$

$2 - 1\frac{6}{15} = \frac{9}{15}$

Shelby used $\frac{9}{15}$ ounce to paint the flag.

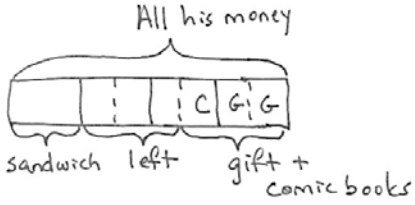
She used $\frac{9}{30}$ tube or $\frac{3}{10}$ tube to paint the flag.

Problem 4

Jim sold 3/4 gallon of lemonade. Dwight sold some lemonade too. Together, they sold 1 5/12 gallons. Who sold more lemonade, Jim or Dwight? How much more? (See the lesson debrief for student work samples.)

Problem 5

Leonard spent 1/4 of his money on a sandwich. He spent 2 times as much on a gift for his brother as on some comic books. He had 3/8 of his money left. What fraction of his money did he spend on the comic books?

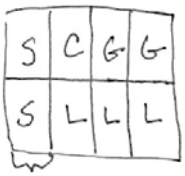


gift $\boxed{\frac{1}{8}}\boxed{\frac{1}{8}}$

comics $\boxed{\frac{1}{8}}$

$1 - \frac{7}{8} = \frac{1}{8}$

Leonard spent $\frac{1}{8}$ of his money on comics.



Leonard spent $\frac{1}{8}$ his money on comics.

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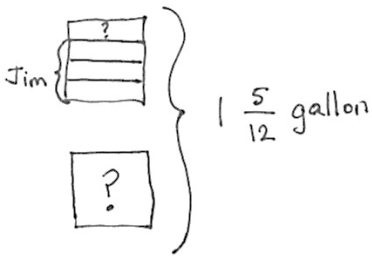
Student Debrief (15 minutes)

T: Bring your activity worksheet to the Debrief. Share, check, and/or explain your answers to your partner.

S: (Students work together for 2 minutes.)

T: (Circulate and listens to students' explanations while they work, then review answers.)

T: Let's take a look at two different strategies for solving problem 4.



$1\frac{5}{12} - \frac{3}{4}$

$= \frac{1}{4} + \frac{5}{12}$

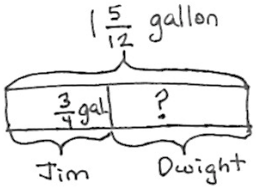
$= \frac{3}{12} + \frac{5}{12}$

$= \frac{8}{12}$ (Dwight's lemonade)

$\frac{3}{4} = \frac{9}{12}$

$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

Jim sold $\frac{1}{12}$ gallon more than Dwight.



$\frac{3}{4} + ? = \frac{17}{12}$

$\frac{9}{12} + \frac{8}{12} = \frac{17}{12}$

Jim Dwight

$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

Jim sold $\frac{1}{12}$ gallon more than Dwight.

T: What do you notice about the 2 different drawings?

S: "The first one draws the containers of lemonade, the second one draws the bar diagram." → "Both drawings are different but they both have the part, part, whole relationship."

T: Let's look at them closely. How is Jim's container of 3/4 gallon of lemonade represented in the bar diagram? Turn and share.

S: "Instead of drawing a container of 3/4 gallon, Jim's lemonade is now a part of a whole in the bar diagram."

T: How is Dwight's container of lemonade represented in the bar diagram? Turn and share.

S: "Since we don't know Dwight's lemonade, we put a question mark in the container. But in the bar

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diagram, it's a missing part of a whole."

T: Look at both drawings. How is the whole represented? Turn and share.

S: "The drawing on the left shows 1 5/12 gallons of lemonade in the containers. The drawing on the right shows the whole in a bar diagram created by Jim and Dwight."

T: What if I change the numbers in this problem and make them into bigger units? For example, Jim has 126 3/4 gallons, and the total is 348 5/12 gallons. Which drawing do you think it's easier to draw and represent the new problem? Turn and share.

S: "That's too many containers to draw." → "It's easier to draw the new problem using the bar diagram." → "It's faster to label the part, part, whole in the bar diagram than drawing all the containers."

T: The bar diagram is much easier to use, even with larger numbers.

MP.3

T: What do you notice about their methods of solving this problem?

S: "The second one started with the addition sentence 3/4 + ? = 1 5/12, but the first one started with subtraction sentence 1 5/12 – 3/4 = ?"

T: Turn and share with your partner and follow each solution strategy through step by step. Share what is the same and different about them.

S: (Students share.)

T: If you have to solve a similar problem again, what kind of drawing and solution strategy would you use? Turn and share.

S: (Students share.)

NOTES ON SCAFFOLDING ELLS:

Make sure ELLs are sitting close to the displayed work during the Debrief, so that even when they lose the conversation's thread, they can be analyzing and learning from the student work.

ELLs may prefer the more concrete drawing of the gallon of lemonade. The explicit connection between the two representations is a strong bridge to understanding the bar diagram.

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A # Correct _____

Circle the equivalent fraction.

1	$\frac{2}{4}=$	$\frac{1}{2}$	$\frac{1}{3}$	23	$\frac{9}{27}=$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
2	$\frac{2}{6}=$	$\frac{1}{2}$	$\frac{1}{3}$	24	$\frac{9}{63}=$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
3	$\frac{2}{8}=$	$\frac{1}{2}$	$\frac{1}{4}$	25	$\frac{8}{12}=$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
4	$\frac{5}{10}=$	$\frac{1}{2}$	$\frac{1}{4}$	26	$\frac{8}{16}=$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
5	$\frac{5}{15}=$	$\frac{1}{2}$	$\frac{1}{3}$	27	$\frac{8}{24}=$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
6	$\frac{5}{20}=$	$\frac{1}{2}$	$\frac{1}{4}$	28	$\frac{8}{64}=$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$
7	$\frac{4}{8}=$	$\frac{1}{2}$	$\frac{1}{4}$	29	$\frac{12}{18}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
8	$\frac{4}{12}=$	$\frac{1}{2}$	$\frac{1}{3}$	30	$\frac{12}{16}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
9	$\frac{4}{16}=$	$\frac{1}{2}$	$\frac{1}{4}$	31	$\frac{9}{12}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
10	$\frac{3}{6}=$	$\frac{1}{2}$	$\frac{1}{3}$	32	$\frac{6}{8}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
11	$\frac{3}{9}=$	$\frac{1}{2}$	$\frac{1}{3}$	33	$\frac{10}{12}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
12	$\frac{3}{12}=$	$\frac{1}{2}$	$\frac{1}{4}$	34	$\frac{15}{18}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
13	$\frac{4}{6}=$	$\frac{2}{3}$	$\frac{1}{3}$	35	$\frac{8}{10}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
14	$\frac{6}{12}=$	$\frac{2}{3}$	$\frac{1}{2}$	36	$\frac{16}{20}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
15	$\frac{6}{18}=$	$\frac{2}{3}$	$\frac{1}{3}$	37	$\frac{12}{15}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
16	$\frac{6}{30}=$	$\frac{1}{5}$	$\frac{1}{3}$	38	$\frac{18}{27}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
17	$\frac{6}{9}=$	$\frac{2}{3}$	$\frac{1}{3}$	39	$\frac{27}{36}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
18	$\frac{7}{14}=$	$\frac{1}{2}$	$\frac{1}{3}$	40	$\frac{32}{40}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
19	$\frac{7}{21}=$	$\frac{1}{2}$	$\frac{1}{3}$	41	$\frac{45}{54}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$
20	$\frac{7}{42}=$	$\frac{1}{6}$	$\frac{1}{7}$	42	$\frac{24}{36}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
21	$\frac{8}{12}=$	$\frac{2}{3}$	$\frac{3}{4}$	43	$\frac{60}{72}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
22	$\frac{9}{18}=$	$\frac{1}{2}$	$\frac{1}{3}$	44	$\frac{48}{60}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$

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B Improvement _____ # Correct _____

Circle the equivalent fraction.

1	$\frac{5}{10}=$	$\frac{1}{2}$	$\frac{1}{3}$	23	$\frac{8}{24}=$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
2	$\frac{5}{15}=$	$\frac{1}{2}$	$\frac{1}{4}$	24	$\frac{8}{56}=$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
3	$\frac{5}{20}=$	$\frac{1}{2}$	$\frac{1}{4}$	25	$\frac{8}{12}=$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
4	$\frac{2}{4}=$	$\frac{1}{2}$	$\frac{1}{3}$	26	$\frac{9}{18}=$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
5	$\frac{2}{6}=$	$\frac{1}{2}$	$\frac{1}{4}$	27	$\frac{9}{27}=$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
6	$\frac{2}{8}=$	$\frac{1}{2}$	$\frac{1}{4}$	28	$\frac{9}{72}=$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$
7	$\frac{3}{6}=$	$\frac{1}{2}$	$\frac{1}{3}$	29	$\frac{12}{18}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
8	$\frac{3}{9}=$	$\frac{1}{2}$	$\frac{1}{4}$	30	$\frac{6}{8}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
9	$\frac{3}{12}=$	$\frac{1}{2}$	$\frac{1}{3}$	31	$\frac{9}{12}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
10	$\frac{4}{8}=$	$\frac{1}{2}$	$\frac{1}{3}$	32	$\frac{12}{16}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
11	$\frac{4}{12}=$	$\frac{1}{2}$	$\frac{1}{4}$	33	$\frac{8}{10}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
12	$\frac{4}{16}=$	$\frac{2}{3}$	$\frac{1}{3}$	34	$\frac{16}{20}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
13	$\frac{4}{6}=$	$\frac{2}{3}$	$\frac{1}{2}$	35	$\frac{12}{15}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
14	$\frac{7}{14}=$	$\frac{2}{3}$	$\frac{1}{3}$	36	$\frac{10}{12}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$
15	$\frac{7}{21}=$	$\frac{1}{5}$	$\frac{1}{3}$	37	$\frac{15}{18}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$
16	$\frac{7}{35}=$	$\frac{2}{3}$	$\frac{1}{3}$	38	$\frac{16}{24}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
17	$\frac{6}{9}=$	$\frac{1}{2}$	$\frac{1}{3}$	39	$\frac{24}{32}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
18	$\frac{6}{12}=$	$\frac{1}{2}$	$\frac{1}{3}$	40	$\frac{36}{45}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
19	$\frac{6}{18}=$	$\frac{1}{6}$	$\frac{1}{7}$	41	$\frac{40}{48}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$
20	$\frac{6}{36}=$	$\frac{2}{3}$	$\frac{3}{4}$	42	$\frac{24}{36}=$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{2}{3}$
21	$\frac{8}{12}=$	$\frac{1}{2}$	$\frac{1}{3}$	43	$\frac{48}{60}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{4}{5}$
22	$\frac{8}{16}=$	$\frac{1}{2}$	$\frac{1}{3}$	44	$\frac{60}{72}=$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{2}{3}$

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Name _____ Date _____

Solve the word problems using the RDW strategy. Show all your work.

1) George weeded 2/5 of the garden and Tiffany weeded some, too. When they were finished, 4/15 of the garden still needed to be weeded. What fraction of the garden did Tiffany weed?

2) Leonard spent 2 times as much on a gift for his brother as on some comic books. He spent 1/4 of his money on a sandwich. If he had 3/8 of his money left, what fraction of his money did he spend on the comic books?

3) Jing spent 1/3 of her money on a pack of pens, 1/2 of her money on a pack of markers, and 1/6 of her money on a pack of pencils. What fraction of Jing's money has been spent? What fraction of her money is left?

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4) Shelby bought 2 cans of blue paint. She used 2/3 of a can painting flowers. Then Aaron used 3/5 of a can for his model car. After painting a picture of the sky, Shelby has 2/15 of a can left. How much paint did Shelby use for her picture of the sky?

5) Jim sold 3/4 gallon of lemonade. Dwight sold some lemonade too. Together they sold 1 5/12 gallons. Who sold more lemonade, Jim or Dwight? How much more?

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Name _____ Date _____

Solve the word problem using the RDW strategy. Show all your work.

1. Mr. Pham mowed 2/7 of a lawn. His son mowed 1/4 of it. Who mowed the most? How much of the lawn still needs to be mowed?

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Name _____ Date _____

Solve the word problem using the RDW strategy. Show all your work.

1. Christine baked a pumpkin pie. She ate 1/6 of the pie. Her brother ate 1/3 of it, and gave the left overs to his friends. What fraction of the pie did he give to his friends?

2. Liang went to the bookstore. He spent 1/3 of his money on a pen and 4/7 of it on books. What fraction of his money did he have left?

3. Tiffany bought 2/5 kg of cherries. Linda bought 1/10 kg of cherries less than Tiffany. How many kg of cherries did they buy altogether?

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4. Mr. Rivas bought a can of paint. He used 3/8 of it to paint a book shelf. He used 1/4 of it to paint a wagon. He used some of it to paint a bird house, and have 1/8 of paint left. How much paint did he use for the bird house?

5. Ribbon A is 1/3 m long. It is 2/5 m shorter than ribbon B. What's the total length of two ribbons?

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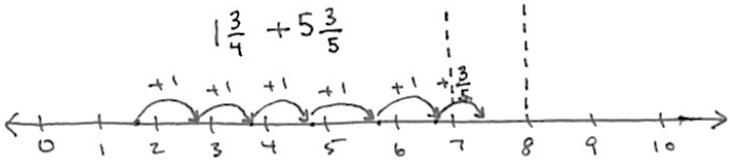
Topic C:

Fraction Addition and Subtraction: Making Like Units Numerically

5.NF.1, 5.NF.2

Focus Standard:	5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.).*
	5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
Instructional Days:	5	
Coherence -Links from:	G4–M5	Order and Operations with Fractions
	G5–M1	Whole Number and Decimal Fraction Place Value to the One Thousandths
-Links to:	G5–M4	Extensions and Applications of Multiplication and Division of Fractions and Decimal Fractions

Topic C uses the number line when adding and subtracting fractions greater than or equal to one. The number line helps students see that fractions are analogous to whole numbers. The number line makes it clear that numbers on the left are smaller than numbers on the right (which helps lead to integers in Grade 6). Using this tool, students recognize and manipulate fractions in relation to larger whole numbers and to each other (e.g., "Between what two whole numbers does the sum of 1 2/3 and 5 ¾ lie?")



___ < $1\frac{3}{4} + 5\frac{3}{5}$ < ___

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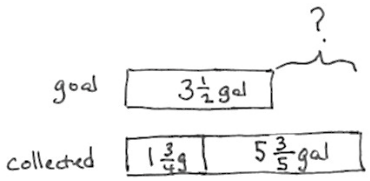
This leads to understanding and skill with solving more interesting problems, often embedded within multi-step word problems:

> Cristina and Matt's goal is to collect a total of 3 ½ gallon of sap from the maple trees. Cristina collected 1 ¾ gallon. Matt collected 5 3/5 gallon. By how much did they beat their goal?



$$1\frac{3}{4}\text{ gal} + 5\frac{3}{5}\text{ gal} - 3\frac{1}{2}\text{gal} = 3 + \left(\frac{3\times 5}{4\times 5}\right) + \left(\frac{3\times 4}{5\times 4}\right) - \left(\frac{1\times 10}{2\times 10}\right)$$

$$= 3 + \frac{15}{20} + \frac{12}{20} - \frac{10}{20} = 3\,\frac{17}{20}\text{ gal}$$

Cristina and Matt beat their goal by $3\frac{17}{20}$ gallons.

Word problems are part of every lesson. Students are encouraged to utilize bar diagrams, which facilitate analysis of the same part/whole relationships they have worked with since Grade 1.

CONCEPT CHART	A Teaching Sequence Towards Mastery of Adding and Subtracting by Partitioning to Make Like Units

Concept 1: Add Fractions to and Subtract Fractions from Whole Numbers Using Equivalence and the Number Line as Strategies (Lesson 8)

Concept 2: Add Fractions Making Like Units Numerically (Lesson 9)

a) $\frac{1}{2} + \frac{1}{3} = \left(\frac{1}{2}\times\frac{3}{3}\right) + \left(\frac{1}{3}\times\frac{2}{2}\right) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

b) $1\frac{1}{5} + \frac{2}{3} = \left(\frac{6}{5}\times\frac{3}{3}\right) + \left(\frac{2}{3}\times\frac{5}{5}\right) = \frac{18}{15} + \frac{10}{15} = \frac{28}{15} = 1\frac{13}{15}$

Concept 3: Add Fractions with Sums Greater than Two (Lesson 10)

Concept 4: Subtract Fractions Making Like Units Numerically (Lesson 11)

a) $\frac{7}{8} - \frac{5}{6} = \left(\frac{7}{8}\times\frac{6}{6}\right) - \left(\frac{5}{6}\times\frac{8}{8}\right) = \frac{42}{48} - \frac{40}{48} = \frac{2}{48} = \frac{1}{24}$

b) $1\frac{1}{7} - \frac{2}{9} = \left(\frac{8}{7}\times\frac{9}{9}\right) - \left(\frac{2}{9}\times\frac{7}{7}\right) = \frac{72}{63} - \frac{14}{63} = \frac{58}{63}$

Concept 5: Subtract Fractions Greater Than or Equal to One (Lesson 12)

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Mathematical Practices Brought to Life

Lessons 8, 10, and 11 highlight segments that exemplify MP.3, "Construct viable arguments and critique the reasoning of others." In Lesson 8, students share their reasoning for the various strategies they prefer to solve addition and subtraction of whole numbers and fractions. In Lesson 10, one student argues in favor of using eighths to find a sum when presented with unlike units, while another argues for sixteenths. In Lesson 11, students assess the reasonableness of their answers based on their work with the number line.

Lessons 9 and 12 highlight segments that demonstrate MP.7, "Look for and make use of structure." In Lesson 9, students first generate a pictorial solution and then analyze the representation to generate and solve abstract equations. In Lesson 12, students look back at the role of the number bond in problems since Grade 1. They analyze similarities between problems at each grade level, noticing how the thought process represented by the number bond consistently applies.

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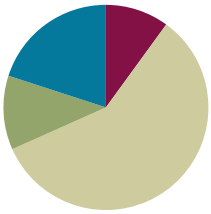
Lesson 8: Add Fractions to and Subtract Fractions from Whole Numbers Using Equivalence and the Number Line as Strategies

Suggested Lesson Structure

Fluency Practice	(6 minutes)
Application Problems	(7 minutes)
Concept Development	(35 minutes)
Student Debrief	(12 minutes)
Total Time	**(60 minutes)**



Fluency Practice (6 minutes)

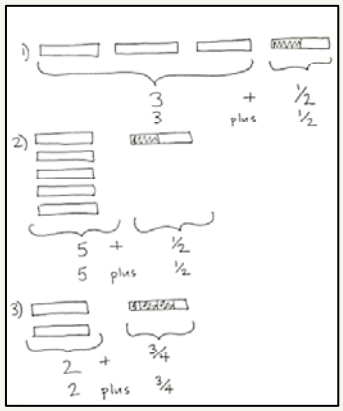
- Adding Whole Numbers and Fractions **4.NF.3a** (3 minutes)
- Subtracting Fractions from Whole Numbers **4.NF.3a** (3 minutes)

Adding Whole Numbers and Fractions (3 minutes)

T: I'll say the answer. You say the addition problem as a whole number and a fraction. 3 and 1 half.
S: 3 + 1 half.
T: 5 and 1 half.
S: 5 + 1 half.
T: 2 and 3 fourths.
S: 2 + 3 fourths.
T: 1 and 5 sixths.
S: 1 + 5 sixths.
T: Let's switch roles. I'll say the addition problem, you say the answer. 2 + 1 fifth.
S: 2 and 1 fifth.
T: 2 + 4 fifths.
S: 2 and 4 fifths.
T: 5 + 7 eighths.
S: 5 and 7 eighths.
T: 3 + 7 twelfths.
S: 3 and 7 twelfths.

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

If necessary, show numbers with bar diagrams to create a visual and slow the pace of the activity.



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Subtracting Fractions From Whole Numbers (3 minutes)

T: I'll say a subtraction sentence. You repeat the sentence and give the answer. 1 – 1 half.
S: 1 – 1 half = 1 half.
T: 2 – 1 half.
S: 2 – 1 half = 1 and 1 half.
T: 2 and 1 half – 1 half.
S: 2 and 1 half – 1 half = 2.
T: 6 – 1 fourth.
S: 6 – 1 fourth = 5 and 3 fourths.
T: 6 and 3 fourths – 3 fourths.
S: 6 and 3 fourths – 3 fourths = 6.

Repeat process with possible sequence:

$$3 - \frac{5}{6}, \quad 3\frac{5}{6} - \frac{5}{6}, \quad 4 - \frac{7}{8}, \quad 4\frac{7}{8} - \frac{7}{8}, \quad 5 - \frac{7}{12}, \quad 5\frac{7}{12} - \frac{7}{12}$$

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

As with the addition activity, have students represent the subtraction sentence with a bar diagram and cross off the subtracted amount.

Application Problem (7 minutes)

Jane found money in her pocket. She went to a convenience store and spent ¼ of her money on chocolate milk, 3/5 of her money on a magazine, and the rest of her money on candy. What fraction of her money did she spend on candy?

T: Let's read the problem together.
S: (Students read chorally.)
T: Quickly share with your partner how to solve this problem. (Circulate and listen.)
T: Malory, will you tell the class your plan?
S: I have to find like units for the cost of the milk and magazine. Then I can add them together. Then I can see how much more I would need to make 1 whole.
T: You have 2 minutes to solve the problem.
T: What like units did you find for the milk and magazine?
S: Twentieths.
T: Say your addition sentence with these like units.
S: 5 twentieths plus 12 twentieths equals 17 twentieths.

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

Add the following question as an extension for students performing above grade level:

How much does the magazine cost if she started with $10?

The question goes beyond the scope of the lesson, but may be an engaging challenge for some students.

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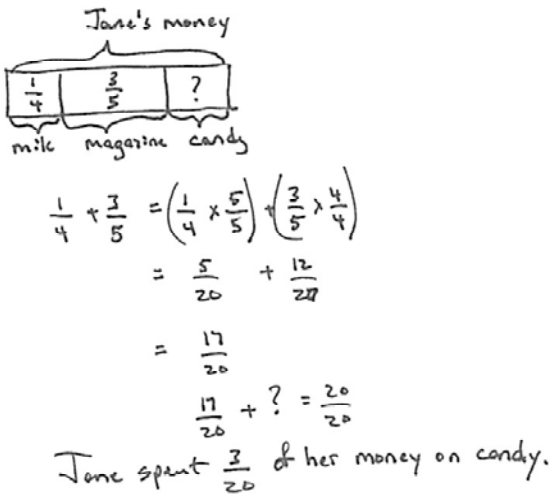
T: How many more twentieths do you need to make a whole?

S: 3 twentieths.

T: Tell your partner the answer in the form of a sentence.

S: Jane spent 3 twentieths of her money on candy.



Concept Development (35 minutes)

Materials: (S) Empty number line worksheet or lined paper

T: Discuss with your partner what addition problem would match this picture.

S: 1 + 1 ¾

Problem 1

$$1 + 1\frac{3}{4}$$

Draw a line or project the number line worksheet.

T: Start at zero. Travel one unit.

T: Start at 1 and travel one more equal unit. Where do we land?

S: 2.

T: How much more do I need to travel?

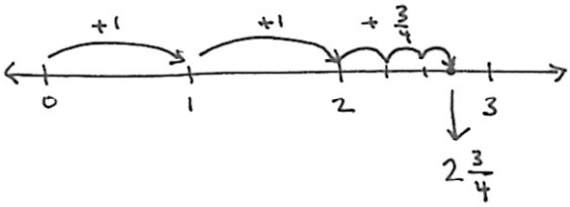
S: 3 fourths.

T: Will that additional distance be less than or more than one whole unit?

S: Less than one whole unit.

T: Make 3 smaller equal units, 1 fourth, 2 fourths, 3 fourths. What is 2 plus 3 fourths?

S: 2 and 3 fourths.



NOTES ON THE EMPTY NUMBER LINE:

It may be easier for your students to use a worksheet with the empty number line on it. However if you are limited with printing worksheets for your class, lined paper will do the same job. Encourage students to make number lines by tracing the blue lines on lined paper. The increments on rulers can be a distraction. Our goal is for students to use the line as a helpful tool for visualizing the addition and subtraction, and to contextualize fractions within the set of whole numbers.

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S: 1 plus 1 and 3 fourths equals 2 and 3 fourths.

$$1 + 1\frac{3}{4}$$
$$= 1 + 1 + \frac{3}{4}$$
$$= 2\frac{3}{4}$$

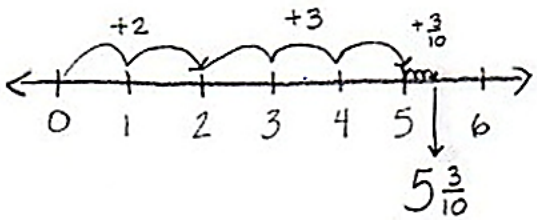
Problem 2

$$\mathbf{2\frac{3}{10} + 3}$$

T: Talk to your partner: How should we solve this?

S: "First add 2." → "3 tenths comes next so add that." → "Adding all the whole numbers first might be easier." → "Adding the numbers as they are written is best so you don't forget the fractions or whole numbers." → "Adding the whole numbers first will make the number line easier to read and it's similar to how we add all the ones, then the tens, then the hundreds. Add like numbers or units first."

T: Let's travel 2, then 3 more units on our number line. (Show on the board.) Can someone explain how to travel 3 tenths?



S: 1 tenth is much smaller than a whole, so make 3 very small units. Label the final one 5 3/10.

T: Say your complete number sentence.

S: 2 and 3 tenths plus 3 equals 5 and 3 tenths.

$$\mathbf{2\frac{3}{10} + 3}$$
$$\mathbf{= 2 + 3 + \frac{3}{10}}$$
$$\mathbf{= 5\frac{3}{10}}$$

T: What do you notice about the fractional units when adding it to a whole number?

S: The fraction amount doesn't change. All we have to do is add the whole numbers.

Problem 3

$$\mathbf{1 - \frac{1}{4}}$$

T: Read the problem.

S: 1 minus 1 fourth.

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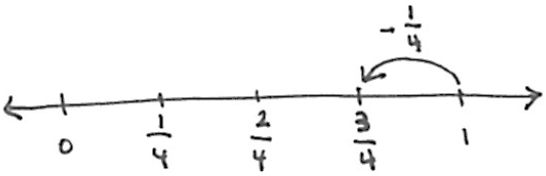
T: On the number line, let's start at 1 because that's the whole.

T: When I subtract ¼ from 1, my answer will lie between what 2 whole numbers?

S: 0 and 1.

T: (Write 0 on the number line.) Because the answer will be between 0 and 1, the whole number will be 0. We will partition the number line into fourths. Starting at 1, let's travel back 1 fourth. (Mark the unit.)

S: 1 minus 1 fourth equals 3 fourths.



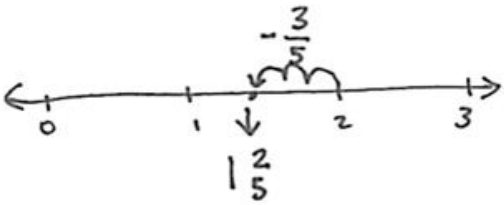
$$1 - \frac{1}{4}$$

$$= \frac{4}{4} - \frac{1}{4}$$

$$= \frac{3}{4}$$

Problem 4

T: Discuss with your partner your strategy for solving this problem. (Listen to students discuss for 30 seconds.)



$$2 - \frac{3}{5}$$

T: I will start at the whole number 2 on my number line. Am I subtracting a whole number?

S: No.

T: My answer will lie between what 2 whole numbers?

S: 1 and 2.

T: If your answer lies between 1 and 2, what is the whole number part of your answer?

S: 1.

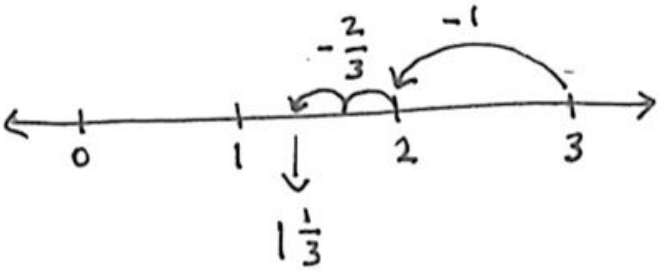
T: With your partner, subtract 3 fifths on the number line.

Allow students 1 minute to solve the problem with their partner using the number line. Review the problem counting back 3 fifths on the number line. Ask for students to submit answers, rather than giving the answers.

$$2 - \frac{3}{5}$$

$$= 1 + (1 - \frac{3}{5})$$

$$= 1\ \frac{2}{5}$$



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Problem 5

T: Let's say this subtraction sentence.

$$3 - 1\frac{2}{3}$$

S: 3 minus 1 and 2 thirds.

T: First, we will subtract the whole number 1 and then subtract the fraction 2 thirds. Start with 3 on the number line and subtract 1 whole. (Show the subtraction of the unit.)

T: When you subtract the fraction 2 thirds, what 2 whole numbers will your answer lie between?

S: Between 1 and 2.

T: You have one minute to complete this problem with your partner.

$$3 - 1\frac{2}{3}$$

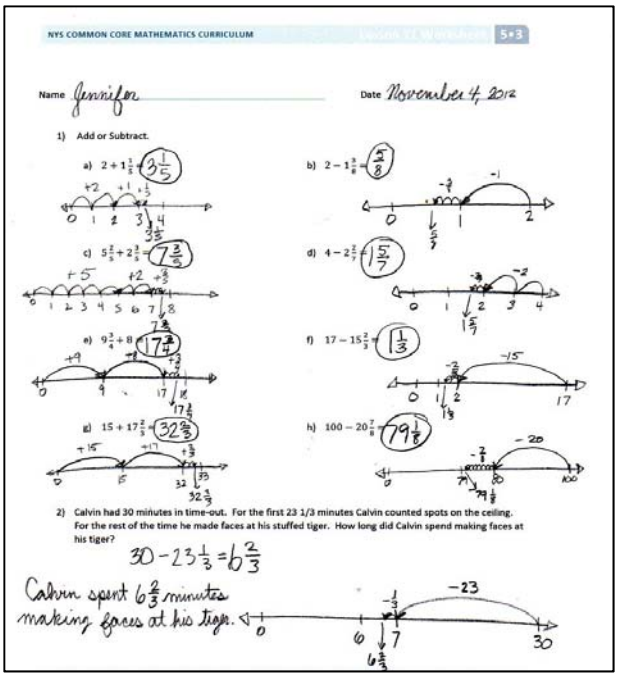
$$= (3 - 1) - \frac{2}{3}$$

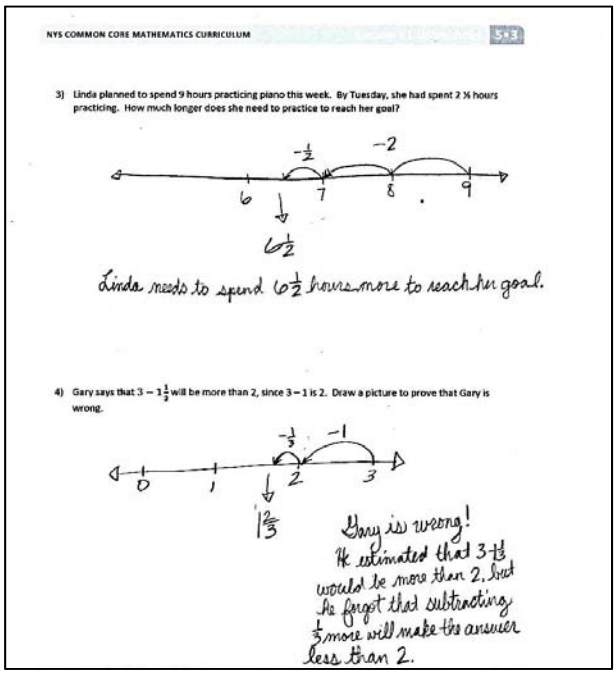
$$= 2 - \frac{2}{3}$$

$$= 1\frac{1}{3}$$

Activity Worksheet (12 minutes)

Students complete the worksheet individually for 12 minutes.





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Student Debrief (12 minutes)

Materials: (S) Worksheet

T: Please take two minutes to check your answers with your partner. Do not change any of your answers. (Allow students to work.)

T: I will say the addition or subtraction problem. Share your answers out loud to check your work. Letter a) 2 plus 1 and 1 fifth equals?

S: 3 and 1 fifth.

Continue with sequence.

T: Take the next two minutes to discuss the worksheet with your partner. Did you notice anything new? Are there any patterns? (Students discuss. Circulate and listen for conversations that can be shared with the whole class.)

T: Carla, will you tell us what you noticed about letter 'c'?

S: I added the whole numbers and got 7, but then I realized that the fractions added up to 5 fifths. That's one whole, so I had to add that to 7 and got 8 for my answer.

MP.3

T: Benjamin, what were you saying about the addition problems compared to the subtraction problems?

S: Addition takes less time and thinking. Just add the whole numbers and write in the fraction. But with subtraction, you have to think harder. First you subtract the whole numbers, but that won't be your whole number answer. You have to make it one number smaller. Like in letter 'e'. 17 minus 15 equals 2 but the answer won't be 2; it will be between 1 and 2. So I write down the whole number 1, and then figure out the fraction.

T: Tammy, how did you find the fraction that Benjamin mentioned?

S: For finding the fraction part of subtraction, I like to count up. For example, in letter 'd' I found the whole number and then said 3/7, 4/7, 5/7, 6/7, 7/7. That's 5 groups of sevenths. So the fraction is 5/7.

T: So many of us are finding our own strategies for solving addition and subtraction of whole numbers and fractions. Share with your partner your own strategies. Listen carefully and see if you learn a new strategy to try.

S: (Students talk. If time permits, ask for two students to share what they heard.)

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

You may want to project work or have students show their strategies visually as they share. Also, give the class an opportunity to ask the presenter questions. You may want to ask students to retell particularly efficient strategies to a partner to help them internalize either language or content, depending on need.

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Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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Name _____ Date _____

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Name _____ Date _____

1) Add or Subtract.

a) $2 + 1\frac{1}{5} =$

b) $2 - 1\frac{3}{8} =$

c) $5\frac{2}{5} + 2\frac{3}{5} =$

d) $4 - 2\frac{2}{7} =$

e) $9\frac{3}{4} + 8 =$

f) $17 - 15\frac{2}{3} =$

g) $15 + 17\frac{2}{3} =$

h) $100 - 20\frac{7}{8} =$

2) Calvin had 30 minutes in time-out. For the first 23 1/3 minutes, Calvin counted spots on the ceiling. For the rest of the time he made faces at his stuffed tiger. How long did Calvin spend making faces at his tiger?

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3) Linda planned to spend 9 hours practicing piano this week. By Tuesday, she had spent 2 ½ hours practicing. How much longer does she need to practice to reach her goal?

