# 8th Grade Common Core State Standards

# Flip Book

**This document is intended to show the connections to the Standards of Mathematical Practices and the content standards and to get detailed information at each level. Resources used: CCSS, Arizona DOE, Ohio DOE and North Carolina DOE. This "Flip Book" is intended to help teachers understand what each standard means in terms of what students must know and be able to do. It provides only a *sample* of instructional strategies and examples. The goal of every teacher should be to guide students in understanding & making sense of mathematics.**

Construction directions:

Print on cardstock. Cut the tabs on each page starting with page 2. Cut the bottom off of this top cover to reveal the tabs for the subsequent pages. Staple or bind the top of all pages to complete your flip book.

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## **Mathematical Practice Standards** (**MP**) summary of each standard

1. **Make sense of problems and persevere in solving them.**
   In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2. **Reason abstractly and quantitatively.**
   In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. **Construct viable arguments and critique the reasoning of others.**
   In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. **Model with mathematics.**
   In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. **Use appropriate tools strategically.**
   Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
6. **Attend to precision.**
   In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
7. **Look for and make use of structure. (Deductive Reasoning)**
   Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
8. **Look for and express regularity in repeated reasoning. (Inductive Reasoning)**
   In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.



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|---|---|---|---|---|---|
| **Mathematics Practice Standards** | | | | | |

## **Critical Areas** for Mathematics in 8th Grade

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality ($m$) is the slope, and the graphs are lines through the origin. They understand that the slope ($m$) of a line is a constant rate of change, so that if the input or $x$-coordinate changes by an amount $A$, the output or $y$-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and $y$-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Domain: **The Number System (NS)**

Cluster: Know that there are numbers that are not rational, and approximate them by rational numbers.

Standard: **8.NS.1**. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

**Standards for Mathematical Practice (MP):**

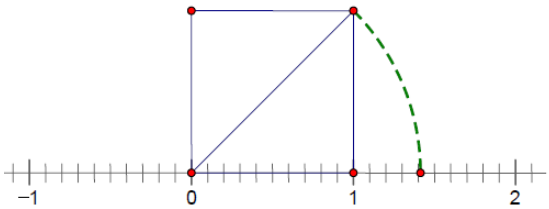
8.MP.2. Reason abstractly and quantitatively.
8.MP.6. Attend to precision.
8.MP.7. Look for and make use of structure.

**Connections**:

This cluster goes beyond the Grade 8 Critical Areas of Focus to address **Working with irrational numbers, integer exponents, and scientific notation**.
This cluster builds on previous understandings from Grades 6-7, The Number System.

**Instructional Strategies**

A rational number is of the form a/b, where a and b are both integers, and b is not 0. In the elementary grades, students learned processes that can be used to locate any rational number on the number line: Divide the interval from 0 to 1 into b equal parts; then, beginning at 0, count out those parts. The surprising fact, now, is that there are numbers on the number line that cannot be expressed as a/b, with a and b both integers, and these are called irrational numbers. Students construct a right isosceles triangle with legs of 1 unit. Using the Pythagorean theorem, they determine that the length of the hypotenuse is $\sqrt{2}$. In the figure right, they can rotate the hypotenuse back to the original number line to show that indeed $\sqrt{2}$ is a number on the number line.




In the elementary grades, students become familiar with decimal fractions, most often with decimal representations that terminate a few digits to the right of the decimal point. For example, to find the exact decimal representation of 2/7, students might use their calculator to find 2/7 = 0.2857142857…, and they might guess that the digits 285714 repeat. To show that the digits do repeat, students in Grade 7 actually carry out the long division and recognize that the remainders repeat in a predictable pattern—a pattern that creates the repetition in the decimal representation (see 7.NS.2.d).

Thinking about long division generally, ask students what will happen if the remainder is 0 at some step. They can reason that the long division is complete, and the decimal representation terminates. If the reminder is never 0, in contrast, then the remainders will repeat in a cyclical pattern because at each step with a given remainder, the process for finding the next remainder is always the same. Thus, the digits in the decimal representation also repeat. When dividing by 7, there are 6 possible nonzero remainders, and students can see that the decimal repeats with a pattern of at most 6 digits. In general, when finding the decimal representation of m/n, students can reason that the repeating portion of decimal will have at most n-1 digits. The important point here is that students can see that the pattern will repeat, so they can imagine the process continuing without actually carrying it out.

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Conversely, given a repeating decimal, students learn strategies for converting the decimal to a fraction. One approach is to notice that rational numbers with denominators of 9 repeat a single digit. With a denominator of 99, two digits repeat; with a denominator of 999, three digits repeat, and so on.
13/99 = 0.13131313…
74/99 = 0.74747474…
237/999 = 0.237237237…
485/999 = 0.485485485…
From this pattern, students can go in the other direction, conjecturing, for example, that the repeating decimal 0.285714285714… = 285714/999999. And then they can verify that this fraction is equivalent to 2/7.
Once students understand that (1) every rational number has a decimal representation that either terminates or repeats, and (2) every terminating or repeating decimal is a rational number, they can reason that on the number line, irrational numbers (i.e., those that are not rational) must have decimal representations that neither terminate nor repeat. And although students at this grade do not need to be able to prove that √2 is irrational, they need to know that √2 is irrational (see 8.EE.2), which means that its decimal representation neither terminates nor repeats. Nonetheless, they can approximate √2 without using the square root key on the calculator. Students can create tables like those below to approximate √2 to one, two, and then three places to the right of the decimal point:

| $x$ | $x^2$ |
|---|---|
| 1.0 | 1.00 |
| 1.1 | 1.21 |
| 1.2 | 1.44 |
| 1.3 | 1.69 |
| 1.4 | 1.96 |
| 1.5 | 2.25 |
| 1.6 | 2.56 |
| 1.7 | 2.89 |
| 1.8 | 3.24 |
| 1.9 | 3.61 |
| 2.0 | 4.00 |

| $x$ | $x^2$ |
|---|---|
| 1.40 | 1.9600 |
| 1.41 | 1.9881 |
| 1.42 | 2.0164 |
| 1.43 | 2.0449 |
| 1.44 | 2.0736 |
| 1.45 | 2.1025 |
| 1.46 | 2.1316 |
| 1.47 | 2.1609 |
| 1.48 | 2.1904 |
| 1.49 | 2.2201 |
| 1.50 | 2.2500 |

| $x$ | $x^2$ |
|---|---|
| 1.410 | 1.988100 |
| 1.411 | 1.990921 |
| 1.412 | 1.993744 |
| 1.413 | 1.996569 |
| 1.414 | 1.999396 |
| 1.415 | 2.002225 |
| 1.416 | 2.005056 |
| 1.417 | 2.007889 |
| 1.418 | 2.010724 |
| 1.419 | 2.013561 |
| 1.420 | 2.016400 |

From knowing that 12 = 1 and 22 = 4, or from the picture above, students can reason that there is a number between 1 and 2 whose square is 2. In the first table above, students can see that between 1.4 and 1.5, there is a number whose square is 2. Then in the second table, they locate that number between 1.41 and 1.42. And in the third table they can locate √2 between 1.414 and 1.415. Students can develop more efficient methods for this work. For example, from the picture above, they might have begun the first table with 1.4. And once they see that 1.422 > 2, they do not need to generate the rest of the data in the second table.
Use set diagrams to show the relationships among real, rational, irrational numbers, integers, and counting numbers. The diagram should show that the all real numbers (numbers on the number line) are either rational or irrational.
Given two distinct numbers, it is possible to find both a rational and an irrational number between them.

**Explanations and Examples**

**8.NS.1** Students distinguish between rational and irrational numbers. Any number that can be expressed as a fraction is a rational number. Students recognize that the decimal equivalent of a fraction will either terminate or repeat. Fractions that terminate will have denominators containing only prime factors of 2 and/or 5. This understanding builds on work in 7th grade when students used long division to distinguish between repeating and terminating decimals.

Students convert repeating decimals into their fraction equivalent using patterns or algebraic reasoning.

One method to find the fraction equivalent to a repeating decimal is shown below.

Change 0.4 to a fraction.

Let x = 0.444444…..

Multiply both sides so that the repeating digits will be in front of the decimal. In this example, one digit repeats so both sides are multiplied by 10, giving 10x = 4.4444444…. Subtract the original equation from the new equation.

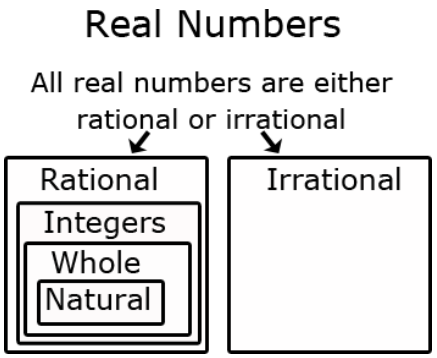
$$10x = 4.4444444\ldots$$
$$x = 0.444444\ldots$$
$$9x = 4$$

Solve the equation to determine the equivalent fraction.

$$\frac{9x}{9} = \frac{4}{9}$$
$$x = \frac{4}{9}$$

Additionally, students can investigate repeating patterns that occur when fractions have a denominator of 9, 99, or 11. For example, 9 is equivalent to 0.4, 9 is equivalent to 0.5, etc.

Students can use graphic organizers to show the relationship between the subsets of the real number system.




**Common Misconceptions**

Some students are surprised that the decimal representation of pi does not repeat. Some students believe that if only we keep looking at digits farther and farther to the right, eventually a pattern will emerge.

A few irrational numbers are given special names (pi and e), and much attention is given to sqrt(2). Because we name so few irrational numbers, students sometimes conclude that irrational numbers are unusual and rare. In fact, irrational numbers are much more plentiful than rational numbers, in the sense that they are "denser" in the real line.

**Domain: The Number System (NS)**

Cluster: Know that there are numbers that are not rational, and approximate them by rational numbers.

Standard: **8.NS.2**. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$). *For example, by truncating the decimal expansion of √2, show that √2 is between 1and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections**:

*See* **8.NS.1**.

**Instructional Strategies:**

See 8.NS.1.

**Explanations and Examples:**

**8.NS.2** Students locate rational and irrational numbers on the number line. Students compare and order rational and irrational numbers. Additionally, students understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational. Students also recognize that square roots may be negative and written as - 28. To find an approximation of 28, first determine the perfect squares 28 is between, which would be 25 and 36. The square roots of 25 and 36 are 5 and 6 respectively, so we know that 28 is between 5 and 6. Since 28 is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27. The estimate of 28 would be 5.27 (the actual is 5.29).

Students can approximate square roots by iterative processes.

Examples:

- Approximate the value of $\sqrt{5}$ to the nearest hundredth.

  Solution: Students start with a rough estimate based upon perfect squares. $\sqrt{5}$ falls between 2 and 3 because 5 falls between $2^2 = 4$ and $3^2 = 9$. The value will be closer to 2 than to 3. Students continue the iterative process with the tenths place value. $\sqrt{5}$ falls between 2.2 and 2.3 because 5 falls between $2.2^2 = 4.84$ and $2.3^2 = 5.29$. The value is closer to 2.2. Further iteration shows that the value of $\sqrt{5}$ is between 2.23 and 2.24 since $2.23^2$