4) Gary says that $3 - 1\frac{1}{3}$ will be more than 2, since 3 – 1 is 2. Draw a picture to prove that Gary is wrong.

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Name _____ Date _____

Directions: Add or Subtract.

1) $5 + 1\frac{7}{8} =$

2) $3 - 1\frac{3}{4} =$

3) $7\frac{3}{8} + 4 =$

4) $4 - 2\frac{3}{7} =$

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Name _____ Date _____

1) Add or Subtract.

a) $3 + 1\frac{1}{4} =$

b) $2 - 1\frac{5}{8} =$

c) $5\frac{2}{5} + 2\frac{3}{5} =$

d) $4 - 2\frac{5}{7} =$

d) $8\frac{4}{5} + 7 =$

e) $18 - 15\frac{3}{4} =$

f) $16 + 18\frac{5}{6} =$

g) $100 - 50\frac{3}{8} =$

2) The total length of two ribbons is 13 meters. If one ribbon is $7\frac{5}{8}$ meters long, what is the length of the other ribbon?

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3) It took Sandy two hours to jog 13 miles. She ran 7 ½ miles in the first hour. How far did she run during the second hour?

4) Andre says that $5\frac{3}{4} + 2\frac{1}{4} = 7\frac{1}{2}$ because $7\frac{4}{8} = 7\frac{1}{2}$. Identify his mistake. Draw a picture to prove that he is wrong.

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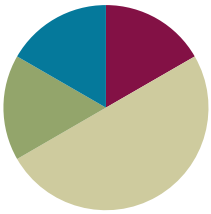
Lesson 9: Add Fractions Making Like Units Numerically

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Application Problem	(10 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (10 minutes)

- Adding and Subtracting Fractions with Like Units **4.NF.3a** (1 minute)
- Sprint **4.NF.3a** (9 minutes)

Adding and Subtracting Fractions with Like Units (1 minute)

T: I'll say an addition or subtraction sentence. You say the answer. 2 fifths + 1 fifth.

S: 3 fifths.

T: 2 fifths – 1 fifth.

S: 1 fifth.

T: 2 fifths + 2 fifths.

S: 4 fifths.

T: 2 fifths – 2 fifths.

S: Zero.

T: 3 fifths + 2 fifths.

S: 1.

T: I'm going to write an addition sentence. You say true or false.

T: (Write) $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$

S: True.

T: (Write) $\frac{3}{7}+\frac{3}{7}=\frac{6}{14}$

S: False.

T: Say the answer that makes this addition sentence true.

S: 3 sevenths + 3 sevenths = 6 sevenths.

NOTES ON SCAFFOLDING ELLS:

Provide written equations along with oral. Colored response cards (green=true and red=false) can help scaffold responses to the statement "Tell me if it's true or false." This statement might also be simplified to "Is it right?" to which ELLS may respond "Yes" or "No."

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T: (Write) $\frac{5}{9}+\frac{2}{9}=\frac{7}{18}$

T: True or false?

S: False.

T: Say the answer that will make this addition sentence true.

S: 5 ninths + 2 ninths = 7 ninths.

T: (Write) $\frac{5}{9}+\frac{4}{9}=1$

T: True or false?

S: True.

T: Great work. You're ready for your sprint!

Sprint (9 minutes)

Materials: (S) Add and Subtract Fractions with Like Units Sprint (See directions for administration of sprints in the **G5-M3 Fluency Progressions.**)

Application Problems (10 minutes)

Hannah and her friend are training to run in a 2 mile race. On Monday, Hannah runs 1/2 mile. On Tuesday, she runs 1/5 mile further than she ran on Monday.

a. How far did Hannah run on Tuesday?

b. If her friend ran ¾ mile on Tuesday, how many miles did the girls run in all on Tuesday?

T: Use the RDW (read, draw, write) process to solve with your partner.

S: (Students read, draw and write an equation, as well as a word sentence.)

T: (Debrief the problem.) Could you use the same units to answer Parts 'a' and 'b'? Why or why not?

S: No. There's no easy way to change fourths to tenths.

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

This problem may feel like review for some students. Consider extending it by asking, "If Hannah keeps to this training pattern, how many days will it take her to reach a distance of 2 miles?"

You might also task students with generating other questions that could be asked about the story. For example:

- How far did Hannah run in 5 days?
- How much farther did Hannah run than her friend on Tuesday?
- How much farther did Hannah run on day 10 than day 1?

If students offer a question for which there is insufficient information, ask how the problem could be altered in order for their question to be answered.

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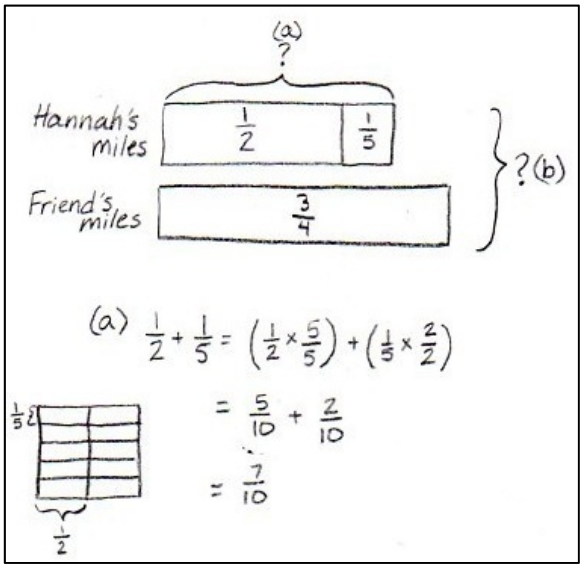

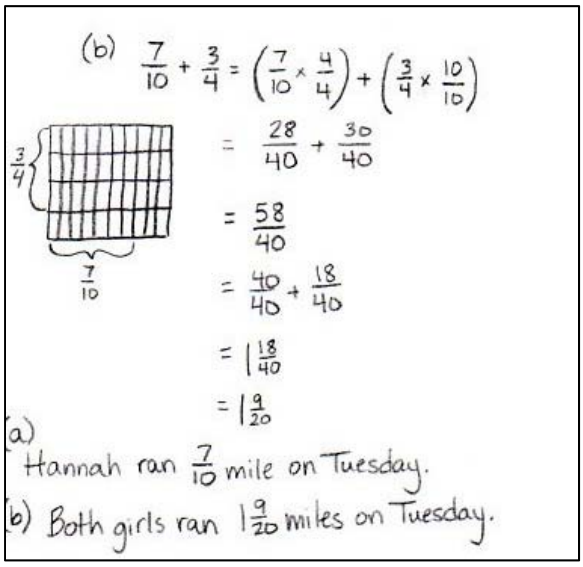

Concept Development (30 minutes)

Materials: (S) Personal white boards

Problem 1

MP.7

T: How did you decide to use tenths in the first part of our application problem? Turn and talk.

S: "We can draw a rectangle and split it using the other unit." → "Since we had halves and fifths, we drew two parts and then split them into 5 parts each. That made 10 parts for the halves. That meant the fifths were each 2 smaller units, too."

T: Turn and talk: What happened to the number of units we selected when we split our rectangle?

S: "Instead of one part, now we have five." → "The number of selected parts is five times more." → "The total number of parts is now 10."

T: What happened to the size?

S: The units got smaller.

T: Let me record what I hear you saying. Does this equation say the same thing?

NOTES ON SCAFFOLDING ELLS:

"Turn and talk" allows ELLS the opportunity to practice academic language in a relatively low-stakes setting. It also allows time and space for them to formulate responses before sharing with the whole class.

Consider pairing ELLS with each other at first and then shift the language composition of groups over the course of the year.

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(Record the following equation.)

$$(\frac{1}{2}\times\frac{5}{5})=\frac{5}{10}$$

5 times as many selected units
5 times as many units in the whole

MP.7

S: Yes!

T: Write an equation like mine to explain what happened to the fifths.

T: (Circulate and listen.) Jennifer, can you share for us?

S: The number of parts we had doubled. The units are half as big as before, but there are twice as many of them.

$$(\frac{1}{5}\times\frac{2}{2})=\frac{2}{10}$$

Number of parts doubled or 2 times as many parts
Number of units in whole doubled or twice as many parts in the whole

T: Then, of course, we could add our two fractions together. (Write)

$$(\frac{1}{2}\times\frac{5}{5})+(\frac{1}{5}\times\frac{2}{2})$$
$$=\frac{5}{10}+\frac{2}{10}$$
$$=\frac{7}{10}$$

T: Are there other units we could have used to make these denominators the same? Another way to ask that question is: Do 2 and 5 have other common multiples?

S: Yes. We could have used 20ths, 30ths, or 50ths….

T: If we had used 20ths, how many slices would we need to change ½? To change 1/5? Turn and talk. Draw a model on your personal board if necessary.

T: Let's hear your ideas.

S: "10 slices for half." → "Ten times as many units in the whole and 10 times as many units that we selected." → "4 slices for fifths." → "4 times as many selected units and 4 times as many units in the whole, but they are smaller in size."

T: Let's record that on our boards in equation form. (Write)

$$(\frac{1}{2}\times\frac{10}{10})+(\frac{1}{5}\times\frac{4}{4})$$
$$=\frac{10}{20}+\frac{4}{20}$$
$$=\frac{14}{20}$$

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T: Is 14/20 the same amount as 7/10?

S: "Yes, they are equivalent." → "7/10 is simplified."

T: Express 1/2 + 1/5 using another unit. Show your thoughts with an equation.

S: (Students draw and write appropriate representations.)

T: Who used the smallest unit? Who used the largest unit? Who had the least or most units in their whole? Turn and talk.

T: (After students share.) Garrett and Pete, please share your findings.

S: "I used 30ths so I had to multiply by 15 to make ½ and multiply by 6 to make 1/5. That's 21/30 in all." → "I used 50ths. I had smaller units in my whole, so I needed 25 to make ½ and 10 to make 1/5. That's 35/50 in all."

T: Look at this statement. What do the types of units we used have in common?

$$\frac{1}{2}+\frac{1}{5}=\frac{7}{10}=\frac{14}{20}=\frac{21}{30}=\frac{35}{50}$$

S: "All of the units are smaller than halves and fifths." → "All are common multiples of 2 and 5." → "All are multiples of ten."

T: Will the new unit always be a multiple of the original units? Try to answer this as we consider the next problem.

Problem 2

$$\frac{1}{2}+\frac{2}{3}=$$

T: How does this problem compare with our first?

S: "It's still adding 1/2 to something else." → "The first problem was two unit fractions." → "This one only has one unit fraction." → "We were adding an amount less than half to 1/2 in the first, but 2/3 is more than half."

T: Great observations! What can we expect to change about our answer? How about the units we use?

S: "We should get a fraction greater than one." → "We won't use most of the units from before."

T: How can you be sure?

S: "We are adding half and more than half." → "Only one of our units from before is a multiple of 3."

T: Imagine the rectangle that helps you find a common unit. Record an equation that explains what you saw in your mind's eye.

T: (Circulate and observe.)

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T: Tia, show us your equation and explain it.

S: (Student displays equation.) I used sixths. My equation shows that for 1/2, the selected pieces tripled and the units in the whole tripled too. For 2/3, the parts doubled and so did the units in the whole.

$$\left(\frac{1}{2}\times\frac{3}{3}\right)+\left(\frac{2}{3}\times\frac{2}{2}\right)$$
$$=\frac{3}{6}+\frac{4}{6}$$
$$=\frac{7}{6} \text{ or } 1\frac{1}{6}$$

T: Was our prediction about the answer correct?

S: Yes! It was greater than one!

T: Did anyone use another unit to find the sum?

(Record sums on board as students respond.)

T: Do these units follow the pattern we saw in our earlier work? Let's keep looking for evidence as we work.

Problem 3

$$\frac{5}{9}+\frac{5}{6}=$$

T: Compare this problem to the others. Turn and talk.

S: "My partner and I see different things. I think this one is like Problem 2 because the addends are more than half. But my partner says this one is like Problem 1 because the numerators are the same, even though they are not unit fractions." → "This one was different because you can find a bigger unit—18ths work as a common unit, but that's not the unit you get if you multiply 6 and 9."

T: Find the sum. Use an equation to show your thinking.

Follow a similar procedure to Problems 1 and 2 for debriefing the solution.

Problem 4

$$\frac{2}{3}+\frac{1}{4}+\frac{1}{2}=$$

T: This problem has three addends. Will this affect our approach to solving?

S: No. We still have to use a common unit. It has to be a multiple of all three denominators.

T: Find the sum using an equation. (Debrief as above.)

T: To wrap up, what patterns have you observed about the common units?

S: "All the new units we found are common multiples of our original units." → "We don't always have to multiply the original units to find a common multiple." → "You can skip count by the largest common unit to find smaller common units."

NOTES ON SCAFFOLDING STUDENTS BELOW GRADE LEVEL:

While the thrust of this lesson is the transition between pictorial and abstract representations of common units, allow students to continue to use the "sliced rectangle" from previous lessons as a scaffold for writing equations.

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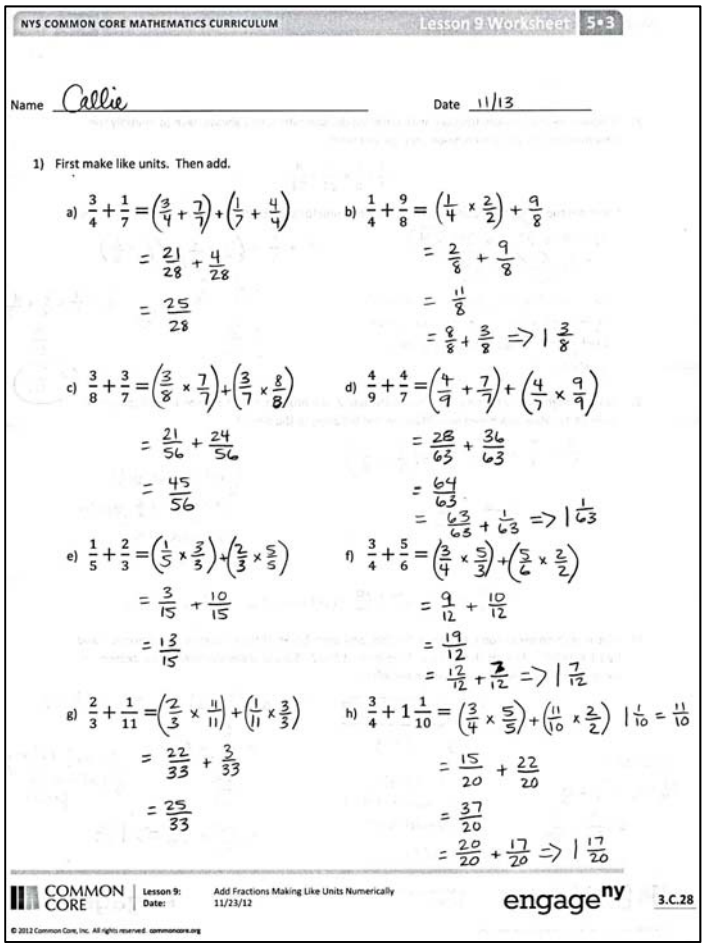
Activity Worksheet (10 minutes)

Distribute the activity worksheet (pictured below). Students work independently until there are 10 minutes remaining in the class period.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 9 Worksheet 5•3

Name Callie Date 11/13

1) First make like units. Then add.

a) $\frac{3}{4}+\frac{1}{7}=\left(\frac{3}{4}+\frac{7}{7}\right)+\left(\frac{1}{7}+\frac{4}{4}\right)$
$=\frac{21}{28}+\frac{4}{28}$
$=\frac{25}{28}$

b) $\frac{1}{4}+\frac{9}{8}=\left(\frac{1}{4}\times\frac{2}{2}\right)+\frac{9}{8}$
$=\frac{2}{8}+\frac{9}{8}$
$=\frac{11}{8}$
$=\frac{8}{8}+\frac{3}{8}\Rightarrow 1\frac{3}{8}$

c) $\frac{3}{8}+\frac{3}{7}=\left(\frac{3}{8}\times\frac{7}{7}\right)+\left(\frac{3}{7}\times\frac{8}{8}\right)$
$=\frac{21}{56}+\frac{24}{56}$
$=\frac{45}{56}$

d) $\frac{4}{9}+\frac{4}{7}=\left(\frac{4}{9}+\frac{7}{7}\right)+\left(\frac{4}{7}\times\frac{9}{9}\right)$
$=\frac{28}{63}+\frac{36}{63}$
$=\frac{64}{63}$
$=\frac{63}{63}+\frac{1}{63}\Rightarrow 1\frac{1}{63}$

e) $\frac{1}{5}+\frac{2}{3}=\left(\frac{1}{5}\times\frac{3}{3}\right)+\left(\frac{2}{3}\times\frac{5}{5}\right)$
$=\frac{3}{15}+\frac{10}{15}$
$=\frac{13}{15}$

f) $\frac{3}{4}+\frac{5}{6}=\left(\frac{3}{4}\times\frac{5}{3}\right)+\left(\frac{5}{6}\times\frac{2}{2}\right)$
$=\frac{9}{12}+\frac{10}{12}$
$=\frac{19}{12}$
$=\frac{12}{12}+\frac{7}{12}\Rightarrow 1\frac{7}{12}$

g) $\frac{2}{3}+\frac{1}{11}=\left(\frac{2}{3}\times\frac{11}{11}\right)+\left(\frac{1}{11}\times\frac{3}{3}\right)$
$=\frac{22}{33}+\frac{3}{33}$
$=\frac{25}{33}$

h) $\frac{3}{4}+1\frac{1}{10}=\left(\frac{3}{4}\times\frac{5}{5}\right)+\left(\frac{11}{10}\times\frac{2}{2}\right)$ $1\frac{1}{10}=\frac{11}{10}$
$=\frac{15}{20}+\frac{22}{20}$
$=\frac{37}{20}$
$=\frac{20}{20}+\frac{17}{20}\Rightarrow 1\frac{17}{20}$

COMMON CORE Lesson 9: Add Fractions Making Like Units Numerically
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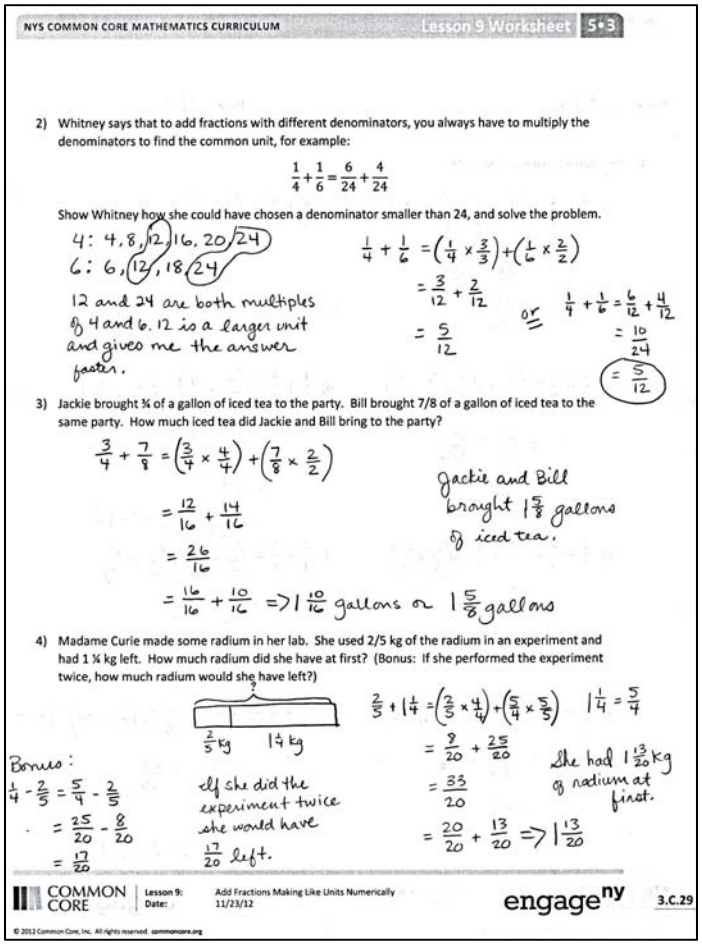

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 9 Worksheet 5•3

2) Whitney says that to add fractions with different denominators, you always have to multiply the denominators to find the common unit, for example:

$$\frac{1}{4}+\frac{1}{6}=\frac{6}{24}+\frac{4}{24}$$

Show Whitney how she could have chosen a denominator smaller than 24, and solve the problem.

4: 4, 8, 12, 16, 20, 24
6: 6, 12, 18, 24

12 and 24 are both multiples of 4 and 6. 12 is a larger unit and gives me the answer faster.

$\frac{1}{4}+\frac{1}{6}=\left(\frac{1}{4}\times\frac{3}{3}\right)+\left(\frac{1}{6}\times\frac{2}{2}\right)$
$=\frac{3}{12}+\frac{2}{12}$
$=\frac{5}{12}$

or $\frac{1}{4}+\frac{1}{6}=\frac{6}{12}+\frac{4}{12}$
$=\frac{10}{24}$
$=\frac{5}{12}$

3) Jackie brought ¾ of a gallon of iced tea to the party. Bill brought 7/8 of a gallon of iced tea to the same party. How much iced tea did Jackie and Bill bring to the party?

$\frac{3}{4}+\frac{7}{8}=\left(\frac{3}{4}\times\frac{4}{4}\right)+\left(\frac{7}{8}\times\frac{2}{2}\right)$
$=\frac{12}{16}+\frac{14}{16}$
$=\frac{26}{16}$
$=\frac{16}{16}+\frac{10}{16}\Rightarrow 1\frac{10}{16}$ gallons or $1\frac{5}{8}$ gallons

Jackie and Bill brought $1\frac{5}{8}$ gallons of iced tea.

4) Madame Curie made some radium in her lab. She used 2/5 kg of the radium in an experiment and had 1¼ kg left. How much radium did she have at first? (Bonus: If she performed the experiment twice, how much radium would she have left?)

$\frac{2}{5}$ kg, $1\frac{1}{4}$ kg, ?

$\frac{2}{5}+1\frac{1}{4}=\left(\frac{2}{5}\times\frac{4}{4}\right)+\left(\frac{5}{4}\times\frac{5}{5}\right)$ $1\frac{1}{4}=\frac{5}{4}$
$=\frac{8}{20}+\frac{25}{20}$
$=\frac{33}{20}$
$=\frac{20}{20}+\frac{13}{20}\Rightarrow 1\frac{13}{20}$

She had $1\frac{13}{20}$ kg of radium at first.

Bonus:
$\frac{1}{4}-\frac{2}{5}=\frac{5}{4}-\frac{2}{5}$
$=\frac{25}{20}-\frac{8}{20}$
$=\frac{17}{20}$

If she did the experiment twice she would have $\frac{17}{20}$ left.

COMMON CORE Lesson 9: Add Fractions Making Like Units Numerically
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Student Debrief (10 minutes)

Materials: (S) Worksheet

T: Take two minutes to check your answers with your partner. Please do not change any of them. (Allow time for students to confer.)

T: Did you notice any patterns in the sums on this sheet?

S: "The answers in the first column were all less than a whole." → "The answers in the second column

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were more than a whole."

T: I noticed that Problem 'b' is different from all the other problems. Can you explain how it is different?

S: "'b' is different because I only had to change the unit of one fraction to be like the other one." → "One unit is a multiple of the other." → "Fourths can be made out of eighths. None of the others were like that."

T: John, please share your answer and your partner's answer to problem 'b'.

S: I got 1 and 3/8, but Kate got 1 and 1/32.

T: Class, is it a problem that John and Kate's answers to 'b' are different?

S: "No. It is the same amount. They just used different units." → "You don't always have to multiply."

T: Did this situation come up more often in some problems than others?

S: Yes. It happened more in 'f' and 'h'.

T: Why?

MP.7

S: "Multiplying the units together in these didn't give us the largest unit they had in common." → "I could find a smaller common multiple than just multiplying them together." → "I could skip-count by the bigger number and say a multiple of the other number that was smaller than multiplying them together." → "If I multiplied them together, I could simplify the answer I got to use a bigger unit."

T: How can these observations help you answer #2?

S: "Number 2 was like 'f' and 'h'. There was a bigger unit in common." → "You can slice the units by the same number to get a common unit."

T: Terrific insights! Put them to use as you complete your Exit Ticket.

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A

Correct_____

Add or subtract.

1	$\frac{1}{5}+\frac{1}{5}=$	/	23	$\frac{1}{9}+\frac{1}{9}+\frac{1}{9}=$	/
2	$\frac{1}{10}+\frac{5}{10}=$	/	24	$\frac{1}{9}+\frac{3}{9}+\frac{1}{9}=$	/
3	$\frac{1}{10}+\frac{7}{10}=$	/	25	$\frac{4}{9}-\frac{1}{9}-\frac{3}{9}=$	/
4	$\frac{2}{5}+\frac{2}{5}=$	/	26	$\frac{1}{4}+\frac{2}{4}+\frac{1}{4}=$	/
5	$\frac{5}{10}-\frac{4}{10}=$	/	27	$\frac{1}{8}+\frac{3}{8}+\frac{2}{8}=$	/
6	$\frac{3}{5}-\frac{1}{5}=$	/	28	$\frac{5}{12}+\frac{1}{12}+\frac{5}{12}=$	/
7	$\frac{3}{10}+\frac{3}{10}=$	/	29	$\frac{2}{9}+\frac{3}{9}+\frac{2}{9}=$	/
8	$\frac{4}{5}-\frac{1}{5}=$	/	30	$\frac{3}{10}-\frac{3}{10}+\frac{3}{10}=$	/
9	$\frac{1}{4}+\frac{1}{4}=$	/	31	$\frac{3}{5}-\frac{1}{5}-\frac{1}{5}=$	/
10	$\frac{1}{4}+\frac{2}{4}=$	/	32	$\frac{1}{6}+\frac{2}{6}=$	/
11	$\frac{3}{12}-\frac{2}{12}=$	/	33	$\frac{3}{12}+\frac{4}{12}=$	/
12	$\frac{1}{4}+\frac{3}{4}=$	/	34	$\frac{3}{12}+\frac{6}{12}=$	/
13	$\frac{1}{12}+\frac{1}{12}=$	/	35	$\frac{4}{8}+\frac{2}{8}=$	/
14	$\frac{1}{3}+\frac{1}{3}=$	/	36	$\frac{4}{12}+\frac{1}{12}=$	/
15	$\frac{3}{12}-\frac{2}{12}=$	/	37	$\frac{1}{5}+\frac{3}{5}=$	/
16	$\frac{5}{12}+\frac{6}{12}=$	/	38	$\frac{2}{5}+\frac{2}{5}=$	/
17	$\frac{7}{12}+\frac{4}{12}=$	/	39	$\frac{1}{6}+\frac{2}{6}=$	/
18	$\frac{4}{6}-\frac{1}{6}=$	/	40	$\frac{5}{12}-\frac{3}{12}=$	/
19	$\frac{1}{6}+\frac{2}{6}=$	/	41	$\frac{7}{15}-\frac{2}{15}=$	/
20	$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=$	/	42	$\frac{7}{15}-\frac{3}{15}=$	/
21	$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=$	/	43	$\frac{11}{15}-\frac{2}{15}=$	/
22	$\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=$	/	44	$\frac{2}{15}+\frac{4}{15}=$	/

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B Improvement_____ # Correct_____

Add or subtract.

1	$\frac{1}{2}+\frac{1}{2}=$	/	23	$\frac{1}{12}+\frac{6}{12}+\frac{2}{12}=$	/
2	$\frac{2}{8}+\frac{1}{8}=$	/	24	$\frac{4}{12}+\frac{3}{12}+\frac{3}{12}=$	/
3	$\frac{2}{8}+\frac{3}{8}=$	/	25	$\frac{8}{12}-\frac{4}{12}-\frac{4}{12}=$	/
4	$\frac{2}{12}-\frac{1}{12}=$	/	26	$\frac{1}{10}+\frac{2}{10}+\frac{4}{10}=$	/
5	$\frac{5}{12}+\frac{2}{12}=$	/	27	$\frac{1}{10}+\frac{1}{10}+\frac{6}{10}=$	/
6	$\frac{4}{8}+\frac{3}{8}=$	/	28	$\frac{4}{6}+\frac{1}{6}+\frac{1}{6}=$	/
7	$\frac{4}{8}-\frac{3}{8}=$	/	29	$\frac{2}{12}+\frac{3}{12}+\frac{4}{12}=$	/
8	$\frac{1}{8}+\frac{5}{8}=$	/	30	$\frac{2}{10}+\frac{4}{10}+\frac{4}{10}=$	/
9	$\frac{3}{4}-\frac{1}{4}=$	/	31	$\frac{3}{10}+\frac{1}{10}+\frac{2}{10}=$	/
10	$\frac{3}{6}-\frac{3}{6}=$	/	32	$\frac{4}{6}-\frac{2}{6}=$	/
11	$\frac{3}{9}+\frac{3}{9}=$	/	33	$\frac{3}{12}-\frac{2}{12}=$	/
12	$\frac{2}{3}+\frac{1}{3}=$	/	34	$\frac{2}{3}+\frac{1}{3}=$	/
13	$\frac{6}{9}-\frac{4}{9}=$	/	35	$\frac{2}{4}+\frac{1}{4}=$	/
14	$\frac{5}{9}-\frac{3}{9}=$	/	36	$\frac{3}{12}+\frac{2}{12}=$	/
15	$\frac{2}{9}+\frac{2}{9}=$	/	37	$\frac{1}{5}+\frac{2}{5}=$	/
16	$\frac{1}{12}+\frac{3}{12}=$	/	38	$\frac{4}{5}-\frac{4}{5}=$	/
17	$\frac{5}{12}-\frac{4}{12}=$	/	39	$\frac{5}{12}-\frac{1}{12}=$	/
18	$\frac{9}{12}-\frac{6}{12}=$	/	40	$\frac{6}{8}+\frac{2}{8}=$	/
19	$\frac{6}{10}-\frac{4}{10}=$	/	41	$\frac{2}{8}+\frac{2}{8}+\frac{2}{8}=$	/
20	$\frac{2}{8}+\frac{2}{8}+\frac{2}{8}=$	/	42	$\frac{9}{10}-\frac{7}{10}-\frac{1}{10}=$	/
21	$\frac{1}{10}+\frac{1}{10}+\frac{1}{10}=$	/	43	$\frac{2}{10}+\frac{5}{10}+\frac{2}{10}=$	/
22	$\frac{7}{10}-\frac{2}{10}-\frac{4}{10}=$	/	44	$\frac{9}{12}-\frac{1}{12}-\frac{4}{12}=$	/

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Name _____ Date _____

1) First make like units. Then add.

a) $\frac{3}{4} + \frac{1}{7} =$

b) $\frac{1}{4} + \frac{9}{8} =$

c) $\frac{3}{8} + \frac{3}{7} =$

d) $\frac{4}{9} + \frac{4}{7} =$

e) $\frac{1}{5} + \frac{2}{3} =$

f) $\frac{3}{4} + \frac{5}{6} =$

g) $\frac{2}{3} + \frac{1}{11} =$

h) $\frac{3}{4} + 1\frac{1}{10} =$

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2) Whitney says that to add fractions with different denominators, you always have to multiply the denominators to find the common unit, for example:

$$\frac{1}{4}+\frac{1}{6}=\frac{6}{24}+\frac{4}{24}$$

Show Whitney how she could have chosen a denominator smaller than 24, and solve the problem.

3) Jackie brought $\frac{3}{4}$ of a gallon of iced tea to the party. Bill brought $\frac{7}{8}$ of a gallon of iced tea to the same party. How much iced tea did Jackie and Bill bring to the party?

4) Madame Curie made some radium in her lab. She used $\frac{2}{5}$ kg of the radium in an experiment and had $1\frac{1}{4}$ kg left. How much radium did she have at first? (Bonus: If she performed the experiment twice, how much radium would she have left?)

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Name _____ Date _____

Make like units, then add.

1. $\frac{1}{6} + \frac{3}{4} =$

2. $1\frac{1}{2} + \frac{2}{5} =$

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Name _____ Date _____

1. Make like units, then add. Use an equation to show your thinking.

a) $\frac{3}{5} + \frac{1}{3} =$

b) $\frac{3}{5} + \frac{1}{11} =$

c) $\frac{2}{9} + \frac{5}{6} =$

d) $\frac{2}{5} + \frac{1}{4} + \frac{1}{10} =$

e) $\frac{1}{3} + \frac{7}{5} =$

f) $\frac{5}{8} + \frac{7}{12} =$

g) $1\frac{1}{3} + \frac{3}{4} =$

h) $\frac{5}{6} + 1\frac{1}{4} =$

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2. On Monday, Ka practices guitar for $\frac{2}{3}$ of one hour. When she's finished, she practices piano for $\frac{3}{4}$ of one hour. How much time did Ka spend practicing instruments on Monday?

3. Ms. How buys a bag of rice to cook dinner. She used $\frac{3}{5}$ kg of rice and still had $2\frac{1}{4}$ kg left. How heavy was the bag of rice that Ms. How bought?

4. Joe spends $\frac{2}{5}$ of his money on a jacket and $\frac{3}{8}$ of his money on a shirt. He spends the rest on a pair of pants. What fraction of his money does he use to buy the pants?

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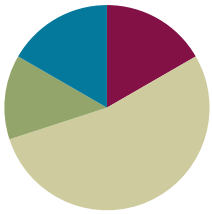
Lesson 10: Add Fractions with Sums Greater than Two

Suggested Lesson Structure

Fluency Practice	(10 minutes)
Concept Development	(32 minutes)
Application Problems	(8 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (10 minutes)

- Sprint **4.NF.3c** (10 minutes)

Sprint (10 minutes)

Materials: (S) Add and Subtract Whole Numbers and Ones with Fraction Units Sprint (See directions for administration of sprints in the **G5-M3 Fluency Progressions.**)

Application Problems (8 minutes)

To make punch for the class party, Mrs. Lui mixed 1 1/3 cups orange juice, 3/4 cup apple juice, 2/3 cup cranberry juice, and 3/4 cup lemon-lime soda. Mixed together, how many cups of punch does the recipe make? (Bonus: Each student drinks 1 cup. How many recipes does Mrs. Lui need to serve her 20 students?)

T: Let's read the problem together.

S: (Students read chorally.)

T: Can you draw something? Use your RDW process to solve the problem.

(Circulate while students work.)

T: Alexis, will you tell the class about your solution?

S: I noticed that Mrs. Lui uses thirds and fourths when measuring. I added the like units together first. Then I add the unlike units last to find the answer.

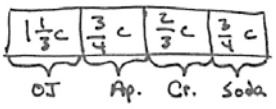
T: Say the addition sentence for the units of thirds.

S: 1 1/3 + 2/3 = 2.

T: 2 what?

S: 2 cups.

T: Say your addition sentence for the units of fourths.



$1\frac{1}{3} + \frac{3}{4} + \frac{2}{3} + \frac{3}{4} =$

$= \left(1\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{3}{4} + \frac{3}{4}\right)$

$= 2 + 1\frac{2}{4}$

$= 3\frac{2}{4} = 3\frac{1}{2}$

The recipe makes $3\frac{1}{2}$ cups punch.

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S: 3 fourths + 3 fourths = 1 and 1 half.

T: 1 and 1 half what?

S: 1 and 1 half cups.

T: How do I finish solving this problem?

S: Add 2 cups + 1 and 1 half cups.

T: Tell your partner your final answer as a sentence.

S: Mrs. Lui's recipe makes 3 and 1 half cups of punch.

If time allows, ask students to share strategies for solving the bonus question.

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

So often during our fraction work, we talk about like units. Develop a visual code for this with your students. It might be as simple as posting a bar model you can reference showing 1/2 subdivided by a dotted line to make fourths. Say "like units" while pointing to and saying: "1/2 = 2/4." Then say, "Like units, 3/4 = 18/24."

Concept Development (32 minutes)

T: Look at the three problems on the board. Discuss with your partner how they are similar and how they are different.

Problem A: $2\frac{1}{5} + 1\frac{1}{5}$ Problem B: $2\frac{1}{5} + 1\frac{1}{2}$ Problem C: $2\frac{4}{5} + 1\frac{1}{2}$

S: "Both add whole numbers plus fractional units." → "The fractional units are different in Problems B and C." → "Both A and B will result in an answer between 3 and 4, but C will be between 4 and 5."

NOTES ON SCAFFOLDING ELLS:

Throughout this lesson, students are asked to work with fractions equations and understand the step by step logic of what is happening to them. Have ELLs with similar home language sit together to support their ongoing analysis of the numbers.

Problem 1

T: Read the expression.

S: 2 and 1/5 + 1 and ½.

T: Discuss with your partner if the following equation is true. (Write)

2 1/5 + 1 ½
= 2 + 1/5 + 1 + ½
= 3 + 1/5 + ½
= 3 + (1/5 + ½)

S: (Students discuss and find it is true using the commutative and associative properties.)

T: What should be done to add 1/5 + 1/2?

S: Change fifths and halves to tenths.

T: Yes. We can create an equivalent fraction of 1/2 using 10 as the denominator.

$$2\frac{1}{5} + 1\frac{1}{2}$$
$$= 2 + \frac{1}{5} + 1 + \frac{1}{2}$$
$$= 3 + (\frac{1}{5} + \frac{1}{2})$$
$$= 3 + \left(\frac{1}{5} \times \frac{2}{2}\right) + (\frac{1}{2} \times \frac{5}{5})$$
$$= 3\frac{2}{10} + \frac{5}{10}$$
$$= 3\frac{7}{10}$$

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T: Say the multiplication sentence for converting 1 fifth to tenths.

S: $\frac{1}{5} \times \frac{2}{2} = \frac{2}{10}$

T: Say the multiplication sentence for converting 1 half to tenths.

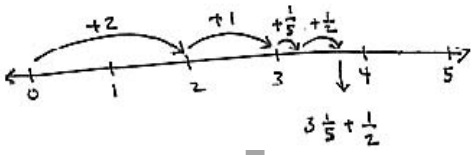
S: $\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$

T: What is our new addition sentence with like units?

S: 3 2/10 + 5/10 = 3 7/10.

T: Look at equations I have written here (pictured to the right). Discuss with your partner the logic of the equalities from top to bottom.

S: (Discuss step by step the logic of each equality.)



$$2\frac{1}{5} + 1\frac{1}{2}$$
$$= 3\frac{1}{5} + \frac{1}{2}$$
$$= 3\frac{2}{10} + \frac{5}{10}$$
$$= 3\frac{7}{10}$$

Problem 2

$$\mathbf{2\frac{4}{5} + 1\frac{1}{2}}$$

T: Compare with your partner how this problem is the same and different from our last problem.

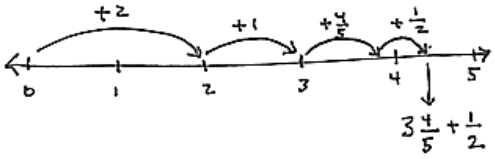
T: (After brief comparison) Joseph, could you share your thoughts?

S: The sum of the fractional units will be greater than 1 this time.

T: Let's compare them on the number line.

T: (Go through the process quickly, including generating the conversion equations. Omit recording them as in the example to the right. Allow 1-2 minutes for solving this problem.)

T: You can record the conversion equations or not. If you are ready to convert mentally, do so. If you need to write the conversions down, do so.



$$2\frac{4}{5} + 1\frac{1}{2}$$
$$= 3\frac{4}{5} + \frac{1}{2}$$
$$= 3\frac{8}{10} + \frac{5}{10}$$
$$= 3\frac{13}{10}$$
$$= 3\frac{10}{10} + \frac{3}{10}$$
$$= 4\frac{3}{10}$$

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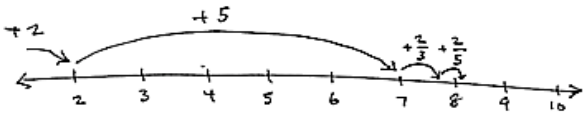
Problem 3

$$2\frac{2}{3} + 5\frac{2}{5}$$

$$___ < 2\frac{2}{3} + 5\frac{2}{5} < ___$$



$2\frac{2}{3} + 5\frac{2}{5}$
$= 7 + \frac{2}{3} + \frac{2}{5}$

T: Discuss with your partner: The sum will be between which two numbers?

S: "It's hard to know because 2 fifths is really close to 2 and 1 third. Is it more or less?" → "One way to think about it is that 2 sixths is the same as 1 third and 2 thirds plus 1 third is 1. Fifths are bigger than sixths so the answer must be between 8 and 9 but kind of close to 8."

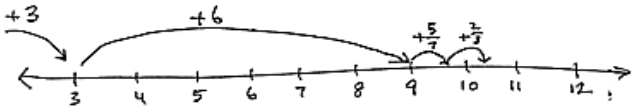
T: Try solving this problem step by step with your partner.

$$2\frac{2}{3} + 5\frac{2}{5}$$
$$= 7\frac{2}{3} + \frac{2}{5}$$
$$= 7\frac{10}{15} + \frac{6}{15}$$
$$= 7\frac{16}{15}$$
$$= 7\frac{15}{15} + \frac{1}{15}$$
$$= 8\frac{1}{10}$$

Problem 4

$$3\frac{5}{7} + 6\frac{2}{3}$$

$$___ < 3\frac{5}{7} + 6\frac{2}{3} < ___$$



$3\frac{5}{7} + 6\frac{2}{3}$
$= 9 + \frac{5}{7} + \frac{2}{3}$

T: Discuss with your partner: The sum will be between which two numbers?

S: "It's greater than 9." → "5/7 and 2/3 are both greater than ½ so the answer must be between 10 and 11." → "5/7 only needs 2/7 to be 1 and 2/3 is much more that so I agree, the answer will be between 10 and 11."

T: Take 2 minutes to solve collaboratively with your partner.

$$3\frac{5}{7} + 6\frac{2}{3}$$
$$= 9\frac{5}{7} + \frac{2}{3}$$
$$= 9\frac{15}{21} + \frac{14}{21}$$
$$= 9\frac{29}{21}$$
$$= 9\frac{21}{21} + \frac{8}{21}$$
$$= 10\frac{8}{21}$$

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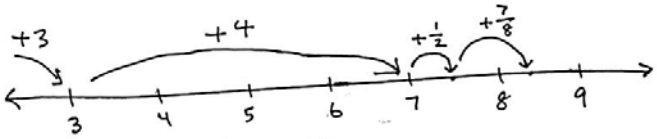
Problem 5

$$3\frac{1}{2} + 4\frac{7}{8}$$

$$___ < 3\frac{1}{2} + 4\frac{7}{8} < ___$$



$3\frac{1}{2} + 4\frac{7}{8}$

$= 7 + \frac{1}{2} + \frac{7}{8}$

T: First discuss with your partner what unit you will use for adding the fractional parts. (Allow 1 minute to discuss.)

T: Julia and Curtis, I heard you disagreeing. Julia, what is your choice?

S: I'm just going to use sixteenths. It's easy for me just to multiply by the denominator of the other addend.

MP.3

T: Curtis, how is your strategy different?

S: I will use eighths. To me that is easier because I only have to change the 1 half into eighths.

T: I will give you 2 minutes to solve the problem. Try using either Julia's or Curtis's strategy of 16 or 8 for your like units. Let's see who is right.

Method 1

$3\frac{1}{2} + 4\frac{7}{8} =$

$7\frac{1}{2} + \frac{7}{8} = 7\frac{4}{8} + \frac{7}{8}$

$= 7\frac{11}{8}$

$= 7\frac{8}{8} + \frac{3}{8}$

$= 8\frac{3}{8}$

Method 2

$3\frac{1}{2} + 4\frac{7}{8} =$

$7\frac{1}{2} + \frac{7}{8} = 7\frac{8}{16} + \frac{14}{16}$

$= 7\frac{22}{16}$

$= 7\frac{16}{16} + \frac{6}{16}$

$= 8\frac{6}{16}$

$= 8\frac{3}{8}$

Allow students two minutes to work together. Note that students should simplify their answers and that both choices of unit yield an equivalent, correct response.

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Problem 6

$$15\frac{5}{6}+7\frac{9}{10}$$

Allow students to solve the last problem individually. Again, note that there are two methods for finding like units. As students work, have 2 pairs come to the board and solve the problems using different units, highlighting that both methods result in the same solution.

Method 1

$$15\frac{5}{6}+7\frac{9}{10}$$
$$=22\frac{5}{6}+\frac{9}{10}$$
$$=22\frac{50}{60}+\frac{54}{60}$$
$$=22\frac{104}{60}$$
$$=22\frac{60}{60}+\frac{44}{60}$$
$$=23\frac{11}{15}$$

Method 2

$$15\frac{5}{6}+7\frac{9}{10}$$
$$=22\frac{5}{6}+\frac{9}{10}$$
$$=22\frac{25}{30}+\frac{27}{30}$$
$$=22\frac{52}{30}$$
$$=22\frac{30}{30}+\frac{22}{30}$$
$$=23\frac{11}{15}$$

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

If students finish early, have them solve the problem using more than one method for finding like units. They might also draw their solutions on the number line to prove the equivalence of different units. Drawings can be shared with the rest of the class to clarify confusion that others may have about the relationship between different methods.

It is worth pointing out that if this were a problem about time, in Method 1 we might want to keep our final fraction as sixtieths. The answer might be 22 hours and 44 minutes.

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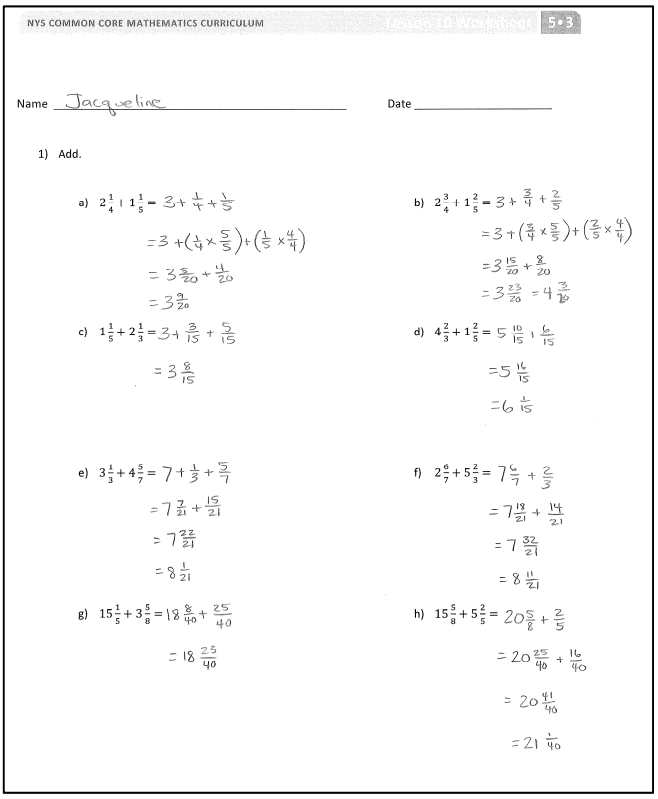
Activity Worksheet (10 minutes)

Students complete the activity worksheet.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 10 Worksheet 5•3

Name Jacqueline Date

1) Add.

a) $2\frac{1}{4} + 1\frac{1}{5} = 3 + \frac{1}{4} + \frac{1}{5}$

$= 3 + \left(\frac{1}{4} \times \frac{5}{5}\right) + \left(\frac{1}{5} \times \frac{4}{4}\right)$

$= 3\frac{5}{20} + \frac{4}{20}$

$= 3\frac{9}{20}$

b) $2\frac{3}{4} + 1\frac{2}{5} = 3 + \frac{3}{4} + \frac{2}{5}$

$= 3 + \left(\frac{3}{4} \times \frac{5}{5}\right) + \left(\frac{2}{5} \times \frac{4}{4}\right)$

$= 3\frac{15}{20} + \frac{8}{20}$

$= 3\frac{23}{20} = 4\frac{3}{20}$

c) $1\frac{1}{5} + 2\frac{1}{3} = 3 + \frac{3}{15} + \frac{5}{15}$

$= 3\frac{8}{15}$

d) $4\frac{2}{3} + 1\frac{2}{5} = 5\frac{10}{15} + \frac{6}{15}$

$= 5\frac{16}{15}$

$= 6\frac{1}{15}$

e) $3\frac{1}{3} + 4\frac{5}{7} = 7 + \frac{1}{3} + \frac{5}{7}$

$= 7\frac{7}{21} + \frac{15}{21}$

$= 7\frac{22}{21}$

$= 8\frac{1}{21}$

f) $2\frac{6}{7} + 5\frac{2}{3} = 7\frac{6}{7} + \frac{2}{3}$

$= 7\frac{18}{21} + \frac{14}{21}$

$= 7\frac{32}{21}$

$= 8\frac{11}{21}$

g) $15\frac{1}{5} + 3\frac{5}{8} = 18\frac{8}{40} + \frac{25}{40}$

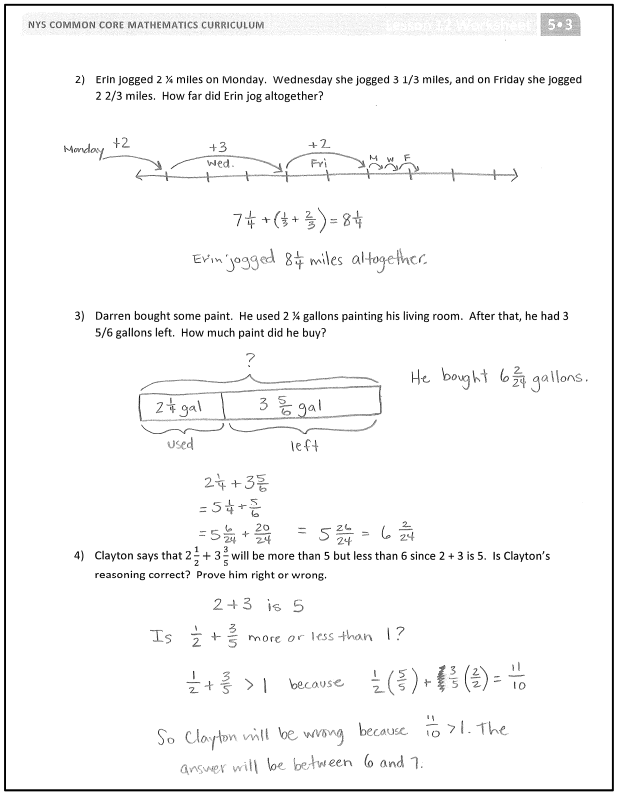
$= 18\frac{25}{40}$

h) $15\frac{5}{8} + 5\frac{2}{5} = 20\frac{5}{8} + \frac{2}{5}$

$= 20\frac{25}{40} + \frac{16}{40}$

$= 20\frac{41}{40}$

$= 21\frac{1}{40}$



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 10 Worksheet 5•3

2) Erin jogged 2 ¼ miles on Monday. Wednesday she jogged 3 1/3 miles, and on Friday she jogged 2 2/3 miles. How far did Erin jog altogether?

Monday +2 +3 Wed. +2 Fri M W F

$7\frac{1}{4} + \left(\frac{1}{3} + \frac{2}{3}\right) = 8\frac{1}{4}$

Erin jogged $8\frac{1}{4}$ miles altogether.

3) Darren bought some paint. He used 2 ¼ gallons painting his living room. After that, he had 3 5/6 gallons left. How much paint did he buy?

? $2\frac{1}{4}$ gal | $3\frac{5}{6}$ gal used left

He bought $6\frac{2}{24}$ gallons.

$2\frac{1}{4} + 3\frac{5}{6}$

$= 5\frac{1}{4} + \frac{5}{6}$

$= 5\frac{6}{24} + \frac{20}{24} = 5\frac{26}{24} = 6\frac{2}{24}$

4) Clayton says that $2\frac{1}{2} + 3\frac{3}{5}$ will be more than 5 but less than 6 since 2 + 3 is 5. Is Clayton's reasoning correct? Prove him right or wrong.

2 + 3 is 5

Is $\frac{1}{2} + \frac{3}{5}$ more or less than 1?

$\frac{1}{2} + \frac{3}{5} > 1$ because $\frac{1}{2}\left(\frac{5}{5}\right) + \frac{3}{5}\left(\frac{2}{2}\right) = \frac{11}{10}$

So Clayton will be wrong because $\frac{11}{10} > 1$. The answer will be between 6 and 7.

Student Debrief (10 minutes)

Materials: (S) Student Activity Worksheet

T: Please take two minutes to check your answers with your partner. Do not change any of your answers. (Allow time for students to work.)

T: I will say the addition problem. Will you please share your answers out loud in response? Letter a) 2 and 1 fourth + 1 and 1 fifth =?

S: 3 and 9 twentieths.

Continue with sequence.

T: Take the next two minutes to discuss with your partner any observations you had while completing this worksheet. What do you notice?

Allow time for students to discuss while you circulate and listen for conversations that can be shared with the whole class.

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T: Myra, can you share what you noticed happening across the page?

S: Sure, the rows going across shared the same units. Letters 'a' and 'b' had units of fourths and fifths, and the like units are twentieths.

T: Victor, what did you see in the right column?

S: On all of the problems in the right column the sum of the fraction was greater than 1. Like in letter 'g' the answer was 20 and 41 fortieths. 41 fortieths is a fraction greater than 1, so I had to change it into a mixed number and add that to the whole number 20. So my final answer was 21 and 1 fortieth.

T: Share with your partner how you realize when it the fraction allows you to make a new whole. (Allow 1 minute for conversation.)

S: "When the top number of the fraction is bigger than the bottom number I know." → "I look at the relationship between the numerator and denominator. If the numerator is larger, I change it to a mixed number." → "The denominator tells us the number of parts in one whole. So if the numerator is greater, the fraction is greater than one."

T: What about Clayton's reasoning in question 4? Discuss your thoughts with your partner.

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A

Correct_____

Add or subtract.

1	$3+1=$	/	23	$3\frac{5}{6}+7=$	/
2	$3+\frac{1}{2}=$	/	24	$7\frac{5}{6}+3=$	/
3	$3\frac{1}{2}+1=$	/	25	$10\frac{5}{6}-3=$	/
4	$3-1=$	/	26	$10\frac{5}{6}-7=$	/
5	$3\frac{1}{2}-1=$	/	27	$3+\frac{4}{5}+2=$	/
6	$4-2=$	/	28	$5+\frac{7}{8}+4=$	/
7	$4\frac{1}{2}-2=$	/	29	$7+\frac{4}{5}-2=$	/
8	$5-2=$	/	30	$9+\frac{5}{12}-5=$	/
9	$5\frac{1}{3}-2=$	/	31	$7+\frac{1}{5}+\frac{1}{5}+2=$	/
10	$5\frac{2}{3}-2=$	/	32	$7+\frac{2}{5}+2=$	/
11	$5\frac{2}{3}+2=$	/	33	$7+\frac{2}{5}+2+\frac{2}{5}=$	/
12	$6+2=$	/	34	$7\frac{2}{5}+2\frac{2}{5}=$	/
13	$6+\frac{3}{4}=$	/	35	$6+\frac{1}{3}+1+\frac{1}{3}=$	/
14	$6\frac{3}{4}+2=$	/	36	$6\frac{1}{3}+1\frac{1}{3}=$	/
15	$6\frac{3}{4}-2=$	/	37	$6+\frac{2}{3}-1=$	/
16	$6\frac{3}{4}-3=$	/	38	$6\frac{2}{3}-1\frac{1}{3}=$	/
17	$6\frac{3}{4}-4=$	/	39	$6\frac{2}{3}-1\frac{2}{3}=$	/
18	$6\frac{3}{4}-6=$	/	40	$3+\frac{4}{7}+1+\frac{2}{7}=$	/
19	$6\frac{3}{4}-\frac{3}{4}=$	/	41	$3\frac{4}{7}+1\frac{2}{7}=$	/
20	$2\frac{5}{6}+3=$	/	42	$7\frac{4}{5}-2\frac{3}{5}=$	/
21	$2\frac{1}{6}+3=$	/	43	$7\frac{4}{5}-2\frac{2}{5}=$	/
22	$2\frac{5}{6}+7=$	/	44	$13\frac{7}{9}-7\frac{5}{9}=$	/

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B Improvement_____ # Correct_____

Add or subtract.

1	$2+1=$	/	23	$4\frac{5}{6}+6=$	/
2	$2+\frac{1}{2}=$	/	24	$6\frac{5}{6}+4=$	/
3	$2\frac{1}{2}+1=$	/	25	$10\frac{5}{6}-4=$	/
4	$2-1=$	/	26	$10\frac{5}{6}-6=$	/
5	$2\frac{1}{2}-1=$	/	27	$4+\frac{4}{5}+2=$	/
6	$5-2=$	/	28	$6+\frac{7}{8}+3=$	/
7	$5\frac{1}{2}-2=$	/	29	$6+\frac{4}{5}-2=$	/
8	$6-2=$	/	30	$9+\frac{5}{12}-4=$	/
9	$6\frac{1}{3}-2=$	/	31	$6+\frac{1}{5}+\frac{1}{5}+2=$	/
10	$6\frac{2}{3}-2=$	/	32	$6+\frac{2}{5}+2=$	/
11	$6\frac{2}{3}+2=$	/	33	$6+\frac{2}{5}+2+\frac{2}{5}=$	/
12	$7+2=$	/	34	$6\frac{2}{5}+2\frac{2}{5}=$	/
13	$7+\frac{3}{4}=$	/	35	$5+\frac{1}{3}+1+\frac{1}{3}=$	/
14	$7\frac{3}{4}+2=$	/	36	$5\frac{1}{3}+1\frac{1}{3}=$	/
15	$7\frac{3}{4}-2=$	/	37	$7+\frac{2}{3}-1=$	/
16	$7\frac{3}{4}-3=$	/	38	$7\frac{2}{3}-1\frac{1}{3}=$	/
17	$7\frac{3}{4}-4=$	/	39	$7\frac{2}{3}-1\frac{2}{3}=$	/
18	$7\frac{3}{4}-7=$	/	40	$5+\frac{4}{7}+1+\frac{2}{7}=$	/
19	$7\frac{3}{4}-\frac{3}{4}=$	/	41	$5\frac{4}{7}+1\frac{2}{7}=$	/
20	$3\frac{5}{6}+2=$	/	42	$6+\frac{4}{5}-2\frac{3}{5}=$	/
21	$3\frac{1}{6}+2=$	/	43	$6\frac{4}{5}-2\frac{3}{5}=$	/
22	$3\frac{5}{6}+6=$	/	44	$13\frac{7}{9}-6\frac{5}{9}=$	/

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Name _____ Date _____

1) Add.

a) $2\frac{1}{4} + 1\frac{1}{5} =$

b) $2\frac{3}{4} + 1\frac{2}{5} =$

c) $1\frac{1}{5} + 2\frac{1}{3} =$

d) $4\frac{2}{3} + 1\frac{2}{5} =$

e) $3\frac{1}{3} + 4\frac{5}{7} =$

f) $2\frac{6}{7} + 5\frac{2}{3} =$

g) $15\frac{1}{5} + 3\frac{5}{8} =$

h) $15\frac{5}{8} + 5\frac{2}{5} =$

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2) Erin jogged $2\frac{1}{4}$ miles on Monday. Wednesday she jogged $3\frac{1}{3}$ miles, and on Friday she jogged $2\frac{2}{3}$ miles. How far did Erin jog altogether?

3) Darren bought some paint. He used $2\frac{1}{4}$ gallons painting his living room. After that, he had $3\frac{5}{6}$ gallons left. How much paint did he buy?

4) Clayton says that $2\frac{1}{2} + 3\frac{3}{5}$ will be more than 5 but less than 6 since 2 + 3 is 5. Is Clayton's reasoning correct? Prove him right or wrong.

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Name _____ Date _____

Solve the problems.

1) $3\frac{1}{2} + 1\frac{1}{3} =$

2) $4\frac{5}{7} + 3\frac{3}{4} =$

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Name _____ Date _____

1) Add.

a) $2\frac{1}{2} + 1\frac{1}{5} =$

b) $2\frac{1}{2} + 1\frac{3}{5} =$

c) $1\frac{1}{5} + 3\frac{1}{3} =$

d) $3\frac{2}{3} + 1\frac{3}{5} =$

e) $2\frac{1}{3} + 4\frac{4}{7} =$

f) $3\frac{5}{7} + 4\frac{2}{3} =$

g) $15\frac{1}{5} + 4\frac{3}{8} =$

h) $18\frac{3}{8} + 2\frac{2}{5} =$

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2) Angela practiced piano for $2\frac{1}{2}$ hours on Friday, $2\frac{1}{3}$ hours on Saturday, and $3\frac{2}{3}$ hours on Sunday. How much time did Angela practice piano during the weekend?

3) String A is $3\frac{5}{6}$ meters long. String B is 2 ¼ long. What's the total length of both strings?

4) Matt says that $5 - 1\frac{1}{4}$ will be more than 4, since 5 – 1 is 4. Draw a picture to prove that Matt is wrong.

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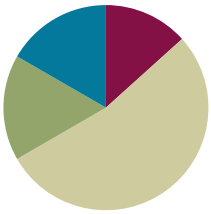
Lesson 11: Subtract Fractions Making Like Units Numerically

Suggested Lesson Structure

Fluency Practice	(8 minutes)
Application Problems	(10 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (8 minutes)

- Subtracting Fractions from Whole Numbers **4.NF.3a** (5 minutes)
- Adding and Subtracting Fractions with Like Units **4.NF.3c** (3 minutes)

Subtracting Fractions from Whole Numbers (5 minutes)

T: I'll say a subtraction sentence. You say the subtraction sentence with answer. 1 – 1 half.

S: 1 – 1 half = 1 half.

T: 2 – 1 half.

S: 2 – 1 half = 1 and 1 half.

T: 3 – 1 half.

S: 3 – 1 half = 2 and 1 half.

T: 7 – 1 half.

S: 7 – 1 half = 6 and 1 half.

Continue with possible sequence:

$$1-\frac{1}{3}, 1-\frac{2}{3}, 2-\frac{2}{3}, 2-\frac{1}{3}, 5-\frac{1}{4}, 5-\frac{3}{4}.$$

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

If students struggle to answer verbally, consider an alternative that includes drawing on personal boards:

T: Draw 2 units. (Students draw.)

T: Subtract 1 half. Are we subtracting 1/2 of 1 unit, or both units?

S: Half of 1 unit!

T: Good. Show it now.

T: Write the number sentence. (Students write 2 – 1/2 = 1 1/2.)

Adding and Subtracting Fractions with Like Units (3 minutes)

T: I'll say an addition or subtraction sentence. You say the answer. 3 sevenths + 1 seventh.

S: 4 sevenths.

T: 3 sevenths – 1 seventh.

S: 2 sevenths.

T: 3 sevenths + 3 sevenths.

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S: 6 sevenths.

T: 3 sevenths – 3 sevenths.

S: 0.

T: 4 sevenths + 3 sevenths.

S: 1.

T: I'll write an addition sentence. You say true or false.

(Write) $\frac{2}{5}+\frac{2}{5}=\frac{4}{10}$

S: False.

T: Say the answer that makes the addition sentence true.

S: 2 fifths + 2 fifths = 4 fifths.

(Write) $\frac{5}{8}+\frac{3}{8}=1$

S: True.

(Write) $\frac{5}{6}+\frac{1}{6}=\frac{6}{12}$

S: False.

T: Say the answer that makes the addition sentence true.

S: 5 sixths + 1 sixth = 1.

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

Assign bonus problems to students who enjoy being challenged. For example assign fraction addition and subtraction problems that include simplest form:

True or false?

3/4 -1/4 = 1/2

4/8 + 2/8 = 3/4

Application Problem (10 minutes)

Meredith went to the movies. She spent 2/5 of her money on a ticket and 3/7 of her money on popcorn. How much of her money did she spend? (Bonus: How much of her money is left?)

T: Today, I want you to try and solve this problem without drawing. Just write an equation.

T: Talk with your partner for 30 seconds about strategies for how to solve this problem using an equation.

Circulate and listen to student responses.

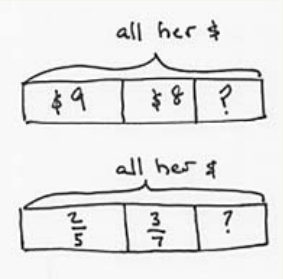
T: Jackie, will you share?

S: I thought about when I go to the movies and buy a ticket and popcorn. I have to add those two things up. So I am going to add to solve this problem.

NOTES ON SCAFFOLDING STUDENTS BELOW GRADE LEVEL:

The language of whole numbers is much more familiar to ELLs and students below grade level. Possibly start by presenting the question with whole numbers.

Meredith went to the movies. She spent $9 of her money on a movie and $8 of her money on popcorn. How much money did she spend? If she started with $20, how much is left?



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T: Good. David, can you expand on Jackie's comment with your strategy?

S: The units don't match. I need to make like units first, then I can add the price of the ticket and popcorn together.

T: Nice observation. I will give you 90 seconds to work with your partner to solve this problem.

Students work.

T: Using the strategies that we learned about adding fractions with unlike units, how can I make like units from fifths and sevenths?

S: Multiply 2 fifths by 7 sevenths and multiply 3 sevenths by 5 fifths.

T: Everyone, say your addition sentence with your new like units.

S: 14 thirty-fifths plus 15 thirty-fifths equals 29 thirty-fifths.

T: Share, please, a sentence about the money Meredith spent.

S: Meredith spent 29 thirty-fifths of her money at the theater.

T: Is 29 thirty-fifths more than or less than a whole? How do you know?

S: Less than a whole because the numerator is less than the denominator.

T: (If time allows.) Did anyone answer the bonus question?

S: Yes!

T: Please share your solution method and statement. Come to the board.

$$\frac{2}{5} + \frac{3}{7} = \left(\frac{2}{5} \times \frac{7}{7}\right) + \left(\frac{3}{7} \times \frac{5}{5}\right)$$

$$\frac{14}{35} + \frac{15}{35} = \frac{29}{35}$$

Meredith spent $\frac{29}{35}$ of her money at the theater.

Concept Development (32 minutes)

T: Look at this problem. Tell your partner how you might solve it. (Display and give 30 seconds for discussion.)

$$\frac{1}{3} - \frac{1}{5}$$

S: I would draw a rectangular model. First I would divide the model into thirds. Then I would horizontally divide the model into fifths and bracket one fifth. That way I would create like units. Then I would subtract.

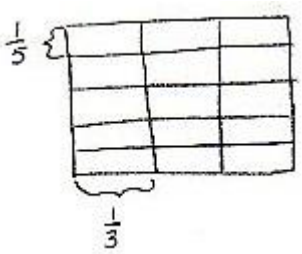
T: What is our like unit for thirds and fifths?

S: Fifteenths.

T: Since we know how to find like units for addition using an equation, let's use that knowledge to subtract using an equation instead of a picture.

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

The dialogue modeled before problem 1 may be more conceptual review than your students need. If so, move right into problem 1.



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Problem 1

$$\frac{1}{3} - \frac{1}{5}$$

T: How many fifteenths are equal to 1 third?

S: 5 fifteenths.

$\left(\frac{1}{3} \times \frac{5}{5}\right)$ 5 times as many selected units.
5 times as many units in the whole.

T: How many fifteenths are equal to 1 fifth?

S: 3 fifteenths.

$\left(\frac{1}{5} \times \frac{3}{3}\right)$ 3 times as many selected units.
3 times as many units in the whole.

$$\left(\frac{1}{3} \times \frac{5}{5}\right) - \left(\frac{1}{5} \times \frac{3}{3}\right) = \frac{5}{15} - \frac{3}{15} =$$

T: As with addition, the equation supports what we drew in our model. Say the subtraction sentence with like units.

S: 5 fifteenths – 3 fifteenths = 2 fifteenths.

$$\frac{5}{15} - \frac{3}{15} = \frac{2}{15}$$

NOTES ON SCAFFOLDING ELLS:

Be aware of cognates – words that sound similar and have the same meaning – between English and ELLs' home languages. Related cognates for Spanish speakers, for example, are:

- fraction = fracción
- find the sum (add) = sumar
- numerator = numerador
- denominator = denominador

Encourage students to listen for them and share them with the class. This will help ELLs actively listen, and also boost auditory comprehension as they make links between prior knowledge and new learning.

Problem 2

$$\frac{3}{5} - \frac{1}{6}$$

T: To make 3 fifths into smaller units we will multiply by?

S: 6 sixths.

T: To make 1 sixth into smaller units we will multiply by?

S: 5 fifths.

$$\left(\frac{3}{5} \times \frac{6}{6}\right) - \left(\frac{1}{6} \times \frac{5}{5}\right) =$$

T: What happened to each fraction?

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S: "The fractions are still equivalent but just smaller units." → "We are changing the fractions to be the same size so we can subtract them." → "We are partitioning our original fractions into smaller units. The value of the fraction doesn't change though."

T: Say your subtraction sentence with the like units.

S: 18 thirtieths – 5 thirtieths = 13 thirtieths.

$$\frac{18}{30} - \frac{5}{30} = \frac{13}{30}$$

Problem 3

$$1\frac{3}{4} - \frac{3}{5}$$

T: What are some different ways we can solve this problem?

S: "You can solve it as 2 fifths plus ¾. Just take the 3/5 from 1 to get 2 fifths and add the 3 fourths." → "You can add 1 + 3 fourths + 3 fifths. Just add the fractional units and then add the whole number." → "The whole number can be represented as 4 fourths and added to 3 fourths to equal 7 fourths. Then subtract."

Method 1

$1\frac{3}{4} - \frac{3}{5}$

$\frac{5}{5} \quad \frac{3}{4}$

$= \frac{2}{5} + \frac{3}{4}$

$= \frac{8}{20} + \frac{15}{20}$

$= \frac{23}{20}$

$= 1\frac{3}{20}$

Method 2

$1\frac{3}{4} - \frac{3}{5}$

$= 1 + \frac{15}{20} \quad \frac{12}{20}$

$= 1\frac{3}{20}$

Method 3

$1\frac{3}{4} - \frac{3}{5}$

$\frac{7}{4} - \frac{3}{5}$

$= \frac{35}{20} - \frac{12}{20}$

$= \frac{23}{20}$

$= 1\frac{3}{20}$

S: I noticed before we started that 3 fifths is less than 3 fourths, so I changed only the fractional units to twentieths.

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Problem 4

$$3\frac{3}{5} - 2\frac{1}{2}$$

Method 1

$3\frac{3}{5} - 2\frac{1}{2}$

$3 \quad \frac{3}{5}$

$= \frac{1}{2} + \frac{3}{5}$

$= \frac{5}{10} + \frac{6}{10}$

$= 1\frac{1}{10}$

Method 2

$3\frac{3}{5} - 2\frac{1}{2}$

$= 1\frac{3}{5} - \frac{1}{2}$

$= 1 + \frac{6}{10} - \frac{5}{10}$

$= 1\frac{1}{10}$

Method 3

$3\frac{3}{5} - 2\frac{1}{2}$

$= \frac{18}{5} - \frac{5}{2}$

$= \frac{36}{10} - \frac{25}{10}$

$= \frac{11}{10}$

$= 1\frac{1}{10}$

MP.3

T: (After students work.) Let's confirm the reasonableness of our answer using the number line to show 2 of our methods.

T: For Method 1, draw a number line from 0 to 4.

T: (Support students to see that they would start at 3. Subtract 2 1/2 and add back the 3/5.)

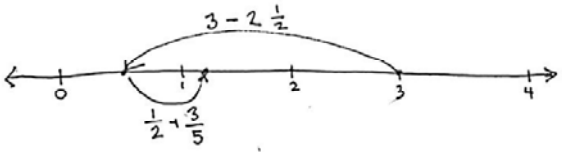
T: (Pause as students work. Circulate and observe.)

T: To show Method 2, draw your number line from 0 to 4, then estimate the location of 3 and 3 fifths.

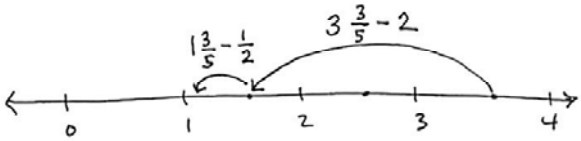
S: Take away 2 first, then take away the half.

T: Is our answer of 1 1/10 reasonable based on both your number lines?

Method 1



Method 2



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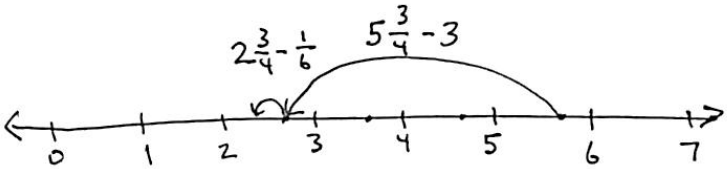
Problem 5

$$5\frac{3}{4} - 3\frac{1}{6}$$

T: Estimate the answer first by drawing a number line. The difference between 5 3/4 and 3 1/6 will be between which 2 whole numbers?

S: 3/4 fourths is much bigger than a sixth so the answer will be between 2 and 3.

T: Will it be closer to 2 or 2 ½? Discuss your thinking with a partner.

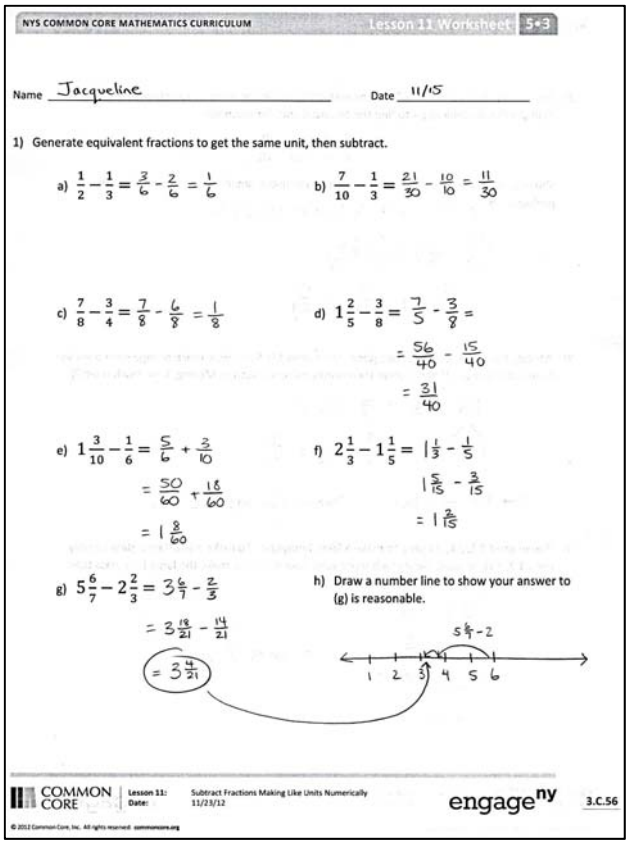


T: Some of you used twenty-fourths and some of you used twelfths to solve this problem. Were your answers the same?

S: They had the same value. 14/24 can be made into larger units of twos. 7 twos out of 12 twos.

Activity Worksheet (10 minutes)

Materials: (S) Worksheet



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 11 Worksheet 5•3

Name Jacqueline Date 11/15

1) Generate equivalent fractions to get the same unit, then subtract.

a) $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

b) $\frac{7}{10} - \frac{1}{3} = \frac{21}{30} - \frac{10}{10} = \frac{11}{30}$

c) $\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{1}{8}$

d) $1\frac{2}{5} - \frac{3}{8} = \frac{7}{5} - \frac{3}{8} = \frac{56}{40} - \frac{15}{40} = \frac{31}{40}$

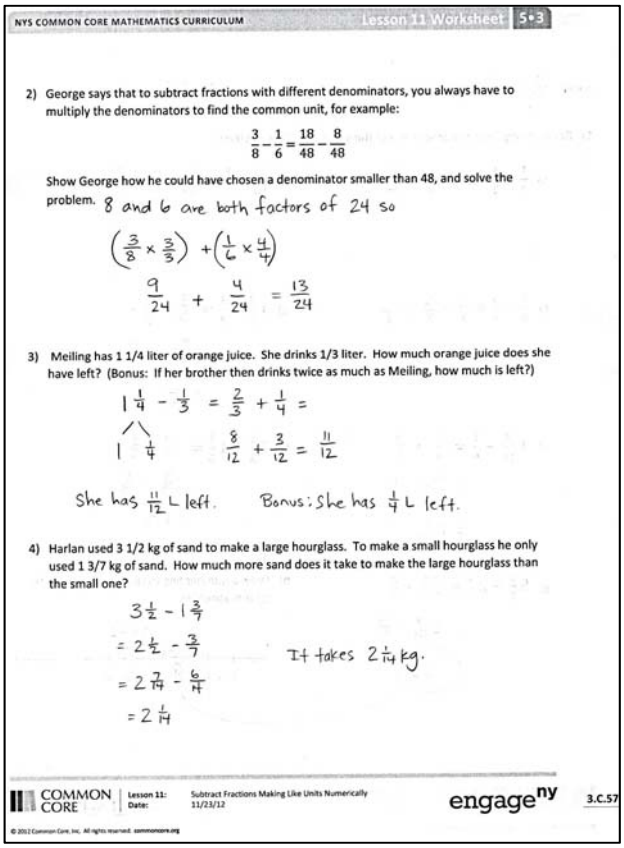
e) $1\frac{3}{10} - \frac{1}{6} = \frac{5}{6} + \frac{3}{10} = \frac{50}{60} + \frac{18}{60} = 1\frac{8}{60}$

f) $2\frac{1}{3} - 1\frac{1}{5} = 1\frac{1}{3} - \frac{1}{5}$ $1\frac{5}{15} - \frac{3}{15} = 1\frac{2}{15}$

g) $5\frac{6}{7} - 2\frac{2}{3} = 3\frac{6}{7} - \frac{2}{3} = 3\frac{18}{21} - \frac{14}{21} = 3\frac{4}{21}$

h) Draw a number line to show your answer to (g) is reasonable.

COMMON CORE Lesson 11: Subtract Fractions Making Like Units Numerically Date: 11/23/12 engage^ny 3.C.56



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 11 Worksheet 5•3

2) George says that to subtract fractions with different denominators, you always have to multiply the denominators to find the common unit, for example:

$$\frac{3}{8} - \frac{1}{6} = \frac{18}{48} - \frac{8}{48}$$

Show George how he could have chosen a denominator smaller than 48, and solve the problem. 8 and 6 are both factors of 24 so

$\left(\frac{3}{8} \times \frac{3}{3}\right) + \left(\frac{1}{6} \times \frac{4}{4}\right)$

$\frac{9}{24} + \frac{4}{24} = \frac{13}{24}$

3) Meiling has 1 1/4 liter of orange juice. She drinks 1/3 liter. How much orange juice does she have left? (Bonus: If her brother then drinks twice as much as Meiling, how much is left?)

$1\frac{1}{4} - \frac{1}{3} = \frac{2}{3} + \frac{1}{4} =$

$1 \quad \frac{1}{4}$ $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

She has $\frac{11}{12}$ L left. Bonus: She has $\frac{1}{4}$ L left.

4) Harlan used 3 1/2 kg of sand to make a large hourglass. To make a small hourglass he only used 1 3/7 kg of sand. How much more sand does it take to make the large hourglass than the small one?

$3\frac{1}{2} - 1\frac{3}{7}$

$= 2\frac{1}{2} - \frac{3}{7}$

$= 2\frac{7}{14} - \frac{6}{14}$

$= 2\frac{1}{14}$

It takes $2\frac{1}{14}$ kg.

COMMON CORE Lesson 11: Subtract Fractions Making Like Units Numerically Date: 11/23/12 engage^ny 3.C.57

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Student Debrief (10 minutes)

Materials: (S) Worksheet

T: Please take two minutes to check your answers with your partner.

T: I will say the subtraction problem. Please say your answers out loud. Letter a) 1 half – 1 third = ?

S: 1 sixth.

T: (Continue.)

T: Now take the next 2 minutes to discuss with your partner any insights you had while solving these problems.

Allow for students to discuss, circulating and listening for conversations that can be shared with the whole class.

T: Sandy, will you share your thinking about problem 2?

S: George is wrong. He just learned a rule and thinks it is the only way. It's a good way but you can also make eighths and sixths into twenty fourths or ninety sixths.

T: Discuss in pairs if there are advantages to using twenty fourths or forty eighths.

S: "Sometimes it's easier to multiply by the opposite denominator." → "Sometimes bigger denominators just get in the way." → "Sometimes they are right. Like if you have to find the minutes, you want to keep your fraction out of 60."

S: An example is I saw that on letter 'c' I didn't need to multiply both fractions. I could have just multiplied 3 fourths by 2 halves. Then I would have had 8 as the like unit for both fractions. And then I wouldn't have had to simplify my answer.

T: Did anyone notice George's issue applying to any of the other problems on the worksheet?

MP.7

S: "Yes, letter 'c'. You could use eighths or thirty seconds. It was just so much easier to use eighths." → "Yes, on letter 'e' the unit of sixtieths is big but easy. 30 is smaller and a multiple of both 6 and 10. I used sixty-eths because I don't have to think as hard!"

T: I notice that many of you are becoming so comfortable with this equation when subtracting unlike units that you don't have to write the multiplication. You are doing it mentally. However, you still have to check your answers to see if they are reasonable. Discuss with your partner how you use mental math, and also how you make sure your methods and answers are reasonable.

S: "It's true. I just look at the other denominator and multiply. It's easy." → "I added instead of subtracted and wouldn't have even noticed if I hadn't checked my answer to see that it was bigger than the whole amount I started with!" → "We are learning to find like units, and we may not always need to multiply both fractions. If I don't slow down, I won't even notice there are other choices for solving the problem." → "I like choosing the strategy I want to use. Sometimes it's easier to use the number bond method and sometimes it's just easier to subtract from the whole."

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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Name _____ Date _____

1) Generate equivalent fractions to get the same unit, then subtract.

a) $\frac{1}{2} - \frac{1}{3} =$

b) $\frac{7}{10} - \frac{1}{3} =$

c) $\frac{7}{8} - \frac{3}{4} =$

d) $1\frac{2}{5} - \frac{3}{8} =$

e) $1\frac{3}{10} - \frac{1}{6} =$

f) $2\frac{1}{3} - 1\frac{1}{5} =$

g) $5\frac{6}{7} - 2\frac{2}{3} =$

h) Draw a number line to show your answer to (g) is reasonable.

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2) George says that to subtract fractions with different denominators, you always have to multiply the denominators to find the common unit, for example:

$$\frac{3}{8} - \frac{1}{6} = \frac{18}{48} - \frac{8}{48}$$

Show George how he could have chosen a denominator smaller than 48, and solve the problem.

3) Meiling has $1\frac{1}{4}$ liter of orange juice. She drinks $\frac{1}{3}$ liter. How much orange juice does she have left? (Bonus: If her brother then drinks twice as much as Meiling, how much is left?)

4) Harlan used $3\frac{1}{2}$ kg of sand to make a large hourglass. To make a small hourglass he only used $1\frac{3}{7}$ kg of sand. How much more sand does it take to make the large hourglass than the small one?

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Name _____ Date _____

Find the common unit and then subtract.

1. $\frac{3}{4} - \frac{3}{10} =$

2. $3\frac{1}{2} - 1\frac{1}{3} =$

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Name _____ Date _____

1) First find a common unit, then subtract.

a. $\frac{1}{2} - \frac{1}{5} =$

b. $\frac{7}{8} - \frac{1}{3} =$

c. $\frac{7}{10} - \frac{3}{5} =$

d. $1\frac{5}{6} - \frac{2}{3} =$

e. $2\frac{1}{4} - 1\frac{1}{5} =$

f. $5\frac{6}{7} - 3\frac{2}{3} =$

g. $15\frac{7}{8} - 5\frac{3}{4} =$

h. $15\frac{5}{8} - 3\frac{1}{3} =$

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2) Sandy ate $\frac{1}{6}$ of a candy bar. John ate $\frac{3}{4}$ of it. How much more of the candy bar did John eat than Sandy?

3) $4\frac{1}{2}$ yards of cloth are needed to make a woman's dress. $2\frac{2}{7}$ yards of cloth are needed to make a girl's dress. How much more cloth is needed to make a woman's dress than a girl's dress?

4) Bill reads $\frac{1}{5}$ of a book on Monday. He reads $\frac{2}{3}$ of the book on Tuesday. If he finishes reading the book on Wednesday, what fraction of the book did he read on Wednesday?

5) Tank A has a capacity of 9.5 gallons. $6\frac{1}{3}$ gallons of the tank's water are poured out. How much water is left in the tank?

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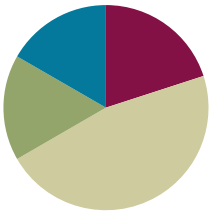
Lesson 12: Subtract Fractions Greater Than or Equal to One

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problems	(10 minutes)
Concept Development	(28 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Sprint **5.NF.1** (12 minutes)

Sprint (12 minutes)

Materials: (S) Subtracting Fractions with Unlike Units Sprint
(See directions for administration of sprints in the **G5-M3 Fluency Progressions.**)

NOTES ON SCAFFOLDING STUDENTS ABOVE AND BELOW GRADE LEVEL:

Below Grade Level: Before delivering the Sprint, consider reviewing some of the equivalent fractions that appear consistently throughout it.

It will be difficult for all students to experience adrenaline while doing this Sprint. Consider modifying the routine accordingly by giving the students a minute and a half on each sprint.

Above Grade Level: For students who want a challenge, ask them to skip count by 1/8s on the back when they finish and simplify when necessary. These counts can be part of their total number of points.

Application Problems (10 minutes)

Problem 1

Max's reading assignment was to read $15\frac{1}{2}$ pages. After reading $4\frac{1}{3}$ pages, he took a break. How many more pages does he need to read to finish his assignment?

T: Let's read the problem together.

S: (Students read chorally.)

T: With your partner, share your thoughts about how to solve this problem. (Circulate and listen.)

T: Clara, can you please share your approach?

S: I said that you need to subtract $4\frac{1}{3}$ from $15\frac{1}{2}$ to find the part that is left.

T: Tell me the subtraction problem we need to solve.

S: $15\frac{1}{2} - 4\frac{1}{3}$.

T: Good. This is the same kind of subtraction problem we have been doing since first grade. A part is

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missing: the pages he has to read to finish.

T: Maggy, read your answer using a complete sentence.

S: Max needed to read 11 and 1 sixth more pages.

NOTES ON SCAFFOLDING ELLS:

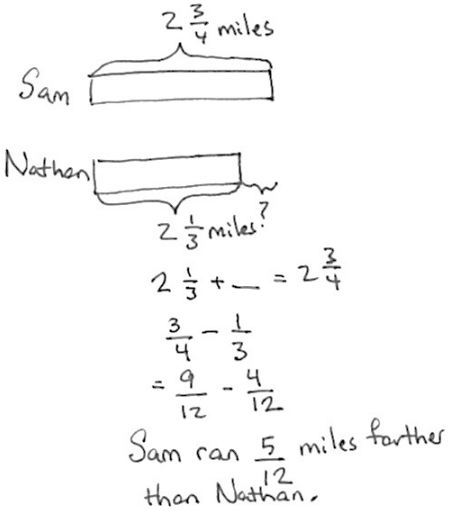
For problems involving "how many more," tape diagrams are very useful for helping ELLs to parse the language of the problem into manageable chunks. Guide students to use a part/whole model if necessary.

Problem 2

Sam and Nathan are training for a race. Monday, Sam ran $2\frac{3}{4}$ miles and Nathan ran $2\frac{1}{3}$ miles. How much farther did Sam run than Nathan?

T: (After students work.) Max, will you come to the board and show us your solution?

T: (Student can present solution, or the class can analyze it.) Does anyone have questions for Max?



Concept Development (28 minutes)

Materials: (S) Number line worksheet or lined paper

Problem 1

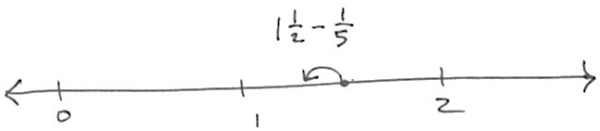
T: Look at these 2 problems and discuss them with your partner.

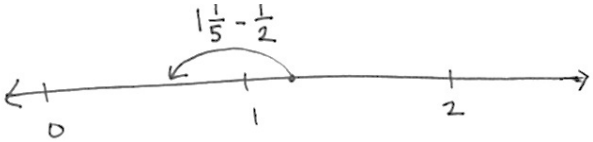
$$1\frac{1}{2} - \frac{1}{5} \qquad\qquad 1\frac{1}{5} - \frac{1}{2}$$

T: What do you notice?

S: They are the same except the half and the fifth are switched around.

T: Quickly sketch a number line to show each. Discuss the difference with your partner.





T: (After drawing and discussing number lines.) Now you know how to make like units by multiplying. With your partner, show 2 methods for writing the equation. Show one way taking the half from 1, and the other taking the half from 1 and 1 fifth.

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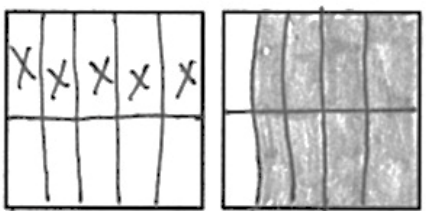
Method 1



Method 1

$1\frac{1}{5} - \frac{1}{2}$

$1 \quad \frac{1}{5}$

$= \frac{1}{2} + \frac{1}{5}$

$= \frac{5}{10} + \frac{2}{10}$

$= \frac{7}{10}$

Method 2

$1\frac{1}{5} - \frac{1}{2}$

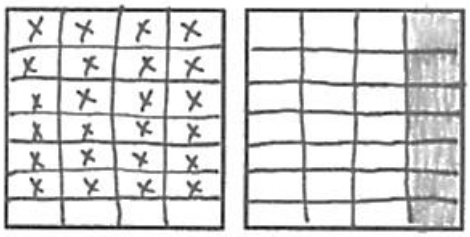
$= \frac{6}{5} - \frac{1}{2}$

$= \frac{12}{10} - \frac{5}{10}$

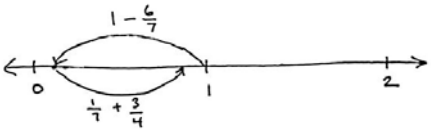
$= \frac{7}{10}$

Problem 2

$$1\frac{3}{4} - \frac{6}{7}$$



Method 1



$1\frac{3}{4} - \frac{6}{7}$

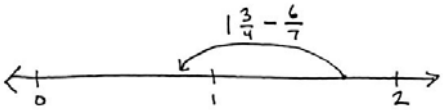
$1 \quad \frac{3}{4}$

$= \frac{1}{7} + \frac{3}{4}$

$= \frac{4}{28} + \frac{21}{28}$

$= \frac{25}{28}$

Method 2



$1\frac{3}{4} - \frac{6}{7}$

$= \frac{7}{4} - \frac{6}{7}$

$= \frac{49}{28} - \frac{24}{28}$

$= \frac{25}{28}$

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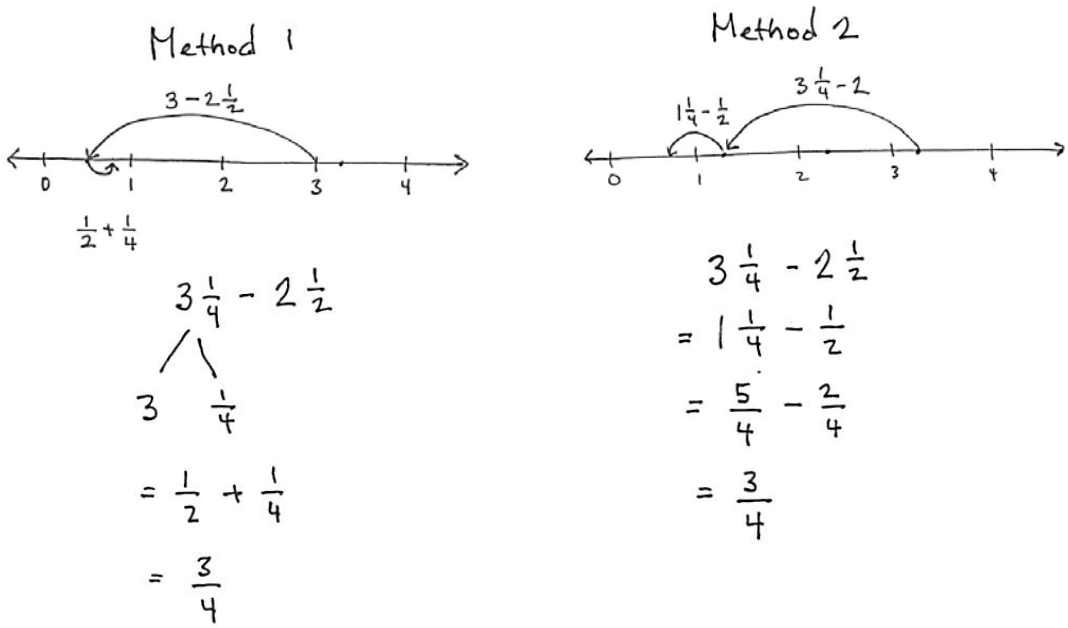
Problem 3

$$3\frac{1}{4} - 2\frac{1}{2}$$

T: Draw a number line. Determine what two numbers your difference will be between.

T: Work with your partner to make sure you understand how each step relates to the number lines.



Problem 4

$$\mathbf{4\,\frac{1}{2} - 3\,\frac{2}{3}}$$

T: Which is bigger, 1 half or 2 thirds?

S: Two thirds.

T: Are you sure? How do you know?

S: "Because you just know that 2 thirds is bigger than 1 half." → "Because if you get like units you can see that 1 half is the same as 3 sixths and 2 thirds is the same as 4 sixths."

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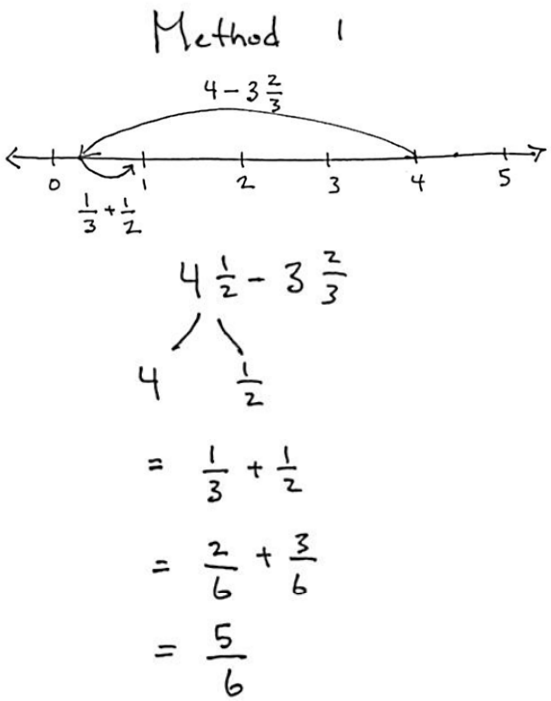
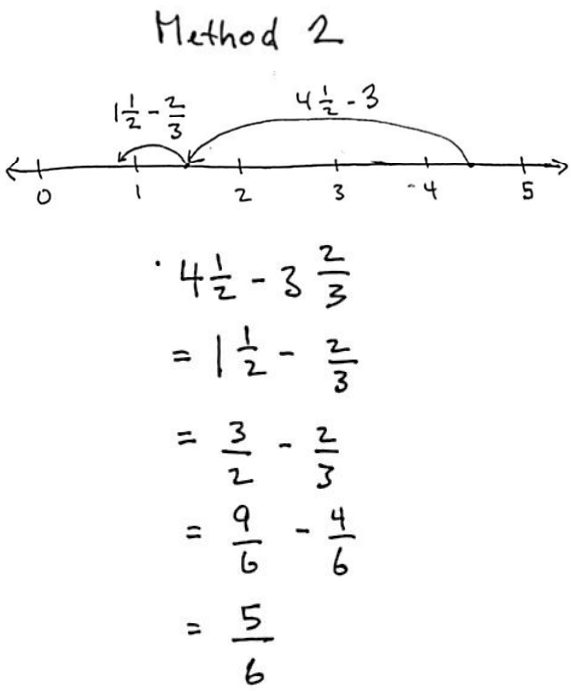
Students can work with the problem or you can guide them, depending on their skill and understanding.





T: Analyze the 2 methods.

S: Method 1 means taking 3 and 2 thirds from 4 and adding back the half.

S: Method 2 takes the whole 3 away from $4\frac{1}{2}$ and then subtracts $\frac{2}{3}$ again.

NOTES ON SCAFFOLDING ELLS:

ELLs may require use of the number line model to communicate reasoning about mixed number subtraction long after other students are able to communicate their reasoning without pictorial representations. Continue to allow the option until students independently move away from it.

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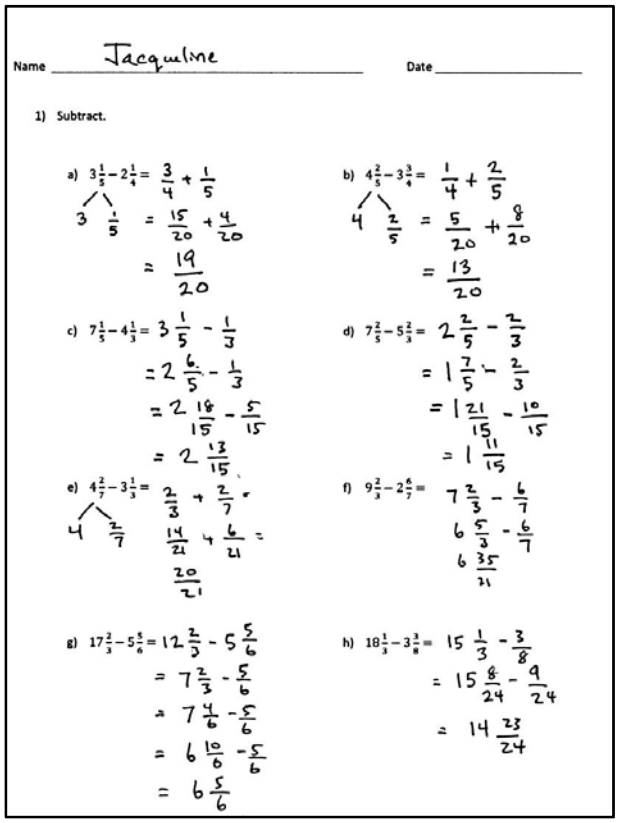
Activity Worksheet (10 minutes)

Materials: (S) Worksheet



Name Jacqueline Date

1) Subtract.

a) $3\frac{1}{5}-2\frac{1}{4}=\frac{3}{4}+\frac{1}{5}$ (3, $\frac{1}{5}$) $=\frac{15}{20}+\frac{4}{20}$ $=\frac{19}{20}$

b) $4\frac{2}{5}-3\frac{3}{4}=\frac{1}{4}+\frac{2}{5}$ (4, $\frac{2}{5}$) $=\frac{5}{20}+\frac{8}{20}$ $=\frac{13}{20}$

c) $7\frac{1}{5}-4\frac{1}{3}=3\frac{1}{5}-\frac{1}{3}$ $=2\frac{6}{5}-\frac{1}{3}$ $=2\frac{18}{15}-\frac{5}{15}$ $=2\frac{13}{15}$

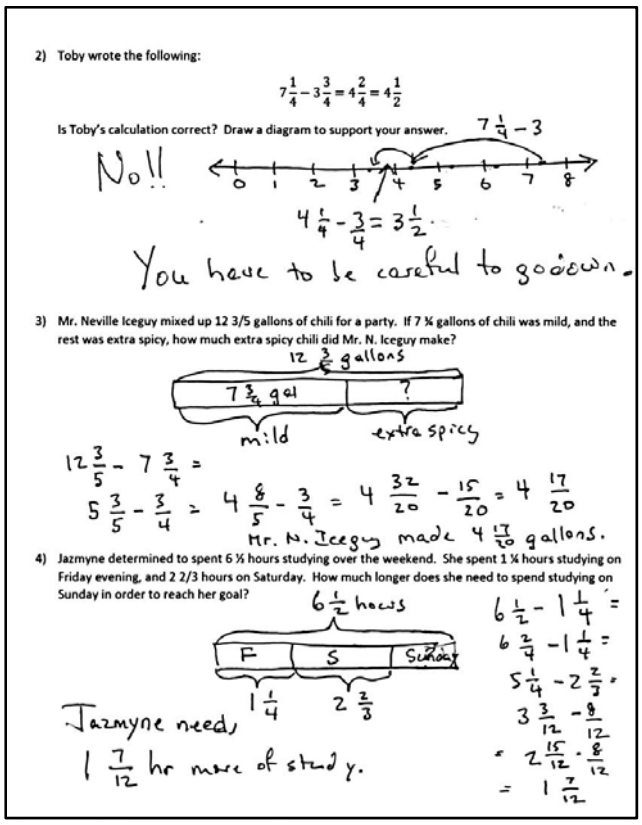
d) $7\frac{2}{5}-5\frac{2}{3}=2\frac{2}{5}-\frac{2}{3}$ $=1\frac{7}{5}-\frac{2}{3}$ $=1\frac{21}{15}-\frac{10}{15}$ $=1\frac{11}{15}$

e) $4\frac{2}{7}-3\frac{1}{3}=\frac{2}{3}+\frac{2}{7}$ (4, $\frac{2}{7}$) $\frac{14}{21}+\frac{6}{21}=$ $\frac{20}{21}$

f) $9\frac{2}{3}-2\frac{6}{7}=7\frac{2}{3}-\frac{6}{7}$ $6\frac{5}{3}-\frac{6}{7}$ $6\frac{35}{21}$

g) $17\frac{2}{3}-5\frac{5}{6}=12\frac{2}{3}-5\frac{5}{6}$ $=7\frac{2}{3}-\frac{5}{6}$ $=7\frac{4}{6}-\frac{5}{6}$ $=6\frac{10}{6}-\frac{5}{6}$ $=6\frac{5}{6}$

h) $18\frac{1}{3}-3\frac{3}{8}=15\frac{1}{3}-\frac{3}{8}$ $=15\frac{8}{24}-\frac{9}{24}$ $=14\frac{23}{24}$



2) Toby wrote the following:

$7\frac{1}{4}-3\frac{3}{4}=4\frac{2}{4}=4\frac{1}{2}$

Is Toby's calculation correct? Draw a diagram to support your answer.

No!! $7\frac{1}{4}-3$ 0 1 2 3 4 5 6 7 8

$4\frac{1}{4}-\frac{3}{4}=3\frac{1}{2}$

You have to be careful to go down.

3) Mr. Neville Iceguy mixed up 12 3/5 gallons of chili for a party. If 7 ¾ gallons of chili was mild, and the rest was extra spicy, how much extra spicy chili did Mr. N. Iceguy make?

$12\frac{3}{5}$ gallons; $7\frac{3}{4}$ gal (mild); ? (extra spicy)

$12\frac{3}{5}-7\frac{3}{4}=$

$5\frac{3}{5}-\frac{3}{4}=4\frac{8}{5}-\frac{3}{4}=4\frac{32}{20}-\frac{15}{20}=4\frac{17}{20}$

Mr. N. Iceguy made $4\frac{17}{20}$ gallons.

4) Jazmyne determined to spent 6 ½ hours studying over the weekend. She spent 1 ¼ hours studying on Friday evening, and 2 2/3 hours on Saturday. How much longer does she need to spend studying on Sunday in order to reach her goal?

$6\frac{1}{2}$ hours; F $1\frac{1}{4}$, S $2\frac{2}{3}$, Sunday

$6\frac{1}{2}-1\frac{1}{4}=$ $6\frac{2}{4}-1\frac{1}{4}=$ $5\frac{1}{4}-2\frac{2}{3}=$ $3\frac{3}{12}-\frac{8}{12}$ $=2\frac{15}{12}-\frac{8}{12}$ $=1\frac{7}{12}$

Jazmyne needs $1\frac{7}{12}$ hr more of study.

Student Debrief (10 minutes)

T: Take two minutes to check your answers with your partner. (Allow students to work.)

T: I will say the problem solutions. Correct your work. (Read solutions.)

T: Today we saw different methods for subtracting. I drew a number bond in some solutions to emphasize how I was thinking about the numbers. Like the number line, it shows a way of thinking. In my work, the number bond shows how I break numbers into parts to make the mathematics easier. Please share with your partner when you used a number bond on your worksheet.

T: (After sharing.) Jacqueline, explain why you chose to make a number bond on letter 'a'.

S: It was clear to me how easy it was to just subtract 2 ¼ from 3. That's just ¾. I like adding better anyway, so then I just added the fifth after making like units of twentieths.

MP.7

T: John, explain why you chose not to make a number bond for 'g'.

S: It just seemed easier to me to subtract the whole numbers first. Right away I know 17 – 5 is 12.

T: I agree. When I was solving the problems, I also subtracted the whole numbers first on that one for

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the same reason you gave.

S: I'm noticing that I drew a bond when the numbers were really easy to subtract and their difference was less than 1. I figured out that I would have a friendly fraction to add to the other part.

T: That is precisely the same process you used starting in grade 1.

T: I'm going to list a set of questions. Talk to your partner about how to solve them with a number bond, and how that relates to our work today with fractions.

Grade 1: 14 – 9

Grade 2: 324 – 198

Grade 3: 1 foot 3 inches – 7 inches

MP.7 Grade 4: 2 kg – 400 g

Grade 5: 1 1/5 – 3/7

S: "In grade 1 there weren't enough ones to take from the ones." → "In grade 2 we bonded 324 as 200 and 124, so the answer was just 124 + 2." → "In grade 3 we had to convert 1 foot to 12 inches to take away 7 inches. 12 inches minus 7 is 5 inches plus the 3 extra inches." → "In grade 4 we had to convert 1 kg to 1000 g to take away 400 g, so we ended up with 1 kg and added back the 600 g." → "In grade 5 we had to convert 1 whole into 5 fifths to take away 3 sevenths and then add back 1/5. So it's 4 sevenths + 1 fifth!

T: Do you notice that every one of the problems has more than one unit? Grade 1 has tens and ones. Grade 2 has hundreds, tens and ones. Grade 3 has feet and inches. Grade 4 has kilograms and grams, and grade 5 has whole numbers and fractions. It's important to understand how to play with the units!

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A

Correct_____

Subtract.

1	$\frac{2}{4}-\frac{1}{4}=$	/	23	$\frac{4}{5}-\frac{7}{10}=$	/
2	$\frac{1}{2}-\frac{1}{4}=$	/	24	$\frac{2}{12}-\frac{1}{12}=$	/
3	$\frac{2}{6}-\frac{1}{6}=$	/	25	$\frac{1}{6}-\frac{1}{12}=$	/
4	$\frac{1}{3}-\frac{1}{6}=$	/	26	$\frac{6}{12}-\frac{1}{12}=$	/
5	$\frac{2}{8}-\frac{1}{8}=$	/	27	$\frac{1}{2}-\frac{1}{12}=$	/
6	$\frac{1}{4}-\frac{1}{8}=$	/	28	$\frac{1}{2}-\frac{5}{12}=$	/
7	$\frac{6}{8}-\frac{1}{8}=$	/	29	$\frac{10}{12}-\frac{5}{12}=$	/
8	$\frac{3}{4}-\frac{1}{8}=$	/	30	$\frac{5}{6}-\frac{5}{12}=$	/
9	$\frac{3}{4}-\frac{3}{8}=$	/	31	$\frac{1}{3}-\frac{3}{12}=$	/
10	$\frac{5}{10}-\frac{2}{10}=$	/	32	$\frac{2}{3}-\frac{1}{12}=$	/
11	$\frac{1}{2}-\frac{2}{10}=$	/	33	$\frac{2}{3}-\frac{3}{12}=$	/
12	$\frac{1}{2}-\frac{2}{10}=$	/	34	$\frac{2}{3}-\frac{7}{12}=$	/
13	$\frac{4}{10}-\frac{1}{10}=$	/	35	$\frac{1}{4}-\frac{2}{12}=$	/
14	$\frac{2}{5}-\frac{1}{10}=$	/	36	$\frac{1}{5}-\frac{1}{15}=$	/
15	$\frac{2}{5}-\frac{3}{10}=$	/	37	$\frac{1}{3}-\frac{1}{15}=$	/
16	$\frac{6}{10}-\frac{3}{10}=$	/	38	$\frac{2}{3}-\frac{3}{15}=$	/
17	$\frac{3}{5}-\frac{3}{10}=$	/	39	$\frac{2}{5}-\frac{4}{15}=$	/
18	$\frac{3}{5}-\frac{5}{10}=$	/	40	$\frac{3}{4}-\frac{2}{12}=$	/
19	$\frac{8}{10}-\frac{1}{10}=$	/	41	$\frac{3}{4}-\frac{5}{16}=$	/
20	$\frac{4}{5}-\frac{1}{10}=$	/	42	$\frac{4}{5}-\frac{5}{15}=$	/
21	$\frac{4}{5}-\frac{5}{10}=$	/	43	$\frac{3}{4}-\frac{4}{12}=$	/
22	$\frac{4}{5}-\frac{5}{10}=$	/	44	$\frac{3}{4}-\frac{7}{16}=$	/

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B Improvement_____ # Correct_____

Subtract.

1	$\frac{2}{10}-\frac{1}{10}=$	/	23	$\frac{3}{4}-\frac{3}{8}=$	/
2	$\frac{1}{5}-\frac{1}{10}=$	/	24	$\frac{5}{15}-\frac{1}{15}=$	/
3	$\frac{2}{4}-\frac{1}{4}=$	/	25	$\frac{1}{3}-\frac{1}{15}=$	/
4	$\frac{1}{2}-\frac{1}{4}=$	/	26	$\frac{3}{15}-\frac{1}{15}=$	/
5	$\frac{5}{10}-\frac{2}{10}=$	/	27	$\frac{1}{5}-\frac{1}{15}=$	/
6	$\frac{1}{2}-\frac{2}{10}=$	/	28	$\frac{1}{5}-\frac{2}{15}=$	/
7	$\frac{1}{2}-\frac{4}{10}=$	/	29	$\frac{12}{15}-\frac{4}{15}=$	/
8	$\frac{4}{10}-\frac{1}{10}=$	/	30	$\frac{4}{5}-\frac{4}{15}=$	/
9	$\frac{2}{5}-\frac{1}{10}=$	/	31	$\frac{1}{4}-\frac{2}{12}=$	/
10	$\frac{2}{5}-\frac{3}{10}=$	/	32	$\frac{3}{4}-\frac{2}{12}=$	/
11	$\frac{6}{10}-\frac{3}{10}=$	/	33	$\frac{3}{4}-\frac{4}{12}=$	/
12	$\frac{3}{5}-\frac{3}{10}=$	/	34	$\frac{3}{4}-\frac{8}{12}=$	/
13	$\frac{3}{5}-\frac{5}{10}=$	/	35	$\frac{1}{3}-\frac{3}{12}=$	/
14	$\frac{8}{10}-\frac{1}{10}=$	/	36	$\frac{1}{6}-\frac{1}{12}=$	/
15	$\frac{4}{5}-\frac{1}{10}=$	/	37	$\frac{1}{3}-\frac{3}{15}=$	/
16	$\frac{4}{5}-\frac{5}{10}=$	/	38	$\frac{2}{3}-\frac{2}{15}=$	/
17	$\frac{4}{5}-\frac{5}{10}=$	/	39	$\frac{2}{5}-\frac{2}{15}=$	/
18	$\frac{4}{5}-\frac{7}{10}=$	/	40	$\frac{3}{4}-\frac{4}{12}=$	/
19	$\frac{2}{8}-\frac{1}{8}=$	/	41	$\frac{3}{4}-\frac{7}{16}=$	/
20	$\frac{1}{4}-\frac{1}{8}=$	/	42	$\frac{4}{5}-\frac{4}{15}=$	/
21	$\frac{6}{8}-\frac{1}{8}=$	/	43	$\frac{3}{4}-\frac{2}{12}=$	/
22	$\frac{3}{4}+\frac{1}{8}=$	/	44	$\frac{3}{4}-\frac{5}{16}=$	/

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Name _____ Date _____

1) Subtract.

a) $3\frac{1}{5} - 2\frac{1}{4} =$

b) $4\frac{2}{5} - 3\frac{3}{4} =$

c) $7\frac{1}{5} - 4\frac{1}{3} =$

d) $7\frac{2}{5} - 5\frac{2}{3} =$

e) $4\frac{2}{7} - 3\frac{1}{3} =$

f) $9\frac{2}{3} - 2\frac{6}{7} =$

g) $17\frac{2}{3} - 5\frac{5}{6} =$

h) $18\frac{1}{3} - 3\frac{3}{8} =$

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2) Toby wrote the following:

$$7\frac{1}{4} - 3\frac{3}{4} = 4\frac{2}{4} = 4\frac{1}{2}$$

Is Toby's calculation correct? Draw a diagram to support your answer.

3) Mr. Neville Iceguy mixed up $12\frac{3}{5}$ gallons of chili for a party. If $7\frac{3}{4}$ gallons of chili was mild, and the rest was extra spicy, how much extra spicy chili did Mr. N. Iceguy make?

4) Jazmyne determined to spent $6\frac{1}{2}$ hours studying over the weekend. She spent $1\frac{1}{4}$ hours studying on Friday evening and $2\frac{2}{3}$ hours on Saturday. How much longer does she need to spend studying on Sunday in order to reach her goal?

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Name _____ Date _____

Solve the problems.

1) $5\frac{1}{2} - 1\frac{1}{3} =$

2) $8\frac{3}{4} - 5\frac{5}{6} =$

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Name _____ Date _____

1) Subtract.

a) $3\frac{1}{4} - 2\frac{1}{3} =$

b) $3\frac{2}{3} - 2\frac{3}{4} =$

c) $6\frac{1}{5} - 4\frac{1}{4} =$

d) $6\frac{3}{5} - 4\frac{3}{4} =$

e) $5\frac{2}{7} - 4\frac{1}{3} =$

f) $8\frac{2}{3} - 3\frac{5}{7} =$

g) $18\frac{3}{4} - 5\frac{7}{8} =$

h) $17\frac{1}{5} - 2\frac{5}{8} =$

2) Tony wrote the following:

$$7\frac{1}{4} - 3\frac{3}{4} = 4\frac{1}{4} - \frac{3}{4}$$

Is Tony's statement correct? Draw a diagram to support your answer.

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3) Ms. Sanger blended $8\frac{3}{4}$ gallons of iced tea with some lemonade for a picnic. If there were $13\frac{2}{5}$ gallons in the mixture, how many gallons of lemonade did she use?

4) A carpenter has a $10\frac{1}{2}$ foot wood plank. He cuts off $4\frac{1}{4}$ feet to replace the slat of a deck and $3\frac{2}{3}$ feet to repair a bannister. He uses the rest of the plank to fix a stair. How many feet of wood does the carpenter use to fix the stair?

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Topic D:

Fraction Addition and Subtraction: Further Applications

5.NF.1, 5.NF.2

Focus Standard:	5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12 . (In general, a/b + c/d = (ad + bc)/bd.)*
	5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
Instructional Days:	4	
Coherence -Links from:	G4–M5	Order and Operations with Fractions
	G5–M1	Whole Number and Decimal Fraction Place Value to the One Thousandths
-Links to:	G5–M4	Extensions and Applications of Multiplication and Division of Fractions and Decimal Fractions

Topic D opens with students estimating the value of expressions involving sums and differences with fractions. "Is your sum less than or greater than one half? One? How do you know?" Though these conversations have been embedded within almost every debrief up to this point, by setting aside an instructional day to dig deeply into logical arguments, students see that it is very easy to forget to make sense of numbers when calculating. This is really the theme of this Topic: Reasoning while using fractions. Concept 2 encourages students to look for relationships before calculating, e.g., to use the tool of the associative property or what they know about parts and wholes. Looking for relationships allows them to see short cuts and connections that are so often by-passed in the rush to "get the answer." In Concept 3, students solve multi-step word problems and actively assess the reasonableness of their answers. In Concept 5 they explore part-to-whole relationships while solving a challenging problem: "One half of Nell's money is equal to 2 thirds of Jennifer's." This lesson challenges the underlying assumption of all fraction arithmetic: That when adding and subtracting, fractions are always defined in relationship to the same whole amount. The beauty of this exploration is to see students grasp that 1/2 of one thing can be equivalent to 2/3 of another!

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CONCEPT CHART	Fraction Addition and Subtraction: Further Applications

Concept 1: Use Fraction Benchmark Numbers to Assess Reasonableness of Addition and Subtraction Equations
(Lesson 13)

Concept 2: Strategize to Solve Multi-Term Problems e.g. $\frac{24}{5} + _____ + \frac{8}{7} = 9\frac{1}{3}$
(Lesson 14)

Concept 3: Solve Multi-Step Word Problems: Assess Reasonableness of Solutions Using Benchmark Numbers
(Lesson 15)

Explore Part to Whole Relationships: "1/2 of Robert's Wire is Equal to 2/3 of Maria's"
(Lesson 16)

Mathematical Practices Brought to Life

In lesson 13, MP.3, "Construct viable arguments and critique the reasoning of others," is modeled as students share reasoning to justify estimations. In Lesson 15, a segment is highlighted that exemplifies MP.5, "Use appropriate tools strategically," as Ming and her classmates develop a method of representing a word problem with the bar diagram and discuss how it allows them to think about and solve the problem. Lesson 16 is an exploration, an example of MP.1, "Make sense of problems and persevere in solving them."

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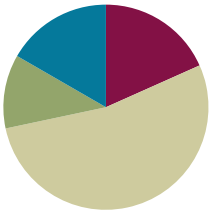
Lesson 13: Use Fraction Benchmark Numbers to Assess Reasonableness of Addition and Subtraction Equations

Suggested Lesson Structure

Fluency Practice	(11 minutes)
Application Problems	(7 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (11 minutes)

- From Fractions to Decimals **4.NF.6** (5 minutes)
- Adding and Subtracting Fractions with Unlike Units **5.NF.1** (6 minutes)

From Fractions to Decimals (5 minutes)

Materials: (T) Dry-erase board or projector

T: (Project) $\frac{1}{10} =$

T: Say the fraction in unit form.

S: 1 tenth.

T: Say the fraction in decimal form.

S: Zero point one.

T: I'll say a fraction in unit form. You say the fraction in decimal form. Ready? 3 tenths.

S: 0.3

T: 7 tenths.

S: 0.7.

T: (Project) $\frac{1}{2} = \frac{\quad}{10}$

T: Say the equivalent fraction with the missing numerator.

S: 1 half = 5 tenths.

T: Say 5 tenths as a decimal.

S: 0.5

T: Say 1 half as a decimal.

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

If students don't remember how to convert from fractions to decimals, then consider doing a review with the whole class. Fractions can be converted to decimals easily when the denominator is tenths, hundredths, or thousandths. It's just like converting the fractions into equivalent fractions with the denominator of tenths, hundredths, or thousandths.

You can also draw out the fraction bars that clearly show equivalent fractions (i.e., 2/5 = 4/10 = 0.4. Both 2/5 and 4/10 have the same values.) The bar models serve as a great visual.

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S: 0.5

T: Say 3 and 1 half as a decimal.

S: 3.5.

Repeat process for possible sequence:

$$\frac{1}{5},\frac{2}{5},\frac{4}{5},2\frac{4}{5},\frac{1}{4},\frac{3}{4},5\frac{3}{4},\frac{1}{25},\frac{2}{25},\frac{3}{25},5\frac{3}{25},\frac{1}{20},\frac{11}{20},3\frac{11}{20},\frac{1}{50},\frac{3}{50},4\frac{3}{50}$$

Adding and Subtracting Fractions with Unlike Units (6 minutes)

Materials: (S) Personal white boards

T: (Write) $\frac{1}{4}+\frac{1}{2}=\frac{2}{6}$

T: True or false?

S: False.

T: On your personal white boards, write the answer that will make the addition sentence true.

S: (Write) $\frac{1}{4}+\frac{1}{2}=\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$

T: (Project) $\frac{1}{2}+\frac{3}{8}=\frac{7}{8}$

T: True or false?

S: True.

T: Rewrite the addition sentence using like units.

S: (Write)

$$\frac{1}{2}+\frac{3}{8}=\frac{4}{8}+\frac{3}{8}$$
$$=\frac{7}{8}$$

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

Provide think time for students to process the problem before answering true or false. If necessary, you might also give another few seconds for students to discuss the problem with their partners. Perhaps have them explain to their partners why they think a problem is true or false. This pausing allows gives them the time they need to own their ideas.

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T: (Project) $\frac{2}{3}-\frac{2}{9}=\frac{4}{9}$

T: True or false?

S: True.

T: Rewrite the subtraction sentence using like units.

S: (Write)

$$\frac{2}{3}-\frac{2}{9}=\frac{6}{9}-\frac{2}{9}$$
$$=\frac{4}{9}$$

T: (Project) $\frac{5}{6}-\frac{2}{3}=\frac{3}{3}$

T: True or false?

S: False.

T: Write the answer that will make the subtraction sentence true.

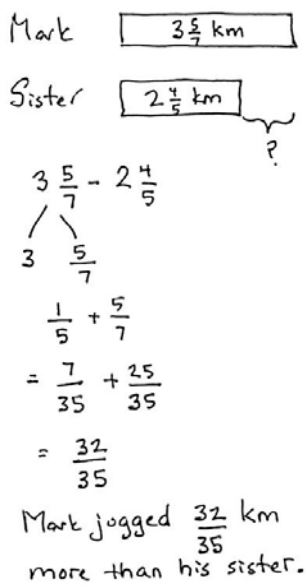
S: (Write)

$$\frac{5}{6}-\frac{2}{3}=\frac{5}{6}-\frac{4}{6}$$
$$=\frac{1}{6}$$

Application Problem (7 minutes)

Mark jogged 3 5/7 km. His sister jogged 2 4/5 km. How much farther did Mark jog than his sister?

Remind students to approach the problem with their RDW strategy. This is a very brief application problem. As you circulate while students work, quickly assess which work you will select for a short two or three minute debrief.



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Concept Development (32 minutes)

T: Look at this problem. Tell your partner how you would solve it. (Display the problem and allow 30 seconds for discussion.)

$$\frac{1}{2}+\frac{3}{4}$$

T: (After listening to partner sharing.) For the past two weeks we have been learning different strategies to add and subtract unlike fractions. Today we will focus on mental math—using reasoning without actually solving using paper and pencil.

Problem 1

$$\frac{1}{2}+\frac{3}{4}$$

T: Think about this expression without solving it using paper and pencil. Share your analysis with a partner.

S: "1/2 could be 50% of something or 50 cents of a dollar." → "I know that 1/2 is the same as 5 tenths or 0.5 as a decimal." → "I know 1/2 is more than half because half of a whole is 2 fourths." → "3/4 is the same as 75%, just like money. 3 quarters equal 75 cents."

MP.3

T: What do you know about the total value of this expression without solving?

S: Since 3/4 is more than half and we need to add 1/2 more, the answer will be greater than 1.

$$\frac{1}{2}+\frac{3}{4}>1$$

S: "It's like adding 50 cents and 75 cents. The answer will be more than 1 dollar. → "1/2 + 2/4 = 1, but there's still a leftover of 1/4." → "The total answer is 1 1/4."

NOTES ON SCAFFOLDING STUDENTS BELOW GRADE LEVEL:

If students are not ready to estimate the sum/difference of a fraction sentence, consider doing a mini pre-lesson or a fluency activity on estimating a single fraction before moving onto this lesson.

For example:

1. Is ¾ closer to 0, ½, or 1 whole?
2. Is 2/7 closer to 0, ½, or 1 whole?
3. Is 9/10 closer to 0, ½, or 1 whole?
4. Is 1 6/7 closer to 1, 1 ½, or 2?
5. Is 3 4/7 closer to 3, 3 ½, or 4?

If necessary, write each fraction on a sentence strip (using it as a number line) and label it. This way, students can easily see whether the fraction is closer to 0, ½, or 1 whole.

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Problem 2

$$1\frac{2}{5}-\frac{2}{3}$$

T: Without calculating, what do you know about value of this expression? Talk to your partner.

S: "I see that it's a subtraction problem. 2/5 is less than 1/2 and 2/3 is more than 1/2." → "I know that 2/3 can't be subtracted from 2/5 because 2/3 is larger, so we'll need to subtract from 1 whole." → "I can convert 1 2/5 to 7/5 in my head."

T: Do you think the answer is more than 1 or less than 1? Turn and share.

$$1\frac{2}{5}-\frac{2}{3}<1$$

S: "Less than 1 because 1/5 is less than 1/3 so 2/5 is less than 2/3." → "The answer is less than 1 because I can create equivalent fractions in my head and solve. 1/5 = 3/15, 1/3 =5/15, 3/15 + 5/15 = 8/15."

Problem 3

$$\frac{4}{10}+\frac{1}{3}$$

T: Use reasoning skills decide if the answer is more than 1 or less than 1/2. Work with your partner.

Allow a minute for students to analyze and discuss the problem. Circulate and listen. If students seem to be lost or off track with their thinking, then you might use some of the following questions:

- Is 4/10 more than 1 half or less than 1 half?
- Is 4/10 closer to 0 or 1 whole?
- What's half of 10 tenths?
- What's 4/10 as a decimal?
- How much money is 4 tenths?
- Is 1/3 closer to 0 or 1 whole?
- Is 1/3 more than 1 half or less than 1 half?

NOTES ON SCAFFOLDING STUDENTS BELOW GRADE LEVEL:

When students are solving problems with partners and continue to struggle even with guided questions, consider asking them to use personal boards to draw a number line. They can use it to estimate one fraction at a time, then estimate the final answer.

For example:

4/10 + 1/3

Draw a number line for 4/10 → 4/10 ≈ less than 1 half.

Draw a number line for 1/3 → 1/3 ≈ less than 1 half.

Less than 1/2 + less than 1/2 < 1

4/10 + 1/3 < 1

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

Because both addends are clearly less than half, this is an easy question. For advanced students, let them determine if 3/10 + 2/3 is less than, equal to, or greater than 1. Encourage them NOT to solve the problem until they have determined their reasoning.

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Problem 4

$$\frac{4}{10}+\frac{2}{9}$$

T: Share your analysis of this expression with your partner.

S: "I see that it's an addition problem. 4/10 is less than 1/2, because 4/10 = 0.4." → "I agree. I also noticed that 2/9 is less than 1 half because half of 9 is 4.5, and 2 is less than 4.5. → "Both fractions are closer to 0 than closer to 1 whole."

MP.3

T: Is the answer less than or greater than 1 half?

S: "4 tenths is really close to a half. It only needs 1/10 to be one half." → "I'm asking myself: 'Is 2/9 greater than 1/10?' If it is, the answer will be greater than 1/2." → "2/9 has to be greater than 1/10 because it's close to 1 fourth or 2/8."

T: Verify to see.

$$\frac{4}{10}+\frac{2}{9} > 1/2$$

NOTES ON SCAFFOLDING STUDENTS ABOVE AND BELOW GRADE LEVEL:

Below grade level: Try 4/10 + 1/9, which is more obviously less than ½.

Above grade level: Try 3/10 + 3/9.

Problem 5

$$1\frac{4}{7}-\frac{9}{10}$$

T: Think about this expression with your partner.

S: "4/7 is more than 1/2, and 9/10 is 1/10 away from 1 whole." → "I know that 9/10 can't be subtracted from 4/7, because 9/10 is larger, so we'll need to subtract from 1 whole." → "I would use 1 – 9/10 = 1/10." → "I agree. Now we have a leftover of 1/10 + 5/7."

T: Is the value of this expression greater than or less than 1/2?

S: I think it's more than 1 half because I know 4/7 alone is already more than 1 half.

S: 1 less than 1 and 4/7 is going to be more than half. 9/10 is less than 1, so 9/10 less than 1 and 4/7 is going to be greater than 1/2.

$$1\frac{5}{7}-\frac{9}{10} > 1/2$$

NOTES ON SCAFFOLDING STUDENTS ABOVE AND BELOW GRADE LEVEL:

Below grade level: Try 1 4/7 – 1 first.

Above grade level: Try 1 3/7 -9/10.

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Problem 6

$$\frac{4}{5} - \frac{1}{8}$$

T: Reason about this problem with your partner. Is the value of the expression more than 1/2 or less?

Use the following questions to support:

- Is 4/5 more than 1 half or less than 1 half?
- Is 4/5 closer to 0 or 1 whole?
- What's half of 5 fifths?
- Can you convert 4 fifths to tenths or a decimal in your head? What is it?
- Is 1/8 more than 1 half or less than 1 half?
- Is 1/8 closer to 0 or 1 whole?

$$\frac{4}{5} - \frac{1}{8} > 1/2$$

Problem 7

$$2\frac{1}{3} + 3\frac{1}{5} ____ 6 + \frac{7}{8}$$

S: "I first need to estimate the total for both equations, then I can compare them." → "I'll first add up the whole numbers on the left, then compare them because they're the larger place values. If the wholes are equal, then I'll estimate the fractions and compare them."

Allow two minutes for students to analyze and discuss the problem. Circulate and listen. If students seem to be lost or off track with their thinking, use the following questions to guide discussion and thinking.

- What do you think about 2 1/3 and 3 1/5?
- What's the total of the whole numbers on the left?
- How do you compare the whole numbers?
- What do you think about 1/3 and 1/5?
- Are 1/3 and 1/5 closer to 0 or 1 whole?
- What is your estimation of 1/3 + 1/5? More than 1 or less than 1?

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- Is 7/8 closer to 0 or 1 whole?
- What's 6 + 7/8 equal to?

$$2\frac{1}{3}+3\frac{1}{5}<6+\frac{7}{8}$$

Problem 8

Have students work in partners or individually for the last problem, and then review as a class.

$$4\frac{9}{10}-1\frac{1}{8} ____ 2\frac{1}{2}+\frac{2}{7}$$

Solution:

$$4\frac{9}{10}-1\frac{1}{8}>2\frac{1}{2}+\frac{2}{7}$$

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

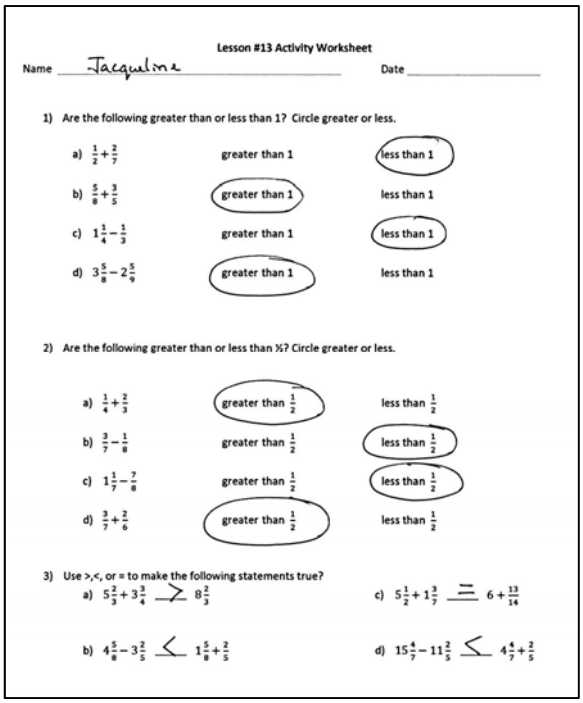
ELLS and SWD might require more examples and more time to process. If necessary, when the class is working on classwork independently, pull out a small group to do more examples.

Allow students to use the actual fraction pieces to estimate if you have them. If not, allow them to draw out the fractions on personal white boards.

Activity Worksheet (12 minutes)

Materials: (S) Worksheet

T: Take out your activity sheet. You have 12 minutes to work on it. You may work independently or with a partner.



Lesson #13 Activity Worksheet

Name Jacquline Date

1) Are the following greater than or less than 1? Circle greater or less.

a) $\frac{1}{2}+\frac{2}{7}$ greater than 1 less than 1

b) $\frac{5}{8}+\frac{3}{5}$ greater than 1 less than 1

c) $1\frac{1}{4}-\frac{1}{3}$ greater than 1 less than 1

d) $3\frac{5}{8}-2\frac{5}{9}$ greater than 1 less than 1

2) Are the following greater than or less than ½? Circle greater or less.

a) $\frac{1}{4}+\frac{2}{3}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

b) $\frac{3}{7}-\frac{1}{8}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

c) $1\frac{1}{7}-\frac{7}{8}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

d) $\frac{3}{7}+\frac{2}{6}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

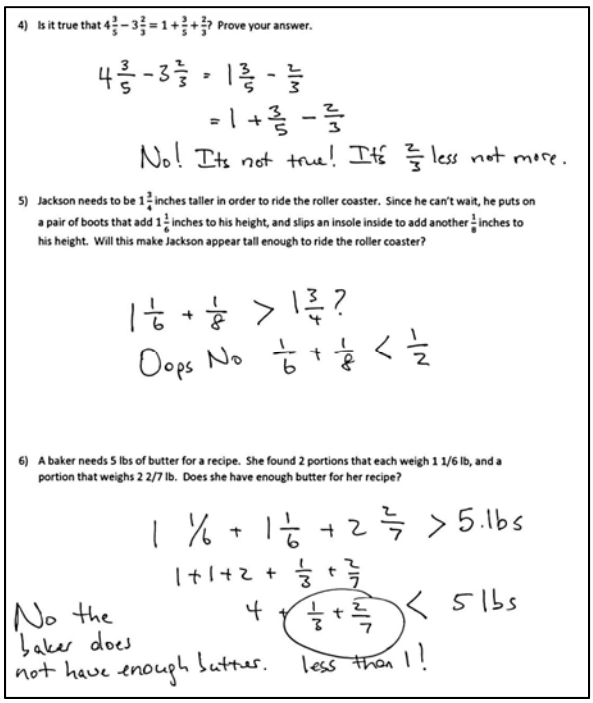
3) Use >,<, or = to make the following statements true?

a) $5\frac{2}{3}+3\frac{3}{4}$ > $8\frac{2}{3}$

b) $4\frac{5}{8}-3\frac{2}{5}$ < $1\frac{5}{8}+\frac{2}{5}$

c) $5\frac{1}{2}+1\frac{3}{7}$ = $6+\frac{13}{14}$

d) $15\frac{4}{7}-11\frac{2}{5}$ < $4\frac{4}{7}+\frac{2}{5}$



4) Is it true that $4\frac{3}{5}-3\frac{2}{3}=1+\frac{3}{5}+\frac{2}{3}$? Prove your answer.

$4\frac{3}{5}-3\frac{2}{3}=1\frac{3}{5}-\frac{2}{3}$

$=1+\frac{3}{5}-\frac{2}{3}$

No! Its not true! Its $\frac{2}{3}$ less not more.

5) Jackson needs to be $1\frac{3}{4}$ inches taller in order to ride the roller coaster. Since he can't wait, he puts on a pair of boots that add $1\frac{1}{6}$ inches to his height, and slips an insole inside to add another $\frac{1}{8}$ inches to his height. Will this make Jackson appear tall enough to ride the roller coaster?

$1\frac{1}{6}+\frac{1}{8}>1\frac{3}{4}$?

Oops No $\frac{1}{6}+\frac{1}{8}<\frac{1}{2}$

6) A baker needs 5 lbs of butter for a recipe. She found 2 portions that each weigh 1 1/6 lb, and a portion that weighs 2 2/7 lb. Does she have enough butter for her recipe?

$1\frac{1}{6}+1\frac{1}{6}+2\frac{2}{7}>5$ lbs

$1+1+2+\frac{1}{3}+\frac{2}{7}$

$4+\frac{1}{3}+\frac{2}{7}<5$ lbs

No the baker does not have enough butter. less than 1!

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Student Debrief (10 minutes)

T: Bring your activity worksheet to the debrief. Share, check, and/or explain your answers to your partner.

S: (Students work together for two minutes. Circulate and listen to explanations. Analyze the work you see to determine which student solutions you will display to support your lesson objective.)

T: (Go over answers or select individual students to explain the thinking process that led them to a correct answer.)

T: What did you learn today? Turn and share with your partner.

S: "I can use my reasoning skills to estimate fraction answers." → "When I estimate fraction answers, I should be thinking about how that fraction is closer to 0, 1/2, or 1 whole. That'll make it easier for me to do mental math." → "I learned to estimate fractions and answers mentally. It reminds me of rounding. If I'm adding 2 fractions that are more than 1/2, then the answer will be more than 1 whole."

T: (Optional as time allows. The following is a suggested list of questions to invite reflection and active processing of the total lesson experience. Use those that resonate for you as you consider what will best support your students' ability to articulate the focus of the lesson.)

- Why do mathematicians agree it is wise to estimate before calculating?
- Think about what happens to your reasoning when you are calculating.
- Why do mathematicians agree it is wise to estimate after calculating?

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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Name _____ Date _____

1) Are the following greater than or less than 1? Circle the correct answer.

a) $\frac{1}{2}+\frac{2}{7}$ greater than 1 less than 1

b) $\frac{5}{8}+\frac{3}{5}$ greater than 1 less than 1

c) $1\frac{1}{4}-\frac{1}{3}$ greater than 1 less than 1

d) $3\frac{5}{8}-2\frac{5}{9}$ greater than 1 less than 1

2) Are the following greater than or less than ½? Circle the correct answer.

a) $\frac{1}{4}+\frac{2}{3}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

b) $\frac{3}{7}-\frac{1}{8}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

c) $1\frac{1}{7}-\frac{7}{8}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

d) $\frac{3}{7}+\frac{2}{6}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

3) Use > , < , or = to make the following statements true.

a) $5\frac{2}{3}+3\frac{3}{4}$ _____ $8\frac{2}{3}$

b) $4\frac{5}{8}-3\frac{2}{5}$ _____ $1\frac{5}{8}+\frac{2}{5}$

c) $5\frac{1}{2}+1\frac{3}{7}$ _____ $6+\frac{13}{14}$

d) $15\frac{4}{7}-11\frac{2}{5}$ _____ $4\frac{4}{7}+\frac{2}{5}$

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4) Is it true that $4\frac{3}{5} - 3\frac{2}{3} = 1 + \frac{3}{5} + \frac{2}{3}$? Prove your answer.

5) Jackson needs to be $1\frac{3}{4}$ inches taller in order to ride the roller coaster. Since he can't wait, he puts on a pair of boots that add $1\frac{1}{6}$ inches to his height, and slips an insole inside to add another $\frac{1}{8}$ inches to his height. Will this make Jackson appear tall enough to ride the roller coaster?

6) A baker needs 5 lbs of butter for a recipe. She found 2 portions that each weigh 1 1/6 lb and a portion that weighs 2 2/7 lb. Does she have enough butter for her recipe?

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Name _____ Date _____

Circle the correct answer.

1) $\frac{1}{2} + \frac{5}{12}$ greater than 1 less than 1

2) $2\frac{7}{8} + 1\frac{7}{9}$ greater than 1 less than 1

3) $1\frac{1}{12} - \frac{7}{10}$ greater than $\frac{1}{2}$ less than

4) $\frac{3}{7} + \frac{1}{8}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

5) Use > , < , or = to make the following statement true.

$4\frac{4}{5} + 3\frac{2}{3}$ _____ $8\frac{1}{2}$

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Name _____ Date _____

1) Are the following greater than or less than 1? Circle the correct answer.

a) $\frac{1}{2}+\frac{4}{9}$ greater than 1 less than 1

b) $\frac{5}{8}+\frac{3}{5}$ greater than 1 less than 1

c) $1\frac{1}{5}-\frac{1}{3}$ greater than 1 less than 1

d) $4\frac{3}{5}-3\frac{3}{4}$ greater than 1 less than 1

2) Are the following greater than or less than ½? Circle the correct answer.

e) $\frac{1}{5}+\frac{1}{4}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

f) $\frac{6}{7}-\frac{1}{6}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

g) $1\frac{1}{7}-\frac{5}{6}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

h) $\frac{4}{7}+\frac{1}{8}$ greater than $\frac{1}{2}$ less than $\frac{1}{2}$

3) Use > , < , or = to make the following statements true.

i) $5\frac{4}{5}+2\frac{2}{3}$ _____ $8\frac{3}{4}$

j) $3\frac{4}{7}-2\frac{3}{5}$ _____ $1\frac{4}{7}+\frac{3}{5}$

k) $4\frac{1}{2}+1\frac{4}{9}$ _____ $5+\frac{13}{18}$

l) $10\frac{3}{8}-7\frac{3}{5}$ _____ $3\frac{3}{8}+\frac{3}{5}$

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4) Is it true that $5\frac{2}{3} - 3\frac{3}{4} = 1 + \frac{2}{3} + \frac{3}{4}$? Prove your answer.

5) A tree limb hangs $5\frac{1}{4}$ feet from a telephone wire. The city trims back the branch before it grows within $2\frac{1}{2}$ feet of the wire. Will the city allow the tree to grow $2\frac{3}{4}$ more feet?

6) Mr. Kreider wants to paint two doors and several shutters. It takes $2\frac{1}{8}$ gallons of paint to coat each door and $1\frac{3}{5}$ gallons of paint to coat his shutters. If Mr. Kreider buys three 2-gallon cans of paint, does he have enough to complete the job?

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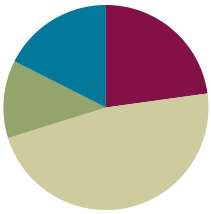
Lesson 14: Strategize to Solve Multi-Term Problems

Suggested Lesson Structure

Fluency Practice	(13 minutes)
Application Problems	(10 minutes)
Concept Development	(27 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (13 minutes)

- Make Larger Units Sprint **4.NF.1** (10 minutes)
- Happy Counting with Mixed Numbers **4.NF.3a** (3 minutes)

Sprints (10 minutes)

Materials: (S) Make Larger Units Sprint (See directions for administration of sprints in the **G5-M3 Fluency Progressions.**)

Example: Make units of 2. 2/6 = 1 x 2/ 3 x 2 = 1/3

Happy Counting with Mixed Numbers (3 minutes)

T: Let's count by ½ with mixed numbers. Ready? (Teacher rhythmically points up until a change is desired. Show a closed hand, then point down. Continue, mixing it up).

S: ½, 1, 1 ½, 2 (stop), 1 ½, 1, ½, 0 (stop), ½, 1, 1 ½, 2, 2 ½, 3, 3 ½, 4 (stop), 3 ½, 3, 2 ½, 2, 1 ½, 1 (stop), 1 ½, 2, 2 ½, 3, 3 ½, 4, 4 ½, 5.

T: Excellent. Try it for 30 seconds with your partner. Partner A, you are the teacher today.

NOTES ON SCAFFOLDING STUDENTS BELOW GRADE LEVEL:

The sprint depends greatly on students' knowledge of their factors. Below grade-level students often are not fluent with their basic facts. The day before administering this sprint, discreetly call below grade-level students to meet with you in order to give them a copy of the next day's sprint. This is very motivating. Now they have a reason to study and practice.

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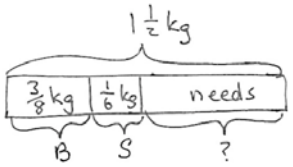
Application Problems (10 minutes)

For a large order, Mr. Magoo made 3/8 kg of fudge in his bakery. He then got 1/6 kg from his sister's bakery. If he needs a total of 1 ½ kg, how much more fudge does he need to make?



$\left(1\frac{1}{2}\text{ kg} - \frac{3}{8}\text{ kg}\right) - \frac{1}{6}\text{ kg} = ___$

$1\frac{4}{8} - \frac{3}{8}$

$= 1\frac{1}{8}$

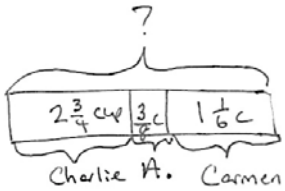
$1\frac{1}{8} - \frac{1}{6}$

$= 1\frac{3}{24} - \frac{4}{24}$

$= \frac{23}{24}$

∴ Mr. Magoo needs $\frac{23}{24}$ kg more fudge.

During lunch, Charlie drinks 2 3/4 cup of milk. Allison drinks 3/8 cup of milk. Carmen drinks 1/6 cup of milk. How much milk do the 3 students drink?



$2\frac{3}{4} + \frac{3}{8} + 1\frac{1}{6} =$

$3\frac{3}{4} + \frac{3}{8} + \frac{1}{6}$

$3\frac{18}{24} + \frac{9}{24} + \frac{4}{24}$

$3\frac{31}{24} = 4\frac{7}{24}$

∴ The students drank $4\frac{7}{24}$ cups of milk.

T: Now that you have solved these two problems, consider how they are the same and how they are different.

S: "Both problems had three parts that we knew." → "True, but actually in the fudge problem, the one part was the whole amount." → "The fudge problem had a missing part but the milk problem was missing the whole amount of milk." → "So, for the fudge problem we had to subtract from 1 ½ kg. For the milk problem we had to add up the three parts to find the total amount of milk."

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Concept Development (27minutes)

Problem 1

$$\frac{2}{3}+\frac{1}{5}+\frac{1}{3}+1\frac{4}{5}=$$

T: Yesterday, we learned to solve fraction problems by estimating the answers without using our pencils. Today we are going to build upon that knowledge by continuing to solve fractions in our heads before using paper and pencil.

T: Look at this problem. What do you notice? Turn and share with a partner.

S: I see that it's an addition problem adding thirds and fifths.

S: I see that I can add up the thirds and I can also add the fifths together.

T: Can you solve this problem mentally? Turn and share.

S: (Students share.)

T: Are we finding a part or whole?

S: Whole.

S: 2/3 plus 1/3 equals 1 whole. 1/5 plus 1 4/5 equals 2 wholes. Finally, 1 plus 2 equals 3.

T: Excellent. We can rearrange the problem and solve it using Sam's strategy.

$$(\frac{2}{3}+\frac{1}{3})+(\frac{1}{5}+1\frac{4}{5})$$
$$= 1 + 2$$
$$= 3$$

Problem 2

$$5\frac{7}{8}-\frac{1}{2}-\frac{7}{8}-1\frac{1}{2}=$$

S: "I see that it's a subtraction problem." → "I see that denominators are in eighths and halves. They need to be the same in order for me to subtract." → "Without looking at the mixed numbers, I see two 7/8's and two 1/2's."

T: Yes. This is a subtraction problem. Analyze the parts and wholes. Turn and share.

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S: "5 7/8 is the whole amount. 7/8 is a part being taken away. That makes 5." → "1 ½ and ½ are both parts being taken away. If I combine them, I'm taking away 2. 5 – 2 = 3." → "We can combine all the parts and make a bigger part, then subtract from the whole."

$$5\frac{7}{8}-\left(\frac{1}{2}+\frac{7}{8}+1\frac{1}{2}\right)$$
$$= 5\frac{7}{8}-2\frac{7}{8}$$
$$=3$$

$$\left(5\frac{7}{8}-\frac{7}{8}\right)-\left(\frac{1}{2}+1\frac{1}{2}\right)$$
$$= 5-2$$
$$= 3$$

NOTES ON SCAFFOLDING STUDENTS ABOVE AND BELOW GRADE LEVEL:

When students are analyzing parts and wholes, relationships, or compatible numbers, resist the temptation to jump in. Wait time is critical. Let them analyze. This allows students that are above grade level to find more complexities and those below grade level to enter at the most basic level.

Problem 3 is more challenging because of the change in sign. Students will see the compatibility of 2 5/6 and 1/6, which will expedite the addition of the two numbers to make a larger whole from which one third is subtracted.

Problem 3

$$2\frac{5}{6}-\frac{1}{3}+\frac{1}{6}$$

$$= (2\tfrac{5}{6}+\tfrac{1}{6})-\tfrac{1}{3}$$

= 3 – 1/3
= 2 2/3

Problem 4

$$\frac{14}{3}+____+\frac{9}{4}=8\frac{11}{12}$$

T: Let's analyze this fraction equation. Share with your partner: What do you notice about this fraction equation?

S: "This is an addition problem and I have the answer of 8 11/12 on the right hand side." → "I'm missing a part that is needed to make the whole amount of 8 11/12." → "14/3 is a part, too."

S: "I can add the parts and subtract them from the whole amount to find that mystery number." → "Find the sum of the parts and take them from the whole."

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T: Go ahead and solve for the missing part. You can use paper and pencil if you wish.

.

$$8\frac{11}{12} - (\frac{14}{3} + \frac{9}{4})$$

= 8 11/12 – (4 2/3 + 2 ¼)
= 8 11/12 – (4 8/12 + 2 3/12)
= 8 11/12 – 6 11/12
= 2

Problem 5

$$____ - 15 - 4\frac{1}{2} = 7\frac{3}{5}$$

S: "I see that it's a subtraction problem. Something minus 15 minus 4 1/2 equals 7 3/5." → "The whole is missing in this problem and everything else is a part." → "I can add up all the parts together to find the whole."

S: The whole is missing, so we'll add up all the parts to find the whole. I can rewrite the problem like this:

$$____ - 15 - 4\frac{1}{2} = 7\frac{3}{5}$$

$$15 + 4\frac{1}{2} + 7\frac{3}{5} = ____$$

= 15 + 4 5/10 + 7 6/10
= 26 11/10
= 27 1/10

Problem 6

T: I would like you to try to solve this problem with your partner. You have two minutes.

$$6\frac{3}{4} + \frac{3}{5} - ____ = 5$$

Allow two minutes for students to analyze and discuss the problem without calculating, just formulating their thoughts about how to solve.

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T: Go ahead and solve the problem.

$$6\frac{3}{4}+\frac{3}{5}-____=5$$

= (6 ¾ + 3/5) – 5

= 6 15/20 + 12/20 – 5

= 6 27/20 – 5

= 7 7/20 – 5

= 2 7/20

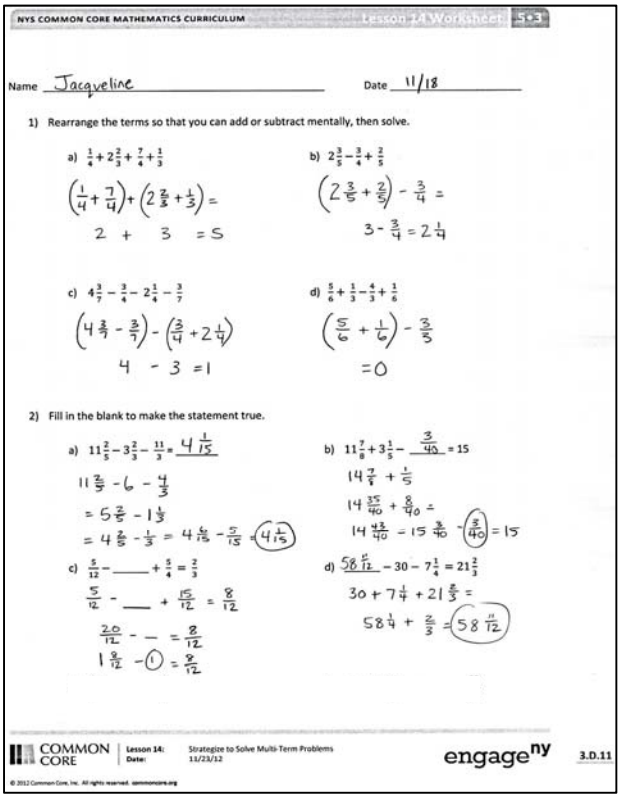
NOTES ON USING THE ACTIVITY SHEET:

The problems on this particular worksheet may require more room than the worksheet offers. Be aware that students do well to have a "math notebook" or "journal." When a worksheet has a set of challenging problems, assign pairs to solve them on the board as others use paper so that they are easier to review. It is also much more engaging for the students to see their peers' solutions.

Activity Worksheet (14 minutes)

Materials: (S) Worksheet

T: Take out your activity sheet. You have 14 minutes to work on it. You may work independently or with a partner.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 14 Worksheet 5•3

Name Jacqueline Date 11/18

1) Rearrange the terms so that you can add or subtract mentally, then solve.

a) $\frac{1}{4}+2\frac{2}{5}+\frac{7}{4}+\frac{1}{3}$

$\left(\frac{1}{4}+\frac{7}{4}\right)+\left(2\frac{2}{5}+\frac{1}{3}\right)=$

$2 + 3 = 5$

b) $2\frac{3}{5}-\frac{3}{4}+\frac{2}{5}$

$\left(2\frac{3}{5}+\frac{2}{5}\right)-\frac{3}{4}=$

$3-\frac{3}{4}=2\frac{1}{4}$

c) $4\frac{3}{7}-\frac{3}{4}-2\frac{1}{4}-\frac{3}{7}$

$\left(4\frac{3}{7}-\frac{3}{7}\right)-\left(\frac{3}{4}+2\frac{1}{4}\right)$

$4 - 3 = 1$

d) $\frac{5}{6}+\frac{1}{3}-\frac{4}{3}+\frac{1}{6}$

$\left(\frac{5}{6}+\frac{1}{6}\right)-\frac{3}{3}$

$=0$

2) Fill in the blank to make the statement true.

a) $11\frac{2}{5}-3\frac{2}{3}-\frac{11}{3}=$ $4\frac{1}{15}$

$11\frac{2}{5}-6-\frac{4}{3}$

$=5\frac{2}{5}-1\frac{1}{3}$

$=4\frac{6}{5}-\frac{1}{3}=4\frac{18}{15}-\frac{5}{15}=4\frac{13}{15}$

b) $11\frac{7}{8}+3\frac{1}{5}-$ $\frac{3}{40}$ $=15$

$14\frac{7}{8}+\frac{1}{5}$

$14\frac{35}{40}+\frac{8}{40}=$

$14\frac{43}{40}=15\frac{3}{40}-\frac{3}{40}=15$

c) $\frac{5}{12}-$ ____ $+\frac{3}{4}=\frac{2}{3}$

$\frac{5}{12}-$ ___ $+\frac{15}{12}=\frac{8}{12}$

$\frac{20}{12}-$ ___ $=\frac{8}{12}$

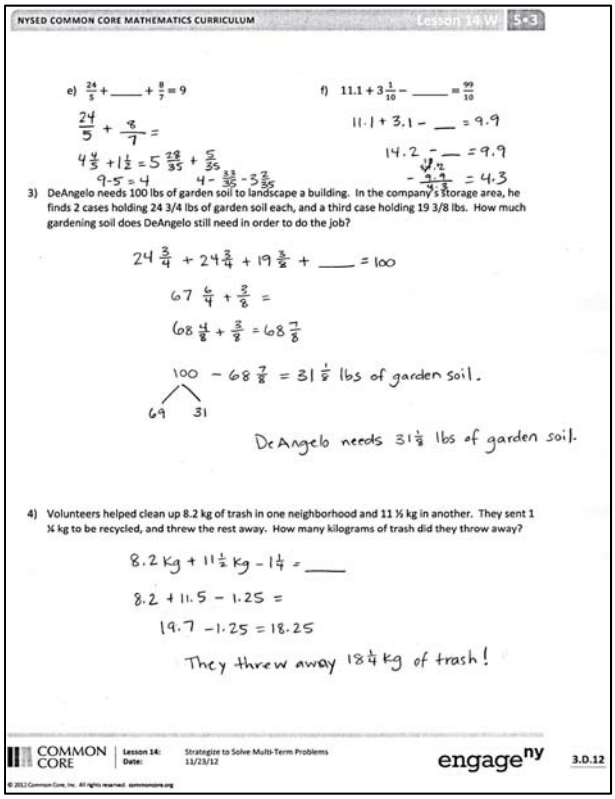
$1\frac{8}{12}-1=\frac{8}{12}$

d) $58\frac{11}{12}-30-7\frac{1}{4}=21\frac{2}{3}$

$30+7\frac{1}{4}+21\frac{2}{3}=$

$58\frac{1}{4}+\frac{2}{3}=58\frac{11}{12}$

COMMON CORE Lesson 14: Strategize to Solve Multi-Term Problems Date: 11/23/12 engage^ny 3.D.11


NYSED COMMON CORE MATHEMATICS CURRICULUM Lesson 14 W 5•3

e) $\frac{24}{5}+$ ____ $+\frac{8}{7}=9$

$\frac{24}{5}+\frac{8}{7}=$

$4\frac{4}{5}+1\frac{1}{2}=5\frac{29}{35}+\frac{5}{35}$

$9-5=4$

f) $11.1+3\frac{1}{10}-$ ____ $=\frac{99}{10}$

$11.1+3.1-$ ___ $=9.9$

$14.2-$ ___ $=9.9$

$-\frac{9.9}{4.3}$

$4-\frac{33}{35}=3\frac{2}{35}$

3) DeAngelo needs 100 lbs of garden soil to landscape a building. In the company's storage area, he finds 2 cases holding 24 3/4 lbs of garden soil each, and a third case holding 19 3/8 lbs. How much gardening soil does DeAngelo still need in order to do the job?

$24\frac{3}{4}+24\frac{3}{4}+19\frac{3}{8}+$ ____ $=100$

$67\frac{6}{4}+\frac{3}{8}=$

$68\frac{4}{8}+\frac{3}{8}=68\frac{7}{8}$

$100-68\frac{7}{8}=31\frac{1}{8}$ lbs of garden soil.

69 31

DeAngelo needs $31\frac{1}{8}$ lbs of garden soil.

4) Volunteers helped clean up 8.2 kg of trash in one neighborhood and 11 ½ kg in another. They sent 1 ¼ kg to be recycled, and threw the rest away. How many kilograms of trash did they throw away?

$8.2\text{ kg}+11\frac{1}{2}\text{ kg}-1\frac{1}{4}=$ ____

$8.2+11.5-1.25=$

$19.7-1.25=18.25$

They threw away $18\frac{1}{4}$ kg of trash!

COMMON CORE Lesson 14: Strategize to Solve Multi-Term Problems Date: 11/23/12 engage^ny 3.D.12

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Student Debrief (10 minutes)

T: When rearranging the terms in the top section of the activity sheet, talk to your partner about what you looked for to help you solve the problems easily.

S: "We grouped to make whole numbers." → "By grouping fractions to make whole numbers it was easy to add or subtract." → "I looked for numbers in different forms. It was harder to see the pairs if the denominators were different or if they weren't written as mixed numbers."

T: What about in section B?

S: That was so much harder! I was really surprised the answer to c) was 1. I didn't expect that so it made me go back and look at the relationships in the problem.

T: Talk to your partner about some of the skills you had to use to solve these problems.

S: "We had to analyze part and whole relationships." → "I had to recognize when there were easy like units." → "We had to move back and forth between decimals and fractions in f) and in the second word problem about the volunteers, too." → "We had to think hard about the problems that had addition and subtraction problems and whether to add or subtract something."

T: This was a challenging activity.

S: We had to really think!

T: Let's go over the last problem about the volunteers. I would say it was related to question b) on the front side. Explain my thinking to your partner.

T: (After students talk.) Which of the problems on the front side of the worksheet would you relate to the problem of the gardening soil?

T: (After students talk.) Review the process you used on two problems. First, review a problem that was very easy for you. Then, review the process on a problem that was very challenging for you.

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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A # Correct _____

1	$\frac{2}{4}=$	/	23	$\frac{9}{27}=$	/
2	$\frac{2}{6}=$	/	24	$\frac{9}{63}=$	/
3	$\frac{2}{8}=$	/	25	$\frac{8}{12}=$	/
4	$\frac{5}{10}=$	/	26	$\frac{8}{16}=$	/
5	$\frac{5}{15}=$	/	27	$\frac{8}{24}=$	/
6	$\frac{5}{20}=$	/	28	$\frac{8}{64}=$	/
7	$\frac{4}{8}=$	/	29	$\frac{12}{18}=$	/
8	$\frac{4}{12}=$	/	30	$\frac{12}{16}=$	/
9	$\frac{4}{16}=$	/	31	$\frac{9}{12}=$	/
10	$\frac{3}{6}=$	/	32	$\frac{6}{8}=$	/
11	$\frac{3}{9}=$	/	33	$\frac{10}{12}=$	/
12	$\frac{3}{12}=$	/	34	$\frac{15}{18}=$	/
13	$\frac{4}{6}=$	/	35	$\frac{8}{10}=$	/
14	$\frac{6}{12}=$	/	36	$\frac{16}{20}=$	/
15	$\frac{6}{18}=$	/	37	$\frac{12}{15}=$	/
16	$\frac{6}{30}=$	/	38	$\frac{18}{27}=$	/
17	$\frac{6}{9}=$	/	39	$\frac{27}{36}=$	/
18	$\frac{7}{14}=$	/	40	$\frac{32}{40}=$	/
19	$\frac{7}{21}=$	/	41	$\frac{45}{54}=$	/
20	$\frac{7}{42}=$	/	42	$\frac{24}{36}=$	/
21	$\frac{8}{12}=$	/	43	$\frac{60}{72}=$	/
22	$\frac{9}{18}=$	/	44	$\frac{48}{60}=$	/

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B Improvement _____ # Correct _____

1	$\frac{5}{10}=$	/	23	$\frac{8}{24}=$	/
2	$\frac{5}{15}=$	/	24	$\frac{8}{56}=$	/
3	$\frac{5}{20}=$	/	25	$\frac{8}{12}=$	/
4	$\frac{2}{4}=$	/	26	$\frac{9}{18}=$	/
5	$\frac{2}{6}=$	/	27	$\frac{9}{27}=$	/
6	$\frac{2}{8}=$	/	28	$\frac{9}{72}=$	/
7	$\frac{3}{6}=$	/	29	$\frac{12}{18}=$	/
8	$\frac{3}{9}=$	/	30	$\frac{6}{8}=$	/
9	$\frac{3}{12}=$	/	31	$\frac{9}{12}=$	/
10	$\frac{4}{8}=$	/	32	$\frac{12}{16}=$	/
11	$\frac{4}{12}=$	/	33	$\frac{8}{10}=$	/
12	$\frac{4}{16}=$	/	34	$\frac{16}{20}=$	/
13	$\frac{4}{6}=$	/	35	$\frac{12}{15}=$	/
14	$\frac{7}{14}=$	/	36	$\frac{10}{12}=$	/
15	$\frac{7}{21}=$	/	37	$\frac{15}{18}=$	/
16	$\frac{7}{35}=$	/	38	$\frac{16}{24}=$	/
17	$\frac{6}{9}=$	/	39	$\frac{24}{32}=$	/
18	$\frac{6}{12}=$	/	40	$\frac{36}{45}=$	/
19	$\frac{6}{18}=$	/	41	$\frac{40}{48}=$	/
20	$\frac{6}{36}=$	/	42	$\frac{24}{36}=$	/
21	$\frac{8}{12}=$	/	43	$\frac{48}{60}=$	/
22	$\frac{8}{16}=$	/	44	$\frac{60}{72}=$	/

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Name _____ Date _____

1) Rearrange the terms so that you can add or subtract mentally, then solve.

a) $\frac{1}{4} + 2\frac{2}{3} + \frac{7}{4} + \frac{1}{3}$

b) $2\frac{3}{5} - \frac{3}{4} + \frac{2}{5}$

c) $4\frac{3}{7} - \frac{3}{4} - 2\frac{1}{4} - \frac{3}{7}$

d) $\frac{5}{6} + \frac{1}{3} - \frac{4}{3} + \frac{1}{6}$

2) Fill in the blank to make the statement true.

a) $11\frac{2}{5} - 3\frac{2}{3} - \frac{11}{3} =$ _____

b) $11\frac{7}{8} + 3\frac{1}{5} -$ _____ = 15

c) $\frac{5}{12} -$ _____ $+ \frac{5}{4} = \frac{2}{3}$

d) _____ $- 30 - 7\frac{1}{4} = 21\frac{2}{3}$

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e) $\frac{24}{5} + _____ + \frac{8}{7} = 9$

f) $11.1 + 3\frac{1}{10} - _____ = \frac{99}{10}$

3) DeAngelo needs 100 lbs of garden soil to landscape a building. In the company's storage area, he finds 2 cases holding 24 3/4 lbs of garden soil each, and a third case holding 19 3/8 lbs. How much gardening soil does DeAngelo still need in order to do the job?

4) Volunteers helped clean up 8.2 kg of trash in one neighborhood and 11 ½ kg in another. They sent 1 ¼ kg to be recycled and threw the rest away. How many kilograms of trash did they throw away?

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Name _____ Date _____

Fill in the blank to make the statement true.

1) $1\frac{3}{4} + \frac{1}{6} + _____ = 7\frac{1}{2}$

2) $8\frac{4}{5} - \frac{2}{3} - _____ = 3\frac{1}{10}$

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Name _____ Date _____

1) Rearrange the terms so that you can add or subtract mentally, then solve.

a) $1\frac{3}{4}+\frac{1}{2}+\frac{1}{4}+\frac{1}{2}$

b) $3\frac{1}{6}-\frac{3}{4}+\frac{5}{6}$

d) $5\frac{5}{8}-2\frac{6}{7}-\frac{2}{7}-\frac{5}{8}$

d) $\frac{7}{9}+\frac{1}{2}-\frac{3}{2}+\frac{2}{9}$

2) Fill in the blank to make the statement true.

g) $7\frac{3}{4}-1\frac{2}{7}-\frac{3}{2}=$ _____

h) $9\frac{5}{6}+1\frac{1}{4}-$ _____ = 14

i) $\frac{7}{10}-$ _____ $+\frac{3}{2}=\frac{6}{5}$

j) _____ $-20-3\frac{1}{4}=14\frac{5}{8}$

k) $\frac{17}{3}+$ _____ $+\frac{5}{2}=10\frac{4}{5}$

l) $23.1+1\frac{7}{10}-$ _____ $=\frac{66}{10}$

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3) Laura bought $8\frac{3}{10}$ yd of ribbon. She used $1\frac{2}{5}$ yd to tie a package and $2\frac{1}{3}$ to make a bow. Joe later gave her $4\frac{3}{5}$ yd. How much ribbon does she now have?

4) Mia bought $10\frac{1}{9}$ lb of flour. She used $2\frac{3}{4}$ lb of flour to bake a banana cake and some to bake a chocolate cake. After baking the two cakes, she had $3\frac{5}{6}$ lb of flour left. How much flour did she use to bake the chocolate cake?

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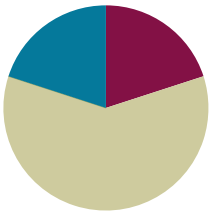
Lesson 15: Solve Multi-Step Word Problems: Assess Reasonableness of Solutions Using Benchmark Numbers

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Concept Development	(36 minutes)
Student Debrief	(12 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Sprint **4.NF.2** (12 minutes)

Sprint (12 minutes)

Materials: (S) Circle the Smallest Fraction Sprint (See directions for administration of sprints in the **G5-M3 Fluency Progressions.**)

Concept Development (36 minutes)

Materials: (S) Worksheet, personal white boards

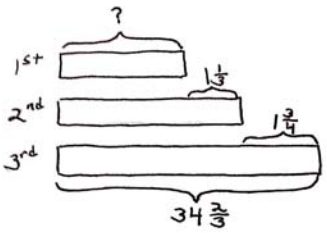
Problem 1

In a race, the second place finisher crossed the finish line 1 1/3 minutes after the winner. The third place finisher was 1 3/4 minutes behind the second place finisher. The third place finisher took 34 2/3 minutes. How long did the winner take?

T: Let's read the problem together. (Students read chorally.) Now, share with your partner: What do you see when you hear the story?

T: (Students share.) Explain to your partner how you are going to draw this problem.

S: (Students share.)



The 1st place time was 31 min 35 s.

$$34\tfrac{2}{3} - 1\tfrac{3}{4} = 33\tfrac{2}{3} - \tfrac{3}{4}$$
$$= 33\tfrac{8}{12} - \tfrac{9}{12}$$
$$= 32\tfrac{20}{12} - \tfrac{9}{12}$$
$$= 32\tfrac{11}{12}$$
$$32\tfrac{11}{12} - 1\tfrac{1}{3} = 31\tfrac{11}{12} - \tfrac{1}{3}$$
$$= 31\tfrac{11}{12} - \tfrac{4}{12}$$
$$= 31\tfrac{7}{12}$$
$$31\tfrac{7}{12}\text{ min} = 31\tfrac{35}{60}\text{ min} = 31\text{ min } 35\text{ s}$$

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T: Ming, could you share your method of drawing?

S: "The first sentence tells me that the second finisher took 1 1/3 minutes longer than the winner. So I'll draw 2 bars. The second bar represents the second finisher with a longer bar and with the difference of 1 1/3 minutes."

T: Betty, can you add more to Ming's drawing?

S: "The second sentence says the third finisher took 1 ¾ minutes longer than the second finisher. So I'll draw a longer bar for the third finisher, and label the difference of 1 ¾ minutes."

T: Steven, can you add anything else to the drawing?

S: "The third sentence tells us the third finisher's minutes. So I can label the third bar with 34 2/3 minutes."

MP.5

T: Excellent. The question now is to find the winner's time. How you are going to solve this problem? Turn and share with your partner.

S: "We have to find the second finisher's time first, then we can find the winner's time." → "We know the third finisher's time but don't know the second finisher's time. We can solve it by subtracting." → "Use the third finisher's time to subtract 1 ¾ to find the second finisher's time. Then use the second finisher's time to subtract 1 1/3 to find the winner's time."

T: Great. Let's first find the second finisher's time. What's the subtraction sentence?

S: 34 2/3 – 1 3/4

= 34 8/12 – 1 9/12

= 33 20/12 – 1 9/12

= 32 11/12

T: What does 32 11/12 mean?

S: The second finisher's time is 32 11/12 minutes.

T: Let's now find the winner's time. What's the subtraction sentence?

S: 32 11/12 – 1 1/3

= 32 11/12 – 1 4/12

= 31 7/12

T: What's the word sentence to answer the question?

S: The winner's time was 31 7/12 minutes.

T: How do I convert 31 7/12 minutes to minutes and seconds? Turn and share with your partner.

T: Alanzo, can you share your thinking with us?

S: 31 7/12 minutes means there are 31 minutes and 7/12 of a minute. I need to convert 7/12 into seconds.

NOTES ON SCAFFOLDING ELLS:

When doing a word problem lesson, be sure to provide many opportunities for students to turn and talk, or repeat what a peer has said.

If possible, pair students that speak the same home language together. For example, a non-ELL Chinese student with a strong math background can be paired with an ELL Chinese student. Teachers can also encourage them to converse in Chinese.

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

A bonus question can be to convert the winner's time into seconds.

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T: Linda, what do you think?

S: I agree with Alanzo. I know there are 60 seconds in a minute, so I'll convert 7 twelfths to 35 sixtieths.

T: Very good. 7/12 = 35/60. What's the winner's time in minutes and seconds?

S: The winner's time was 31 minutes and 35 seconds.

Problem 2

John used 1 3/4 kg of salt to melt the ice on his sidewalk. He then used another 3 4/5 kg on the driveway. If he originally bought 10 kg of salt, how much does he have left?

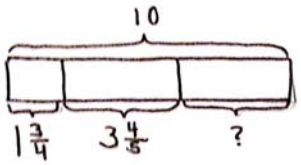
T: Let's read the problem together. (Students read chorally.) What do you see when you hear the story? Turn and share.

T: (Students share.) How you are going to draw this problem? Turn and share.

S: (Students share.)

T: I'll give you one minute to draw.

T: Explain to your partner what conclusions you can make from your drawing.



$$1\tfrac{3}{4} + 3\tfrac{4}{5} = 4\tfrac{3}{4} + \tfrac{4}{5}$$
$$= 4\tfrac{15}{20} + \tfrac{16}{20}$$
$$= 4\tfrac{31}{20}$$
$$= 5\tfrac{11}{20}$$
$$10 - 5\tfrac{11}{20} = 5 - \tfrac{11}{20}$$
$$= 4\tfrac{9}{20}$$

He had $4\frac{9}{20}$ kg of salt left.

T: (After a brief exchange.) May, could your share your method of drawing?

S: "Since I know he bought 10 kg of salt, I'll draw a whole bar and label it 10 kg. He used some salt for the sidewalk and some for the driveway. I'll draw two shorter bars under the whole bar and label them 1 3/4 kg and 3 4/5 kg."

T: How much salt does he have left? How do we solve this problem? Turn and share.

S: "I can use the total of 10 kg to subtract the two parts to find the left over part." → "I can add up the two parts to make them a bigger part, then I'll subtract that from the whole of 10 kg."

T: You have four minutes to solve the problem.

NOTES ON SCAFFOLDING ELLS AND STUDENTS BELOW GRADE LEVEL:

If necessary, when the class is working independently, the teacher can pull out a small group of students to work on the activity sheet together. Ask questions to guide students' thinking on solving the word problems. Provide more or less support as needed.

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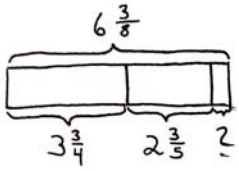
Activity Worksheet (20 minutes)

Materials: (S) Worksheet

Have students work on the rest of the problems independently.

Problem 3

Sinister Stan stole 3 3/4 oz of slime from Messy Molly, but his evil plans required 6 3/8 oz of slime. He stole another 2 3/5 oz from Rude Ralph. How much more slime does Sinister Stan need for his evil plan?



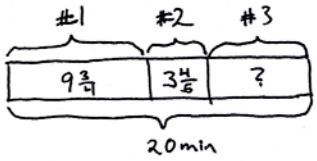
$$3\tfrac{3}{4} + 2\tfrac{3}{5} = 5\tfrac{3}{4} + \tfrac{3}{5}$$
$$= 5\tfrac{15}{20} + \tfrac{12}{20}$$
$$= 5\tfrac{27}{20}$$
$$= 6\tfrac{7}{20}$$

$$6\tfrac{3}{8} - 6\tfrac{7}{20} = \tfrac{3}{8} - \tfrac{7}{20}$$
$$= \tfrac{15}{40} - \tfrac{14}{40}$$
$$= \tfrac{1}{40}$$

Sinister Stan needs $\frac{1}{40}$ ounce of slime.

Problem 4

Gavin had 20 minutes to do a 3-problem quiz. He spent 9 3/4 minutes on question 1 and 3 4/5 minutes on question 2. How much time did he have left for question 3? Write the answer in minutes and seconds.



$$20 - 9\tfrac{3}{4} - 3\tfrac{4}{5}$$
$$= 19\tfrac{20}{20} - 9\tfrac{15}{20} - 3\tfrac{16}{20}$$
$$= 10\tfrac{5}{20} - 3\tfrac{16}{20}$$
$$= 9\tfrac{25}{20} - 3\tfrac{16}{20}$$
$$= 6\tfrac{9}{20}$$

$$6\tfrac{9}{20} = 6\tfrac{27}{60}$$
$$= 6\text{min } 27\text{seconds}$$

He had 6 minutes 27 seconds for question 3.

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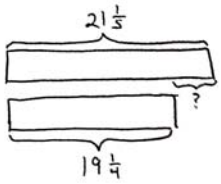
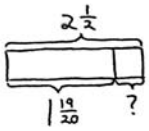
Problem 5

Matt wants to save 2 1/2 minutes on his 5K race time. After a month of hard training, he managed to lower his overall time from 21 1/5 minutes to 19 1/4 minutes. By how many more minutes does Matt need to lower his race time?



$$21\tfrac{1}{5} - 19\tfrac{1}{4} = 2\tfrac{1}{5} - \tfrac{1}{4}$$
$$= 2\tfrac{4}{20} - \tfrac{5}{20}$$
$$= 1\tfrac{24}{20} - \tfrac{5}{20}$$
$$= 1\tfrac{19}{20}$$



$$2\tfrac{1}{2} - 1\tfrac{19}{20} = 1\tfrac{1}{2} - \tfrac{19}{20}$$
$$= 1\tfrac{10}{20} - \tfrac{19}{20}$$
$$= \tfrac{30}{20} - \tfrac{19}{20}$$
$$= \tfrac{11}{20}$$

Matt needs to save $\frac{11}{20}$ min off his race time.

Student Debrief (12 minutes)

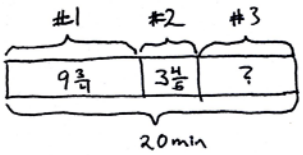
T: Bring your activity worksheet to the debrief. Share, check, and/or explain your answers to your partner.

S: (Students work together for 2 minutes. Circulate and listen to explanations. Analyze the work you see to determine which student solutions you will display to support your lesson objective.)

T: (Teacher goes over answers.) Let's read problem 4 together and we'll take a look at 2 different solution strategies.

S: Gavin had 20 minutes to do a 3-problem quiz. He spent 9 3/4 minutes on question 1 and 3 4/5 minutes on question 2. How much time did he have left for question 3? Write the answer in minutes and seconds.

Student A's Work



$$20 - 9\tfrac{3}{4} - 3\tfrac{4}{5}$$
$$= 19\tfrac{20}{20} - 9\tfrac{15}{20} - 3\tfrac{16}{20}$$
$$= 10\tfrac{5}{20} - 3\tfrac{16}{20}$$
$$= 9\tfrac{25}{20} - 3\tfrac{16}{20}$$
$$= 6\tfrac{9}{20}$$

$$6\tfrac{9}{20} = 6\tfrac{27}{60}$$
$$= 6\text{min } 27\text{seconds}$$

He had 6 minutes 27 seconds for question 3.

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T: Discuss what you notice about the two different drawings. (Allow time for students to share.)

T: Jaron, would you share your thinking?

S: "The first drawing labeled the whole on the bottom. The second drawing labeled it on the side."

T: How are the drawings similar? Turn and share.

S: (Students share.)

T: Keri, what do think?

S: Both drawings labeled the time for the 3 questions. They also labeled the total amount of time, which is 20 minutes.

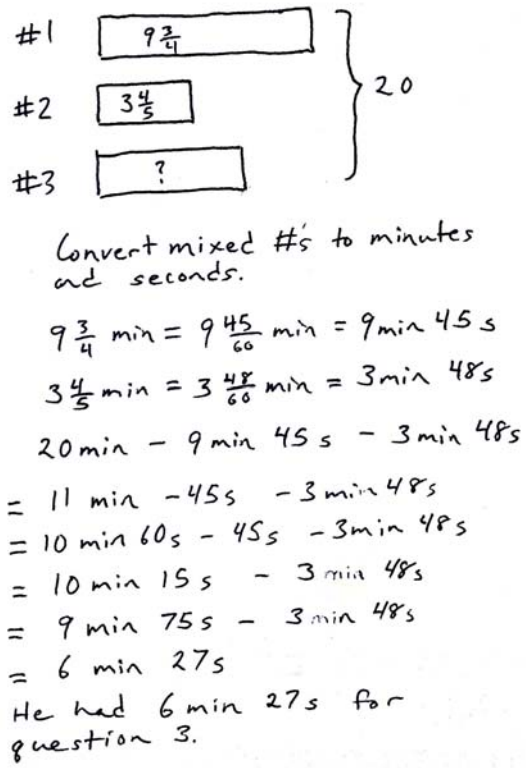
T: Let's look at them closely. How did student A solve the problem? Turn and share.

S: "Student A used the total of 20 minutes to subtract the time spent on questions 1 and 2 to find the left over time. Then the student converted 6 9/20 to 6 minutes and 45 seconds.

T: How did student B solve the problem? Turn and share.

S: Student B converted all the mixed numbers into minutes and seconds. Then they used the 20 minutes to subtract the time spent on question 1, which is 9 minutes 45 seconds, and the time spent for question 2, which is 3 minutes 48 seconds. 6 minutes 27 seconds were left over for question 3.

Student B's Work



T: Which solution strategies did you like better?

S: "The first one." → "The first one is a lot shorter than the second one." → "The second seems like it should be easy, but it took a long time to write it out with all of the minutes and seconds." → "Because it was twentieths, it was really easy to change it to minutes and seconds from 6 9/20 minutes: I just multiplied the fraction by 3 thirds."

Optional as time allows:

T: The following is a suggested list of questions to invite reflection and active processing of the total lesson experience. Use those that resonate for you as you consider what will best support your students' ability to articulate the focus of the lesson.)

- Did anyone else solve the problem differently? (Students come up and explain their solution strategies to the class.)
- What did you get better at today?

Exit Ticket

After the debrief, instruct students to complete the exit ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Remind students to use the RDW strategy while solving the problem.

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A

Correct_____

Circle the smallest fraction.

1	$\frac{1}{2}$	$\frac{1}{4}$	23	$\frac{1}{4}$	$\frac{1}{8}$
2	$\frac{1}{2}$	$\frac{3}{4}$	24	$\frac{1}{4}$	$\frac{3}{8}$
3	$\frac{1}{2}$	$\frac{5}{8}$	25	$\frac{1}{4}$	$\frac{7}{12}$
4	$\frac{1}{2}$	$\frac{7}{8}$	26	$\frac{1}{4}$	$\frac{11}{12}$
5	$\frac{1}{2}$	$\frac{1}{10}$	27	$\frac{1}{6}$	$\frac{7}{12}$
6	$\frac{1}{2}$	$\frac{3}{10}$	28	$\frac{1}{6}$	$\frac{11}{12}$
7	$\frac{1}{2}$	$\frac{5}{12}$	29	$\frac{2}{3}$	$\frac{1}{6}$
8	$\frac{1}{2}$	$\frac{11}{12}$	30	$\frac{2}{3}$	$\frac{5}{6}$
9	$\frac{1}{2}$	$\frac{7}{10}$	31	$\frac{2}{3}$	$\frac{2}{9}$
10	$\frac{1}{5}$	$\frac{9}{10}$	32	$\frac{2}{3}$	$\frac{4}{9}$
11	$\frac{2}{5}$	$\frac{1}{10}$	33	$\frac{2}{3}$	$\frac{1}{12}$
12	$\frac{2}{5}$	$\frac{3}{10}$	34	$\frac{2}{3}$	$\frac{5}{12}$
13	$\frac{3}{5}$	$\frac{3}{10}$	35	$\frac{2}{3}$	$\frac{11}{12}$
14	$\frac{3}{5}$	$\frac{7}{10}$	36	$\frac{2}{3}$	$\frac{7}{12}$
15	$\frac{4}{5}$	$\frac{1}{10}$	37	$\frac{3}{4}$	$\frac{1}{8}$
16	$\frac{4}{5}$	$\frac{9}{10}$	38	$\frac{3}{4}$	$\frac{1}{8}$
17	$\frac{1}{3}$	$\frac{1}{9}$	39	$\frac{5}{6}$	$\frac{7}{12}$
18	$\frac{1}{3}$	$\frac{2}{9}$	40	$\frac{5}{6}$	$\frac{5}{12}$
19	$\frac{1}{3}$	$\frac{4}{9}$	41	$\frac{6}{7}$	$\frac{38}{42}$
20	$\frac{1}{3}$	$\frac{8}{9}$	42	$\frac{7}{8}$	$\frac{62}{72}$
21	$\frac{1}{3}$	$\frac{1}{12}$	43	$\frac{49}{54}$	$\frac{8}{9}$
22	$\frac{1}{3}$	$\frac{5}{12}$	44	$\frac{67}{72}$	$\frac{11}{12}$

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B Improvement_____ # Correct_____

Circle the smallest fraction.

1	$\frac{1}{2}$	$\frac{1}{6}$	23	$\frac{1}{4}$	$\frac{5}{8}$
2	$\frac{1}{2}$	$\frac{5}{6}$	24	$\frac{1}{4}$	$\frac{7}{8}$
3	$\frac{1}{2}$	$\frac{1}{8}$	25	$\frac{1}{4}$	$\frac{1}{12}$
4	$\frac{1}{2}$	$\frac{3}{8}$	26	$\frac{1}{4}$	$\frac{5}{12}$
5	$\frac{1}{2}$	$\frac{7}{10}$	27	$\frac{1}{6}$	$\frac{1}{12}$
6	$\frac{1}{2}$	$\frac{9}{10}$	28	$\frac{1}{6}$	$\frac{5}{12}$
7	$\frac{1}{2}$	$\frac{1}{12}$	29	$\frac{2}{3}$	$\frac{1}{9}$
8	$\frac{1}{2}$	$\frac{7}{12}$	30	$\frac{2}{3}$	$\frac{7}{9}$
9	$\frac{1}{5}$	$\frac{1}{10}$	31	$\frac{2}{3}$	$\frac{5}{9}$
10	$\frac{1}{5}$	$\frac{3}{10}$	32	$\frac{2}{3}$	$\frac{8}{9}$
11	$\frac{2}{5}$	$\frac{7}{10}$	33	$\frac{3}{4}$	$\frac{1}{2}$
12	$\frac{2}{5}$	$\frac{9}{10}$	34	$\frac{3}{4}$	$\frac{5}{12}$
13	$\frac{3}{5}$	$\frac{1}{10}$	35	$\frac{3}{4}$	$\frac{11}{12}$
14	$\frac{3}{5}$	$\frac{9}{10}$	36	$\frac{3}{4}$	$\frac{7}{12}$
15	$\frac{4}{5}$	$\frac{3}{10}$	37	$\frac{5}{6}$	$\frac{1}{12}$
16	$\frac{4}{5}$	$\frac{7}{10}$	38	$\frac{5}{6}$	$\frac{11}{12}$
17	$\frac{1}{3}$	$\frac{1}{6}$	39	$\frac{3}{4}$	$\frac{5}{8}$
18	$\frac{1}{3}$	$\frac{5}{6}$	40	$\frac{3}{4}$	$\frac{3}{8}$
19	$\frac{1}{3}$	$\frac{5}{9}$	41	$\frac{6}{7}$	$\frac{34}{42}$
20	$\frac{1}{3}$	$\frac{7}{9}$	42	$\frac{7}{8}$	$\frac{64}{72}$
21	$\frac{1}{3}$	$\frac{7}{12}$	43	$\frac{47}{54}$	$\frac{8}{9}$
22	$\frac{1}{3}$	$\frac{11}{12}$	44	$\frac{65}{72}$	$\frac{11}{12}$

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Name _____ Date _____

Solve the word problems using the RDW strategy. Show all your work.

1. In a race, the second place finisher crossed the finish line 1 1/3 minutes after the first place finisher. The third place finisher was 1 3/4 minutes behind the second place finisher. The third place finisher took 34 2/3 minutes. How long did the first place finisher take?

2. John used 1 3/4 kg of salt to melt the ice on his sidewalk. He then used another 3 4/5 kg on the driveway. If he originally bought 10 kg of salt, how much does he have left?

3. Sinister Stan stole 3 3/4 oz of slime from Messy Molly, but his evil plans required 6 3/8 oz of slime. He stole another 2 3/5 oz from Rude Ralph. How much more slime does Sinister Stan need for his evil plan?

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4. Gavin went to a book store with $20. He spent 9 3/4 of his money on a book and 3 4/5 on a poster. What fraction of his money did he have left? Write the answer in dollars and cents.

5. Matt wants to save 2 1/2 minutes on his 5K race time. After a month of hard training he managed to lower his overall time from 21 1/5 minutes to 19 1/4 minutes. By how many more minutes does Matt need to lower his race time?

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Name _____ Date _____

Solve the word problems using the RDW strategy. Show all your work.

Cheryl bought a sandwich for $5\frac{1}{2}$ dollars and a drink for $2.60. If she paid for her meal with a $10 bill, how much money did she have left.? Write your answer as a fraction and in dollars and cents.

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Name _____ Date _____

Solve the word problems using the RDW strategy. Show all your work.

1. A baker buys a 5 lb bag of sugar. She uses $1\frac{2}{3}$ lb to make some muffins and $2\frac{3}{4}$ lb to make a cake. How much sugar does she have left?

2. A boxer needs to lose $3\frac{1}{2}$ kg in a month to be able to compete as a flyweight. In three weeks, he lowers his weight from 55.5 kg to 53.8 kg. How many kg must the boxer lose in the final week to be able to compete as a flyweight?

3. A construction company builds a new rail line from Town A to Town B. They complete $1\frac{1}{4}$ miles in their first week of work and $1\frac{2}{3}$ miles in the second week. If they still have $25\frac{3}{4}$ left to build, what is the distance from Town A to Town B?

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4. A catering company needs 8.75 lbs of shrimp for a small party. They buy $3\frac{2}{3}$ lbs of jumbo shrimp, $2\frac{5}{8}$ lbs of medium-sized shrimp, and some mini-shrimp. How many pounds of mini shrimp do they buy?

5. Mark breaks up a 9-hour drive into 3 segments. He drives $2\frac{1}{2}$ hours before stopping for lunch. After driving some more, he stops for gas. If the second segment of his drive was $1\frac{2}{3}$ hours longer than the first segment, how long did he drive after stopping for gas?

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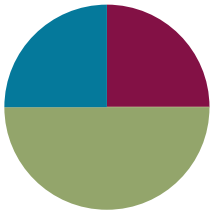
Lesson 16: Exploratory Lesson—Explore Part to Whole Relationships (e.g., 1/2 of Robert's Wire is Equal to 2/3 of Maria's)

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Application Problems	(30 minutes)
Student Debrief	(15 minutes)
Total Time	**(60 minutes)**



Fluency Practice (15 minutes)

- Break Apart the Whole **4. NF.3c** (5 minutes)
- Make a Like Unit **5.NF.1** (5 minutes)
- Add Fractions with Answers Greater Than One **5.NF.1** (5 minutes)

Break Apart the Whole (5 minutes)

Materials: (S) Personal white boards

T: Take out your personal white board. I'll give you a fraction greater than one and you'll break out the whole by writing the addition fraction sentence. For example, I say 3/2, you write 1 + ½. (You can also ask students to write out the improper fraction plus fraction, i.e., 2/2 + 1/2).

T: 4/3.

S: 1 + 1/3.

T: 7/5.

S: 1 + 2/5.

T: 19/17.

S: 1 + 2/17.

T: 13/3.

S: 4+ 1/3.

T: 31/6.

S: 5 + 1/6.

T: (Continue with a sequence appropriate for your students.)

T: Share with your partner. What's your strategy of breaking out the whole?

S: (Students share.)

T: Excellent!

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Make a Like Unit (5 minutes)

Materials: (S) Personal white boards

T: What does 'like unit' mean?

S: When you add or subtract fractions, if the denominators are the same, then they are like units.

T: Tell your partner how we find like units.

S: (Students share.)

T: I'll say two numbers. You make a like unit and write it on your board. Show your board at the signal.

T: 3 and 2. (Pause. Signal.)

S: (Students show 6.)

Continue with possible sequence: 4 and 3; 2 and 4; 2 and 6; 3 and 9; 3 and 12; 3 and 4; 6 and 8.

NOTES ON 'MAKE A LIKE UNIT':

Some students may provide the smallest like unit, others may not. Accept a range of answers. Notice which children consistently do not show the smallest or 'easiest' like unit. It may be that they need extra support.

Add Fractions with Answers Greater than One (5 minutes)

T: I'll say an equation. You write and solve it. If the answer is greater than 1, put a dot next to it. Leave room to write all of the equations on your board without erasing.

T: 3/3 + 1/3 = ? Show your board at the signal. (Pause. Signal.)

S: (Show 3/3 + 1/3 = 4/3)

T: 2/2 + 3/2 = ? (Pause. Signal.)

S: (Show 2/2 + 3/2 = 5/2)

T: 2/4 + 1/4 = ?

S: (Show 2/4 + 1/4 = 3/4)

T: (Continue, mixing up equations that have answers greater than 1 or 2 with those that don't.)

T: What is different about answers that are greater than 1 and those that are less?

S: Greater answers all have a bigger numerator than denominator.

T: Some of these answers are greater than 2. Circle those.

S: (Students look for and circle appropriate answers.)

T: Talk to your partner about the difference between answers that are greater than 1 and those that are greater than 2.

S: "Numerators are bigger than denominators in both kinds." → "Yes, but greater than 2 means the denominator has to fit inside the numerator at least twice, too."

NOTES ON SCAFFOLDING STUDENTS ABOVE GRADE LEVEL:

Depending on your group, make the activity slightly more complex by eliminating the personal board. Have students complete the activity orally.

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Application Problems (30 minutes)

Materials: (S) Worksheet

T: Students, today you are going to work in pairs to solve some ribbon and wire problems. I am going to be an observer for the most part; just listening and watching until the debrief. You will have 30 minutes to reason about and solve 3 problems. I will let you know when you have 10, then 5 minutes remaining. You can use any materials in the classroom, but I ask that you work just with your partner. The work will be scored with a rubric. Each question can earn 4 points.

- Question 1: Each correct answer including the drawing is 1 point.
- Questions 2 and 3: Clear drawing: 1 point. Labeled drawing: 1 point. Correct equation and answer: 1 point. Correct statement of your answer: 1 point. The total possible points are 12.

1. *Draw the following ribbons. When finished, compare your work to your partner's.*

   a) *1 ribbon. The piece shown below is only 1/3 of the whole. Complete the drawing to show the whole ribbon.*

   b) *1 ribbon. The piece shown below is 4/5 of the whole. Complete the drawing to show the whole ribbon.*

   c) *2 ribbons, A and B. One third of A is equal to all of B. Draw a picture of the ribbons.*

   d) *3 ribbons, C, D, and E. C is half the length of D. E is twice as long as D. Draw a picture of the ribbons.*

2. *Half Robert's piece of wire is equal to 2/3 of Maria's wire. The total length of their wires is 10 feet. How much longer is Robert's wire than Maria's?*
3. *Half Sarah's wire is equal to 2/5 of Daniel's. Chris has 3 times as much as Sarah. In all, their wire measures 6 ft. How long is Sarah's wire in feet?*

MP.1 This lesson is an opportunity for students to "Make sense of problems and persevere in solving them" (MP.1). It is recommended that you simply observe with as little interference as possible. For students who have language barriers, support by pairing appropriately for primary language or provide a translation of the problem.

As students work circulate and make decisions about which work to share with the class, and in what order.

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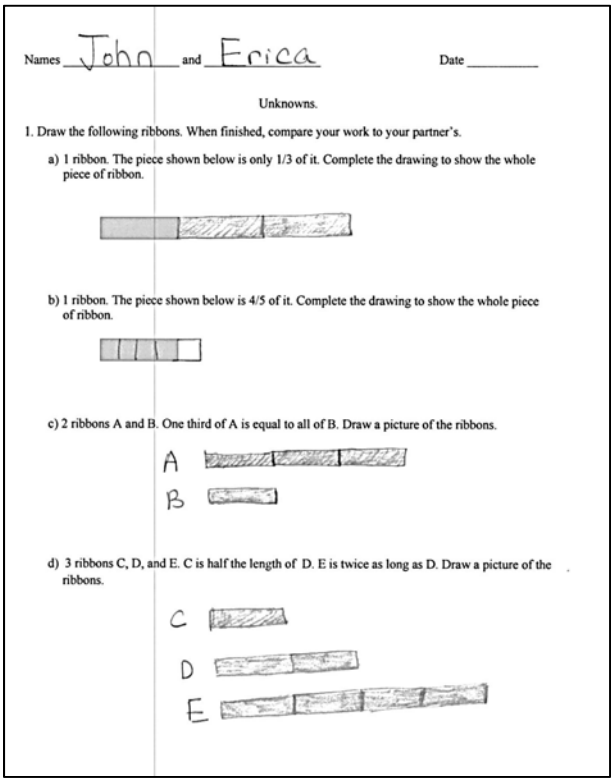

Names John and Erica Date _____

Unknowns.

1. Draw the following ribbons. When finished, compare your work to your partner's.

a) 1 ribbon. The piece shown below is only 1/3 of it. Complete the drawing to show the whole piece of ribbon.

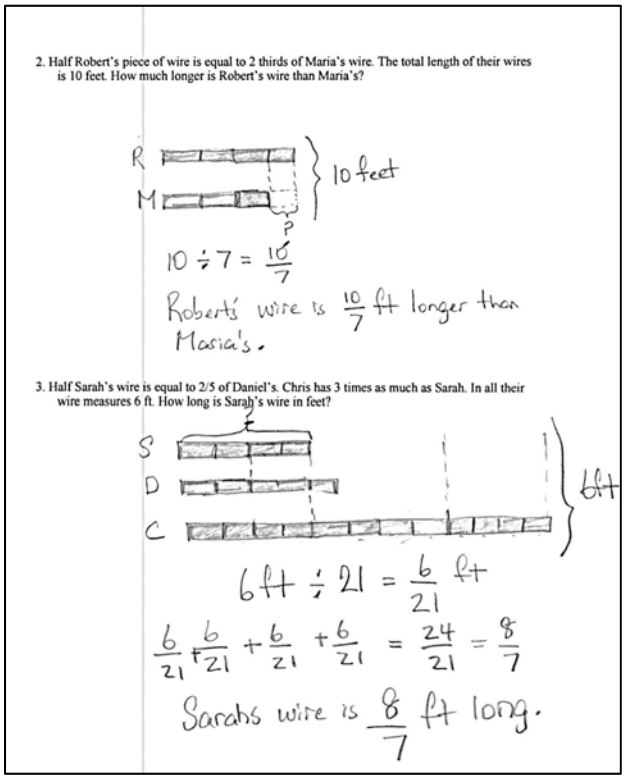
b) 1 ribbon. The piece shown below is 4/5 of it. Complete the drawing to show the whole piece of ribbon.

c) 2 ribbons A and B. One third of A is equal to all of B. Draw a picture of the ribbons.

A
B

d) 3 ribbons C, D, and E. C is half the length of D. E is twice as long as D. Draw a picture of the ribbons.

C
D
E


2. Half Robert's piece of wire is equal to 2 thirds of Maria's wire. The total length of their wires is 10 feet. How much longer is Robert's wire than Maria's?

R
M
10 feet
?

$10 \div 7 = \frac{10}{7}$

Robert's wire is $\frac{10}{7}$ ft longer than Maria's.

3. Half Sarah's wire is equal to 2/5 of Daniel's. Chris has 3 times as much as Sarah. In all their wire measures 6 ft. How long is Sarah's wire in feet?

S
D
C
6ft

$6\text{ft} \div 21 = \frac{6}{21}$ ft

$\frac{6}{21} + \frac{6}{21} + \frac{6}{21} + \frac{6}{21} = \frac{24}{21} = \frac{8}{7}$

Sarahs wire is $\frac{8}{7}$ ft long.

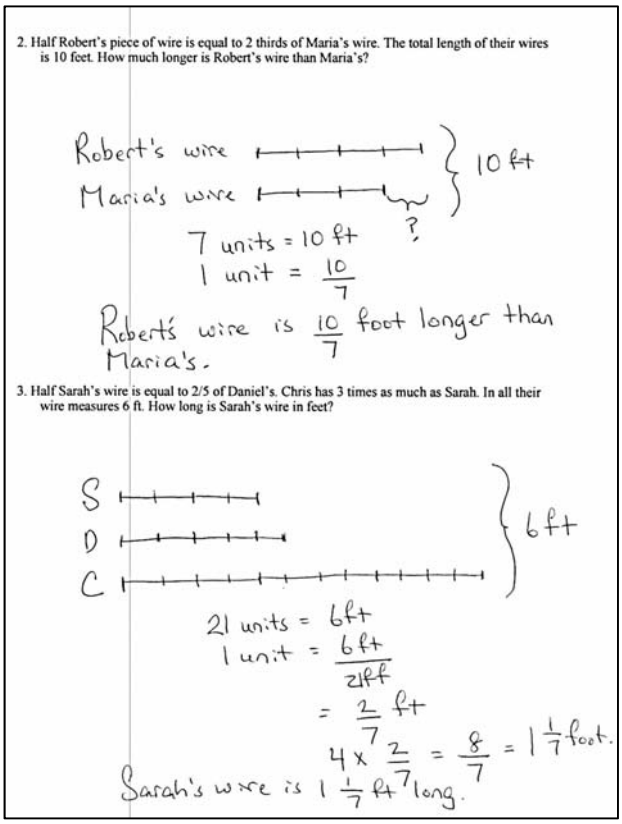
Student Debrief (15 minutes)

T: Let's work together now to analyze your solutions. Compare your solutions from the first page with the solutions of your neighbors to the right.

T: What surprises did you have on this first page?

S: We don't usually think about a shape being a fraction of something. We usually fold it or partition it to make a fraction, so it was new to think of something as a fraction.

T: (After reviewing work from the first page.) Let's analyze John and Erica's work from the two story problems. Take a moment to talk to your partner about precisely what you see when you look at their work and how that relates to the questions.


2. Half Robert's piece of wire is equal to 2 thirds of Maria's wire. The total length of their wires is 10 feet. How much longer is Robert's wire than Maria's?

Robert's wire
Maria's wire
10 ft
?

7 units = 10 ft
1 unit = $\frac{10}{7}$

Robert's wire is $\frac{10}{7}$ foot longer than Maria's.

3. Half Sarah's wire is equal to 2/5 of Daniel's. Chris has 3 times as much as Sarah. In all their wire measures 6 ft. How long is Sarah's wire in feet?

S
D
C
6 ft

21 units = 6ft
1 unit = $\frac{6\text{ft}}{21\text{ft}}$
$= \frac{2}{7}$ ft

$4 \times \frac{2}{7} = \frac{8}{7} = 1\frac{1}{7}$ foot.

Sarah's wire is $1\frac{1}{7}$ ft long.

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T: (After the students have analyzed the work.) Does anyone have a question for John and Erica?

S: Can you explain how you got 21 units in problem 3?

S: (Authors explain.)

T: Retell the explanation to your partner in your own words.

T: (After time for conversation.) Let's compare their work with Jacqueline and Perry's. First, analyze the new team's work by itself for a minute.

T: (After analysis and questioning the authors.)

T: Now, let's compare these two pieces of student work. What is the same and what is different?

S: (Students discuss in pairs and at times in the whole group.)

T: Be sure to compare their work numerically, too. What number sentences did they use? How do their number sentences relate to each other's work? For example, where do we see 4 x 2/7 in John and Erica's work?

Once the analysis is complete, encourage students to score their own work according to the pre-determined rubric. Make sure this work is saved in the portfolio. This task brings students back to a fraction as a quotient and a quotient as a fraction. It also heightens their fraction number sense right as they are about to embark on the multiplication and division module.

Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A quick review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today. Students have two minutes to complete the Exit Ticket. You may read the questions aloud to the students.

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Names _____and _____ Date _____

1. Draw the following ribbons. When finished, compare your work to your partner's.

   a) 1 ribbon. The piece shown below is only 1/3 of the whole. Complete the drawing to show the whole piece of ribbon.

   b) 1 ribbon. The piece shown below is 4/5 of the whole. Complete the drawing to show the whole piece of ribbon.

   c) 2 ribbons, A and B. One third of A is equal to all of B. Draw a picture of the ribbons.

   d) 3 ribbons, C, D, and E. C is half the length of D. E is twice as long as D. Draw a picture of the ribbons.

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2. Half of Robert's piece of wire is equal to 2 thirds of Maria's wire. The total length of their wires is 10 feet. How much longer is Robert's wire than Maria's?

3. Half Sarah's wire is equal to 2/5 of Daniel's. Chris has 3 times as much as Sarah. In all, their wire measures 6 ft. How long is Sarah's wire, in feet?

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Name _____ Date _____

a) 1 ribbon. The piece shown below is only 2/3 of the whole. Complete the drawing to show the whole piece of ribbon.

b) 1 ribbon. The piece shown below is 1/4 of the whole. Complete the drawing to show the whole piece of ribbon.

c) 3 ribbons, A, B, and C. 1 third of A is the same length as B. C is half as long as B. Draw a picture of the ribbons.

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Name _____ Date _____

1. Draw the following ribbons.

   a) 1 road. The piece shown below is only 3/7 of the whole. Complete the drawing to show the whole road.

   b) 1 road. The piece shown below is 1/6 of the whole. Complete the drawing to show the whole road.

   c) 3 roads. B is three times longer than A. C is twice as long as B. Draw the roads. What fraction of the total length of the roads is the length of A? If Road B is 7 miles longer than Road A., what is the length of road C?

   d) Write your own ribbon or road problem with 2 or 3 lengths.

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Name _____ Date _____

1) Lila collected the honey from 4 of her beehives. From the first hive she collected $\frac{5}{6}$ gallon of honey. From the second hive she collected $\frac{2}{3}$ gallon. The last two hives yielded $\frac{1}{12}$ gallon each.

a) How many gallons of honey did Lila collect in all? Draw a diagram to support your answer.

b) After using some of the honey she collected for baking, Lila found that she only had $\frac{3}{4}$ gallon of honey left. How much honey did she use for baking? Explain your answer using a diagram, numbers, and words.

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c) With the remaining $\frac{3}{4}$ gallon of honey, Lila decided to bake some loaves of bread and several batches of cookies for her school bake sale. The bread needed $\frac{1}{6}$ gallon of honey and the cookies needed $\frac{1}{4}$ gallon. How much honey was left over? Explain your answer using a diagram, numbers, and words.

d) Lila decided to make more baked goods for the bake sale. She used $\frac{1}{8}$ lb less flour to make bread than to make cookies. She used $\frac{1}{4}$ lb more flour to make cookies than to make brownies. If she used $\frac{1}{2}$ lb of flour to make the bread, how much flour did she use to make the brownies? Explain your answer using a diagram, numbers and words.

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Mid-Module Assessment Task Standards Addressed
Understand place value.
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Evaluating Student Learning Outcomes

This assessment provides a scoring rubric to determine a numerical representation for how the student completes the task. Scoring rubrics indicate how a score should be determined from the test. However, scoring rubrics are often not detailed enough to provide fine-grained information about what the student can do and what they need to work on next (or where their understanding begins to falter). A Progression Toward Mastery is also provided to describe steps that illuminate the gradually increasing understandings that students develop on their way to proficiency. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for each student is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the student CAN do now, and what they need to work on next.

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A Progression Toward Mastery				
Assessment Task Item	**STEP 1 Little evidence of reasoning without a correct answer.**	**STEP 2 Evidence of some reasoning without a correct answer.**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer.**	**STEP 4 Evidence of solid reasoning with a correct answer.**
1a)	The student shows little evidence of clear reasoning and understanding, resulting with an incorrect answer.	The student shows evidence of beginning to understand addition fractions with unlike denominators, but the answer is incorrect.	The student has the correct answer, but is unable to show evidence accurately using diagrams, numbers, and/or words. Or, the student shows evidence of correctly modeled adding of fractions with unlike denominators, but resulted with an incorrect answer.	The student correctly calculates, explains, and clearly articulates the amount Lila received.
1b)	The student shows little evidence of using a correct strategy and understanding, resulting in the wrong answer.	The student shows evidence of beginning to understand subtracting fractions with unlike denominators, but is unable to get the right answer.	The student has the correct answer, but the model is either omitted or is unable to show evidence accurately using diagrams, numbers, and/or words. Or, the student shows evidence of correctly modeled subtracting fractions with unlike denominators but resulted with an incorrect answer.	The student correctly models and calculates the amount Lila has used.

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A Progression Toward Mastery (continued)				
1c)	The student shows little evidence of using a correct strategy and understanding, resulting in the wrong answer.	The student shows evidence of beginning to understand portions of the solution, such as attempting to add $^1/_6$ and ¼ and then subtract the result from ¾, but is unable to get the right answer.	The student has the correct answer, but the model is either omitted or the student is unable to show evidence accurately using diagrams, numbers, and/or words. Or, the student show evidence of correctly modeling adding and subtracting fractions with unlike denominators, but resulted in an incorrect answer.	The student calculates correctly for every step, and illustrates each calculation with an appropriate fraction model.
1d)	The student shows little evidence of using correct strategies, resulting in the wrong answer.	The student shows evidence of beginning understanding of at least some of the steps involved, but is unable to get the right answer.	The student has the correct answer, but the student does not show sound reasoning. Or, the student demonstrates all steps using appropriate models, but results in an incorrect answer.	The student calculates the amount of flour used for the brownies and demonstrates conceptual understanding through use of appropriate models.

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Assessment Task Item and Standard	Correct Response	Points per Item	Section Total	Progress to Mastery: Steps 1, 2, 3, 4
1a) 5.NF.1	▪ Gives correct response of $\frac{20}{12}$ gal, $\frac{10}{6}$ gal or $1\frac{8}{12}$, $1\frac{4}{6}$, or $1\frac{2}{3}$ gal. ▪ Diagram, words and numbers clearly illustrate the answer.	2 1	3	
1b) 5.NF.1 5.NF.2	▪ Gives correct response of $\frac{11}{12}$ gal. ▪ Diagram, words and numbers clearly illustrate the answer.	2 1	3	
1c) 5.NF.1 5.NF.2	▪ Gives correct response of $\frac{1}{3}$ gal or equivalent fraction, such as $\frac{4}{12}$ gal. ▪ Correctly models $\frac{1}{6}+\frac{1}{4}$ and $\frac{3}{4}-\frac{5}{12}$, or alternatively models $\frac{3}{4}-\frac{1}{6}-\frac{3}{4}$ using words, diagram and numbers.	2 1	3	
1d) 5.NF.1 5.NF.2	▪ Gives correct response of $\frac{3}{8}$ lb. ▪ Diagrams and uses words and numbers to clearly explain the solution.	2 1	3	
Total Points			**12**	

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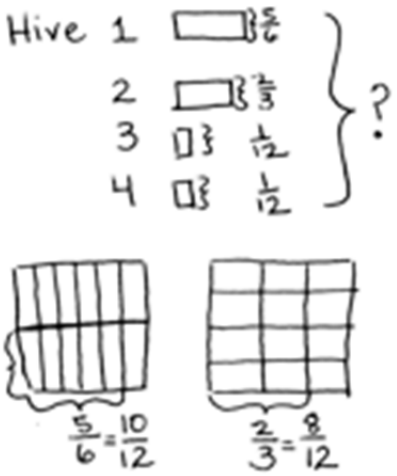
Name John Date 11/23/12

1) Lila collected the honey from 4 of her beehives. From the first hive she collected 5/6 gallon of honey. From the second hive she collected 2/3 gallon. The last two hives yielded 1/12 gallon each.

a) How many gallons of honey did Lila collect in all? Draw a diagram to support your answer.



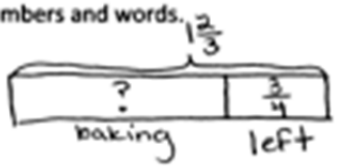
$\frac{5}{6} + \frac{2}{3} + \frac{2}{12} = ?$

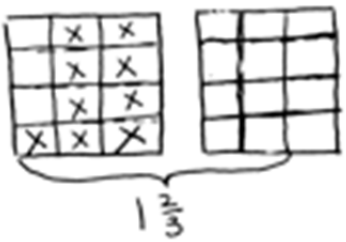
$\frac{10}{12} + \frac{8}{12} + \frac{2}{12} = \frac{20}{12} = \frac{12}{12} + \frac{8}{12} = 1\frac{8}{12}$

or $1\frac{2}{3}$

Lila collected $1\frac{2}{3}$ gal of honey in all.

b) After using some of the honey she collected for baking, Lila found that she only had ¾ gallon of honey left. How much honey did she use for baking? Explain your answer using a diagram, numbers and words.





$\frac{3}{4} = \frac{9}{12}$

I can take away $\frac{9}{12}$ from the whole.

$1\frac{2}{3} - \frac{3}{4} = ?$

$1\frac{8}{12} - \frac{9}{12} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$

Lila used $\frac{11}{12}$ gal for baking.

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c) With the remaining ¾ gallon of honey, Lila decided to bake some loaves of bread and several batches of cookies for her school bake sale. The bread needed $^1/_6$ gallon of honey, and the cookies needed $^1/_4$ gallon. How much honey was left over? Explain your answer using a diagram, numbers and words.

$$\frac{3}{4} - \frac{1}{6} - \frac{1}{4} = ?$$

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} - \frac{1}{6}$$

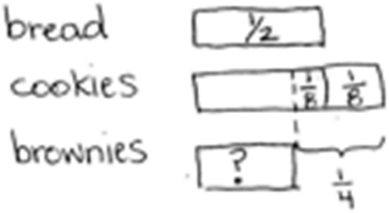
$$= \frac{1}{2} - \frac{1}{6}$$

$$= \frac{3}{6} - \frac{1}{6}$$

$$= \frac{2}{6} = \frac{1}{3}$$

Lila had $\frac{1}{3}$ gal of honey left.

d) Lila decided to make more baked goods for the bake sale. She used 1/8 lb less flour to make bread than to make cookies. She used ¼ lb more flour to make cookies than to make brownies. If she used ½ lb of flour to make the bread, how much flour did she use to make the brownies? Explain your answer using a diagram, numbers and words.



$$\frac{1}{2} - \frac{1}{8}$$

$$= \frac{4}{8} - \frac{1}{8}$$

$$= \frac{3}{8}$$

or

$$\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$

$$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

Lila used $\frac{3}{8}$ lb of flour to make the brownies.

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Name _____ Date _____

1) On Sunday, Sheldon bought $4\frac{1}{2}$ kg of plant food. He used $1\frac{2}{3}$ kg on his strawberry plants, and used $\frac{1}{4}$ kg for his tomato plants.

a) How many kilograms of plant food did Sheldon have left? Write one or more equations to show how you reached your answer.

b) Sheldon wants to feed his strawberry plants 2 more times, and his tomato plants one more time. He will use the same amounts of plant food as before. How much plant food will he need? Does he have enough left to do so? Explain your answer using words, pictures or numbers.

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2) Sheldon harvests the strawberries and tomatoes in his garden.

a) He picks $1\frac{2}{5}$ kg less strawberries in the morning than in the afternoon. If Sheldon picked $2\frac{1}{4}$ kg in the morning, how many kilograms of strawberries does he pick in the afternoon? Explain your answer using words, pictures or equations.

b) Sheldon also picks tomatoes from his garden. He picked $5\frac{3}{10}$ kg but 1.5 kg were rotten. How many kilograms of tomatoes were not rotten? Write an equation that shows how you reached your answer.

c) Did Sheldon's garden produce more kilograms of strawberries or tomatoes? How many more kilograms? Explain your answer using an equation.

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End-of-Module Assessment Task Standards Addressed
Understand place value.
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Evaluating Student Learning Outcomes

This assessment provides a scoring rubric to determine a numerical representation for how the student completes the task. Scoring rubrics indicate how a score should be determined from the test. However, scoring rubrics are often not detailed enough to provide fine-grained information about what the student can do and what they need to work on next (or where their understanding begins to falter). A Progression Toward Mastery is also provided to describe steps that illuminate the gradually increasing understandings that students develop on their way to proficiency. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for each student is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the student CAN do now, and what they need to work on next.

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A Progression Toward Mastery				
Assessment Task Item	**STEP 1 Little evidence of reasoning without a correct answer.**	**STEP 2 Evidence of some reasoning without a correct answer.**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer.**	**STEP 4 Evidence of solid reasoning with a correct answer.**
1a)	The solution is incorrect and there is very little if no evidence of clear reasoning.	The solution is incorrect but there is evidence of clear reasoning.	The solution is correct but the explanation is less than perfectly clear	The student correctly calculates, explains, and clearly articulates the amount left.
1b)	The student was unable to make sense of the problem in any intelligible way.	The student's solution is incorrect and, though showing signs of real thought, is not developed or does not connect to the story's situation.	The student calculates the plant food needed for the strawberries but may omit the plant food needed for the blueberries and explains that Sheldon will have enough.	The student correctly calculates to clearly explain that Sheldon will have enough plant food.
2 a) and b)	The solution is incorrect and shows little evidence of understanding of the need for like units	The student shows evidence of beginning to understand addition fractions with unlike denominators, but the answer is incorrect	The student calculates correctly but may be less than perfectly clear in stating his solution. For example. "The answer is 3 13/20." is not a clearly stated solution. A correct equation is written for b).	The student calculates the weight of the strawberries picked and explains the solution clearly. Writes an equation for b).
2 c)	The solution is incorrect and shows little evidence of understanding of fraction comparison.	The student may have compared correctly but calculated incorrectly and/ or does not explain the meaning of his numerical solution in the context of the story.	The student may have compared correctly but calculated incorrectly and/ or does not explain the meaning of his numerical solution in the context of the story.	The student compares the amounts correctly, determines the difference in their weight and explains the solution clearly.

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Assessment Task Item and Standard	Correct Response	Points per Item	Section Total	Progress to Mastery: Steps 1, 2, 3, 4
1a) 5.NF.1 5.NF.2	▪ Gives the correct answer of $2\frac{19}{24}$ kg. ▪ Partial credit may be assigned for incomplete answers that show the student correctly finding the total amount of plant food used.	2	2	
1b) 5.NF.1 5.NF.2	▪ Student correctly calculates that Sheldon needs $2\frac{11}{12}$ kg of plant food ▪ Student notes that $2\frac{11}{12}$ is more than $2\frac{19}{24}$. (If the student answer from part 'a' is incorrect, then assign credit based on that response).	1 1	2	
2a) **5.NF.1** **5.NF.2**	▪ Gives a correct answer of $3\frac{13}{20}$ kg. ▪ Explains the answer using words, pictures or numbers.	2	2	
2b) 5.NF.1 5.NF.2	▪ Gives a correct equation and correct answer of $3\frac{8}{10}$ kg or $3\frac{4}{5}$ kg. Explains the answer using words, pictures or numbers.	2	2	
2c) 5.NF.1 5.NF.2	▪ Correctly responds that garden produced more tomatoes. ▪ Correctly responds that there were 3/20 kg more tomatoes ▪ Gives correct equation such as: ▪ $3\frac{4}{5} - 3\frac{13}{20} = 3\frac{16}{20} - 3\frac{13}{20} = \frac{3}{20}$	1 1 1	3	
Total Points			**11**	

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Name Jacqueline Date October 28

1) On Sunday, Sheldon bought 4 $^3/_8$ kg of plant food. He used 1 $^1/_3$ kg on his strawberry plants, and used $^1/_4$ kg for his tomato plants.

a) How many kilograms of plant food did Sheldon have left? Write one or more equations to show how you reached your answer.

$4\frac{3}{8} - 1\frac{1}{3} - \frac{1}{4} =$

$3\frac{3}{8} - \frac{1}{3} - \frac{1}{4} =$

$2\frac{11}{8} - \frac{1}{3} - \frac{1}{4} =$

$2\frac{33}{24} - \frac{8}{24} - \frac{6}{24} =$

$2\frac{19}{24}$ Sheldon has $2\frac{19}{24}$ kg plant food left.

b) Sheldon wants to feed his strawberry plants 2 more times, and his tomato plants one more time. He will use the same amounts of plant food as before. How much plant food will he need? Does he have enough left to do so? Explain your answer using words, pictures or numbers.

Strawberries : $1\frac{1}{3} + 1\frac{1}{3} = 2\frac{2}{3}$

Strawberries + tomatos : $2\frac{2}{3} + \frac{1}{4} =$

$1\frac{5}{3} + \frac{1}{4} =$

$1\frac{20}{12} + \frac{3}{12} =$

$1\frac{23}{12} =$

$2\frac{11}{12}$ kg. Sheldon needs $2\frac{11}{12}$ kg food. He doesn't have enough. $2\frac{22}{24}$ is more than $2\frac{19}{24}$ kg.

$2\frac{11}{12} = 2\frac{22}{24}$

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2) Sheldon harvests the strawberries and tomatoes in his garden.

a) He picks 1 $^2/_5$ kg less strawberries in the morning than in the afternoon. If Sheldon picked 2 ¼ kg in the morning, how many kilograms of strawberries does he pick in the afternoon? Explain your answer using words, pictures or equations.

morning $\boxed{2\frac{1}{4}}$

afternoon $\boxed{}\boxed{1\frac{2}{5}}$ } ?

$2\frac{1}{4} + 1\frac{2}{5} = ?$

$2\frac{5}{20} + 1\frac{8}{20} = 3\frac{13}{20}$

Sheldon picks $3\frac{13}{20}$ kg of strawberries in the afternoon

b) Sheldon also picks tomatoes from his garden. He picked 5 $^3/_{10}$ kg but 1.5 kg were rotten. How many kilograms of tomatoes were not rotten? Write an equation that shows how you reached your answer.

$5\frac{3}{10} - 1\frac{1}{2}$

$= 5\frac{3}{10} - 1\frac{5}{10}$

$= 4\frac{3}{10} - \frac{5}{10}$

$= 3\frac{5}{10} + \frac{3}{10} = 3\frac{8}{10}$

or

$5\frac{3}{10} - 1\frac{1}{2}$

$= 5\frac{3}{10} - 1\frac{5}{10}$

$= 4\frac{13}{10} - 1\frac{5}{10}$

$= 3\frac{8}{10} = 3\frac{4}{5}$

Sheldon had $3\frac{8}{10}$ kg that were not rotten.

c) Did Sheldon's garden produce more kilograms of strawberries or tomatoes? How many more kilograms? Explain your answer using an equation.

Strawberries $= 3\frac{13}{20}$

tomatoes $= 3\frac{8}{10} = 3\frac{16}{20}$

$\frac{16}{20} - \frac{13}{20} = \frac{3}{20}$

Sheldon's garden produced more tomatoes.

It produced $\frac{3}{20}$ kg more.

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Bank of Fluency Activities Mapped Through Grade 5•Module 3

Without fluency, students can use too much of their attention and effort on lower concept skills as they try to master essential material. This encumbers their ability to see patterns, relationships, and properties. To empower students with mastery of basic facts and automaticity with previously learned material, a balanced approach to mathematics includes 10-20 minutes of daily fluency practice. The following bank of fluency activities includes: mental math, interactive drills, efficient games, and concept worksheets allowing for individual systematic progress with basic facts.

Topic	Bank of Fluency Activities
A: Equivalent Fractions	**Lesson 1:** ▪ Sprint: Write the Missing Factor - 4.OA.4 ▪ Skip Counting by ¼ Hour - 5.MD.1 **Lesson 2:** ▪ Equivalent Fractions - 5.NF.1 ▪ Sprint: Find the Missing Numerator or Denominator - 4.NF.1
B: Fraction Addition and Subtraction: Making Like Units Pictorially	**Lesson 3:** ▪ Sprint: Equivalent Fractions - 5.NF.1 ▪ Adding Like Fractions - 5.NF.1 ▪ Fractions as Division - 5.NF.3 **Lesson 4:** ▪ Adding Fractions to Make One Whole 4.NF.3a ▪ Division as Fractions 5.NF.3 **Lesson 5:** ▪ Sprint: Subtracting Fractions From a Whole - 4.NF.3a **Lesson 6:** ▪ Name the Fraction to Complete the Whole 4.NF.3b

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	▪ Taking from the Whole - 5.NF.7 ▪ Fraction Units to Ones and Fractions 5.NF.7 **Lesson 7:** ▪ Sprint: Circle the Equivalent Fraction - 4.NF.2
C: Fraction Addition and Subtraction: Making Like Units Numerically	**Lesson 8:** ▪ Adding Whole Numbers and Fractions - 4.NF.3a ▪ Subtracting Fractions from Whole Numbers - 4.NF.3a **Lesson 9:** ▪ Adding and Subtracting Fractions with Like Units - 4.NF.3a ▪ Sprint: Add and Subtract Fractions with Like Units - 4.NF.3a **Lesson 10:** ▪ Sprint: Add and Subtract Whole Numbers and Ones with Fraction Units - 4.NF.3c **Lesson 11:** ▪ Subtracting Fractions from Whole Numbers - 4.NF.3a ▪ Adding and Subtracting Fractions with Like Units - 4.NF.3c **Lesson 12:** ▪ Sprint: Subtracting Fractions with Unlike Units - 5.NF.1
D: Fraction Addition and Subtraction: Further Applications	**Lesson 13:** ▪ From Fractions to Decimals - 4.NF.6 ▪ Adding and Subtracting Fractions with Unlike Units - 5.NF.1 **Lesson 14:** ▪ Sprint: Make Larger Units - 4.NF.1 ▪ Happy Counting with Mixed Numbers - 4.NF.3a **Lesson 15:** ▪ Sprint: Circle the Smallest Fraction - 4.NF.2 **Lesson 16:** ▪ Break Apart the Whole - 4. NF.3c ▪ Make a Like Unit - 5.NF.1 ▪ Add Fractions with Answers Greater Than One - 5.NF.1

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Topic A Fluency Activities

LESSON 1

Sprint **(9 minutes)**

Materials: (S) Write the Missing Factor Sprint

Skip-Counting by ¼ Hour **(2 minutes)**

T: Let's count by ¼ hours. (Rhythmically point up until a change is desired. Show a closed hand and then point down. Continue, mixing it up).

S: 1/4 hour, 2/4 hour, 3/4 hour, 1 hour (stop) 1 ¼ hour, 1 2/4 hour, 1 ¾ hour, 2 hours (stop).

LESSON 2

Equivalent Fractions **(3 minutes)**

T: (Write) $\frac{1}{2} =$

T: Say the fraction.

S: One half.

T: (Write) $\frac{1}{2} = \frac{\quad}{4}$

T: One half is how many fourths?

S: Two fourths.

Continue with possible sequence: $\frac{1}{2} = \frac{\quad}{6}, \frac{1}{3} = \frac{\quad}{6}, \frac{2}{3} = \frac{\quad}{6}, \frac{2}{3} = \frac{\quad}{12}, \frac{3}{4} = \frac{\quad}{16}, \frac{3}{5} = \frac{\quad}{25}$

T: (Write) $\frac{1}{2} =$

T: Say the fraction.

S: One half.

T: (Write) $\frac{1}{2} = \frac{2}{\quad}$

T: One half or one part of two is the same as two parts of what unit?

S: Fourths.

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Continue with possible sequence:

$$\frac{1}{2}=\frac{2}{_},\frac{1}{5}=\frac{2}{_},\frac{2}{5}=\frac{8}{_},\frac{2}{5}=\frac{8}{_},\frac{3}{4}=\frac{9}{_},\frac{4}{5}=\frac{16}{_}$$

Sprint (9 minutes)

Materials: (S) Find the Missing Numerator or Denominator Sprint

Topic B Fluency Activities

LESSON 3

Equivalent Fractions Sprint (10 minutes)

Materials: (S) Equivalent Fractions Sprint

Adding Like Fractions (1 minute)

T: Let's add fractions mentally. Say answers as whole numbers when possible.
T: One third plus one third equals?
S: Two thirds.
T: One fourth plus one fourth equals?
S: Two fourths.
T: 1/5 + 2/5 =
S: 3/5.
T: 3/7 + 4/7 =
S: 1.
T: 1/4 + 1/3 + 3/4 + 2/3 =
S: 2.

Continue and adjust to meet student needs. Use a variety of fraction combinations.

Fractions as Division (1 minute)

Materials: (S) Personal white boards

T: When I show a fraction, you write it as a division statement.
T: (Write) ¾
S: (Write) 3 ÷ 4 = ¾.
T: (Write) 5/2

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S: (Write) 5 ÷ 2 = 5/2.

Continue with fractions that are less than and greater than one. Possible sequence: 1/3, 7/4, 5/8, 9/5, 3/10, 13/6.

LESSON 4

Adding Fractions to Make One Whole (4 minutes)

T: I will name a fraction. You say a fraction with the same denominator so that together our fractions add up to 1 whole. For example, if I say 1 third, you say 2 thirds. 1/3 + 2/3 = 3/3 or 1 whole. Say your answer at the signal.

T: 1 fourth? (Signal)

S: 3 fourths.

T: 1 fifth? (Signal)

S: 4 fifths.

T: 2 tenths? (Signal)

S: 8 tenths.

Continue with possible sequence: 1/3, 3/5, ½, 5/10, 6/7, 3/8.

Division as Fractions (4 minutes)

Materials: (S) Personal white boards and markers

T: I will say a division sentence. You write it as a fraction. At my signal, show your board and say your fraction. (Write 3 ÷ 2.)

S: (Students show 3/2.) 3 halves.

T: (Write 2 ÷ 3.)

S: (Students show 2/3.) 2 thirds.

T: 3 ÷ 4.

S: 3 fourths.

T: 6 ÷ 4.

S: 6 fourths.

Continue with possible sequence: 4 ÷ 5, 7 ÷ 2, 10 ÷ 3, 6 ÷ 8.

LESSON 5

Sprint (12 minutes)

Materials: (S) Subtracting Fractions From a Whole Sprint

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LESSON 6

Name the Fraction to Complete the Whole (4 minutes)

T: I'll say a fraction, you say the missing part to make one whole. Ready?
T: ½.
S: ½.
T: 4/5.
S: 1/5.
T: 1/7.
S: 6/7.
T: 4/9.
S: 5/9.
T: 18/20.
S: 2/20.
T: 147/150.
S: 3/150.
T: Share your strategy for making one whole with a partner.
T: With your partner, take turns giving each other problems to solve. You have one minute.

Taking from the Whole (3 minutes)

Materials: (S) Personal white boards and markers

T: I'll say a subtraction equation. You say the answer.
1 – 1 half.
S: 1 half.
T: 1 – 1 third.
S: 2 thirds.
T: 1 – 2 thirds.
S: 1 third.
T: 1 – 2 fifths.
S: 3 fifths.
T: 1 – 4 fifths.
S: 1 fifth.

Continue with possible sequence:

$1-\frac{1}{3},\ 1-\frac{3}{4},\ 1-\frac{3}{7},\ 1-\frac{5}{9},\ 1-\frac{5}{10}$

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Fraction Units to Ones and Fractions (3 minutes)

Materials: (S) Personal white board and markers

T: I'll say a fraction, you say it as ones and fraction units. Three halves.
S: One and one half.
T: Five halves.
S: Two and one half.
T: Seven halves.
S: Three and one half.
T: Eleven halves.
S: Five and one half.

Continue with possible sequence: $\frac{4}{3}, \frac{5}{3}, \frac{10}{3}, \frac{22}{3}$ and $\frac{5}{4}, \frac{7}{4}, \frac{11}{4}, \frac{39}{4}$.

LESSON 7

Sprint (12 minutes)

Materials: (S) Circle the Equivalent Fraction Sprint

Topic C Fluency Activities

LESSON 8

Adding Whole Numbers and Fractions (3 minutes)

T: I'll say the answer. You say the addition problem as a whole number and a fraction. 3 and 1 half.
S: 3 + 1 half.
T: 5 and 1 half.
S: 5 + 1 half.
T: 2 and 3 fourths.
S: 2 + 3 fourths.
T: 1 and 5 sixths.
S: 1 + 5 sixths.
T: Let's switch roles. I'll say the addition problem, you say the answer. 2 + 1 fifth.
S: 2 and 1 fifth.
T: 2 + 4 fifths.

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S: 2 and 4 fifths.
T: 5 + 7 eighths.
S: 5 and 7 eighths.
T: 3 + 7 twelfths.
S: 3 and 7 twelfths.

Subtracting Fractions From Whole Numbers (3 minutes)

T: I'll say a subtraction sentence. You repeat the sentence and give the answer. 1 – 1 half.
S: 1 – 1 half = 1 half.
T: 2 – 1 half.
S: 2 – 1 half = 1 and 1 half.
T: 2 and 1 half – 1 half.
S: 2 and 1 half – 1 half = 2.
T: 6 – 1 fourth.
S: 6 – 1 fourth = 5 and 3 fourths.
T: 6 and 3 fourths – 3 fourths.
S: 6 and 3 fourths – 3 fourths = 6.

Repeat process with possible sequence:

$3 - \frac{5}{6}$, $3\frac{5}{6} - \frac{5}{6}$, $4 - \frac{7}{8}$, $4\frac{7}{8} - \frac{7}{8}$, $5 - \frac{7}{12}$, $5\frac{7}{12} - \frac{7}{12}$

LESSON 9

Adding and Subtracting Fractions with Like Units (1 minute)

T: I'll say an addition or subtraction sentence. You say the answer. 2 fifths + 1 fifth.
S: 3 fifths.
T: 2 fifths – 1 fifth.
S: 1 fifth.
T: 2 fifths + 2 fifths.
S: 4 fifths.
T: 2 fifths – 2 fifths.
S: Zero.
T: 3 fifths + 2 fifths.
S: 1.
T: I'm going to write an addition sentence. You say true or false.

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T: (Write) $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$

S: True.

T: (Write) $\frac{3}{7}+\frac{3}{7}=\frac{6}{14}$

S: False.

T: Say the answer that makes this addition sentence true.

S: 3 sevenths + 3 sevenths = 6 sevenths.

T: (Write) $\frac{5}{9}+\frac{2}{9}=\frac{7}{18}$

T: True or false?

S: False.

T: Say the answer that will make this addition sentence true.

S: 5 ninths + 2 ninths = 7 ninths.

T: (Write) $\frac{5}{9}+\frac{4}{9}=1$

T: True or false?

S: True.

T: Great work. You're ready for your sprint!

Sprint **(9 minutes)**

Materials: (S) Add and Subtract Fractions with Like Units Sprint

LESSON 10

Sprint **(10 minutes)**

Materials: (S) Add and Subtract Whole Numbers and Ones with Fraction Units Sprint

LESSON 11

Subtracting Fractions from Whole Numbers **(5 minutes)**

T: I'll say a subtraction sentence. You say the subtraction sentence with answer. 1 – 1 half.

S: 1 – 1 half = 1 half.

T: 2 – 1 half.

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S: 2 – 1 half = 1 and 1 half.
T: 3 – 1 half.
S: 3 – 1 half = 2 and 1 half.
T: 7 – 1 half.
S: 7 – 1 half = 6 and 1 half.

Continue with possible sequence:

$1-\frac{1}{3}, 1-\frac{2}{3}, 2-\frac{2}{3}, 2-\frac{1}{3}, 5-\frac{1}{4}, 5-\frac{3}{4}$.

Adding and Subtracting Fractions with Like Units (3 minutes)

T: I'll say an addition or subtraction sentence. You say the answer. 3 sevenths + 1 seventh.
S: 4 sevenths.
T: 3 sevenths – 1 seventh.
S: 2 sevenths.
T: 3 sevenths + 3 sevenths.
S: 6 sevenths.
T: 3 sevenths – 3 sevenths.
S: 0.
T: 4 sevenths + 3 sevenths.
S: 1.
T: I'll write an addition sentence. You say true or false.

(Write) $\frac{2}{5}+\frac{2}{5}=\frac{4}{10}$

S: False.
T: Say the answer that makes the addition sentence true.
S: 2 fifths + 2 fifths = 4 fifths.

(Write) $\frac{5}{8}+\frac{3}{8}=1$

S: True.

(Write) $\frac{5}{6}+\frac{1}{6}=\frac{6}{12}$

S: False.
T: Say the answer that makes the addition sentence true.
S: 5 sixths + 1 sixth = 1.

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LESSON 12

Sprint (12 minutes)

Materials: (S) Subtracting Fractions with Unlike Units Sprint

Topic D Fluency Activities

LESSON 13

From Fractions to Decimals (5 minutes)

Materials: (T) Dry-erase board or projector

T: (Project) $\frac{1}{10} =$

T: Say the fraction in unit form.
S: 1 tenth.
T: Say the fraction in decimal form.
S: Zero point one.
T: I'll say a fraction in unit form. You say the fraction in decimal form. Ready? 3 tenths.
S: 0.3
T: 7 tenths.
S: 0.7.

T: (Project) $\frac{1}{2} = \frac{\quad}{10}$

T: Say the equivalent fraction with the missing numerator.
S: 1 half = 5 tenths.
T: Say 5 tenths as a decimal.
S: 0.5
T: Say 1 half as a decimal.
S: 0.5
T: Say 3 and 1 half as a decimal.
S: 3.5.

Repeat process for possible sequence:

$$\frac{1}{5}, \frac{2}{5}, \frac{4}{5}, 2\frac{4}{5}, \frac{1}{4}, \frac{3}{4}, 5\frac{3}{4}, \frac{1}{25}, \frac{2}{25}, \frac{3}{25}, 5\frac{3}{25}, \frac{1}{20}, \frac{11}{20}, 3\frac{11}{20}, \frac{1}{50}, \frac{3}{50}, 4\frac{3}{50}$$

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Adding and Subtracting Fractions with Unlike Units (6 minutes)

Materials: (S) Personal white boards

T: (Write) $\frac{1}{4}+\frac{1}{2}=\frac{2}{6}$

T: True or false?

S: False.

T: On your personal white boards, write the answer that will make the addition sentence true.

S: (Write) $\frac{1}{4}+\frac{1}{2}=\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$

T: (Project) $\frac{1}{2}+\frac{3}{8}=\frac{7}{8}$

T: True or false?

S: True.

T: Rewrite the addition sentence using like units.

S: (Write)

$$\frac{1}{2}+\frac{3}{8}=\frac{4}{8}+\frac{3}{8}$$
$$=\frac{7}{8}$$

T: (Project) $\frac{2}{3}-\frac{2}{9}=\frac{4}{9}$

T: True or false?

S: True.

T: Rewrite the subtraction sentence using like units.

S: (Write)

$$\frac{2}{3}-\frac{2}{9}=\frac{6}{9}-\frac{2}{9}$$
$$=\frac{4}{9}$$

T: (Project) $\frac{5}{6}-\frac{2}{3}=\frac{3}{3}$

T: True or false?

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S: False.

T: Write the answer that will make the subtraction sentence true.

S: (Write)

$$\frac{5}{6}-\frac{2}{3}=\frac{5}{6}-\frac{4}{6}$$
$$=\frac{1}{6}$$

LESSON 14

Sprint (10 minutes)

Materials: (S) Make Larger Units Sprint

Happy Counting with Mixed Numbers (3 minutes)

T: Let's count by ½ with mixed numbers. Ready? (Teacher rhythmically points up until a change is desired. Show a closed hand, then point down. Continue, mixing it up).

S: ½, 1, 1 ½, 2 (stop), 1 ½, 1, ½, 0 (stop), ½, 1, 1 ½, 2, 2 ½, 3, 3 ½, 4 (stop), 3 ½, 3, 2 ½, 2, 1 ½, 1 (stop), 1 ½, 2, 2 ½, 3, 3 ½, 4, 4 ½, 5.

T: Excellent. Try it for 30 seconds with your partner. Partner A, you are the teacher today.

LESSON 15

Sprint (12 minutes)

Materials: (S) Circle the Smallest Fraction Sprint

LESSON 16

Break Apart the Whole (5 minutes)

Materials: (S) Personal white boards

T: Take out your personal white board. I'll give you a fraction greater than one and you'll break out the whole by writing the addition fraction sentence. For example, I say 3/2, you'll write 1 + ½. (You can also ask students to write out the improper fraction plus fraction, i.e., 2/2 + 1/2).

T: 4/3.

S: 1 + 1/3.

T: 7/5.

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S: 1 + 2/5.
T: 19/17.
S: 1 + 2/17.
T: 13/3.
S: 4+ 1/3.
T: 31/6.
S: 5 + 1/6.
T: (Etc.)
T: Share with your partner. What's your strategy of breaking out the whole?
S: (Students share.)
T: Excellent!

Make a Like Unit (5 minutes)

Materials: (S) Personal white boards

T: What does 'like unit' mean?
S: When you add or subtract fractions, if the denominators are the same, then they are like units.
T: Tell your partner how we find like units.
S: (Students share.)
T: I'll say two numbers. You make a like unit and write it on your board. Show your board at the signal.
T: 3 and 2. (Pause. Signal.)
S: (Students show 6.)

Continue with possible sequence: 4 and 3; 2 and 4; 2 and 6; 3 and 9; 3 and 12; 3 and 4; 6 and 8.

Add Fractions with Answers Greater than One (5 minutes)

T: I'll say an equation. You write and solve it. If the answer is greater than 1, put a dot next to it. Leave room to write all of the equations on your board without erasing.
T: 3/3 + 1/3 = ? Show your board at the signal.
(Pause. Signal.)
S: (Show 3/3 + 1/3 = 4/3)
T: 2/2 + 3/2 = ? (Pause. Signal.)
S: (Show 2/2 + 3/2 = 5/2)
T: 2/4 + 1/4 = ?
S: (Show 2/4 + 1/4 = 3/4)
T: (Continue, mixing up equations that have answers greater than 1 or 2 with those that don't.)
T: What is different about answers that are greater than 1 and those that are less?

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S: Greater answers all have a bigger numerator than denominator.

T: Some of these answers are greater than 2. Circle those.

S: (Students look for and circle appropriate answers.)

T: Talk to your partner about the difference between answers that are greater than 1 and those that are greater than 2.

S: "Numerators are bigger than denominators in both kinds." → "Yes, but greater than 2 means the denominator has to fit inside the numerator at least twice, too."

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Appendix

Directions for Administration of Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is celebrated.

One Sprint has two parts with closely related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

With practice the following routine takes about 8 minutes.

Sprint A

Pass *Sprint A* out quickly, face down on student desks with instructions to not look at the problems until the signal is given. (Some Sprints include words. If necessary, prior to starting the Sprint quickly review the words so that reading difficulty does not slow students down.)

T: You will have 60 seconds to do as many problems as you can.

T: I do not expect you to finish all of them. Just do as many as you can, your personal best. (If some students are likely to finish before time is up, assign a number to "count by" on the back.)

T: Take your mark! Get set! THINK! (When you say THINK, students turn their papers over and work furiously to finish as many problems as they can in 60 seconds. Time precisely.)

After 60 seconds:

T: Stop! Circle the last problem you did. I will read just the answers. If you got it right, call out "Yes!" and give a fist pump. If you made a mistake, circle it. Ready?

T: (Energetically, rapid-fire call the first answer.)

S: Yes!

T: (Energetically, rapid-fire call the second answer.)

S: Yes!

Repeat to the end of *Sprint A*, or until no one has any more correct. If need be, read the "count by" answers in the same way you read Sprint answers. Each number "counted by" on the back is considered a correct answer.

T: Fantastic! Now write the number you got correct at the top of your page. This is your personal goal for *Sprint B*.

T: How many of you got 1 right? (All hands should go up.)

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T: Keep your hand up until I say the number that is 1 more than the number you got right. So, if you got 14 right, when I say 15 your hand goes down. Ready?

T: (Quickly.) How many got 2 right? 3? 4? 5? Etc. (Continue until all hands are down.)

Optional routine, depending on whether or not your class needs more practice with *Sprint A*:

T: I'll give you one minute to do more problems on this half of the Sprint. If you finish, stand behind your chair. (As students work you might have the person who scored highest on Sprint A pass out Sprint B.)

T: Stop! I will read just the answers. If you got it right, call out "Yes!" and give a fist pump. If you made a mistake, circle it. Ready? (Read the answers to the first half again as students stand.)

Movement

To keep the energy and fun going, always do a stretch or a movement game in between Sprint A and B. For example, the class might do jumping jacks while skip counting by 5 for about 1 minute. Feeling invigorated, students take their seats for Sprint B, ready to make every effort to complete more problems this time.

Sprint B

Pass *Sprint B* out quickly, face down on student desks with instructions to not look at the problems until the signal is given. (Repeat the procedure for *Sprint A* up through the show of hands for how many right.)

T: Stand up if you got more correct on the second Sprint than on the first.

S: (Students stand.)

T: Keep standing until I say the number that tells how many more you got right on Sprint B. So if you got 3 more right on Sprint B than you did on Sprint A, when I say 3 you sit down. Ready? (Call out numbers starting with 1. Students sit as the number by which they improved is called. Celebrate the students who improved most with a cheer.)

T: Well done! Now take a moment to go back and correct your mistakes. Think about what patterns you noticed in today's Sprint.

T: How did the patterns help you get better at solving the problems?

T: RallyRobin your thinking with your partner for 1 minute. Go!

RallyRobin is a style of sharing in which partners trade information back and forth, one statement at a time per person, for about 1 minute. This is an especially valuable part of the routine for students who benefit from their friends' support to identify patterns and try new strategies.

Students may take Sprints home.

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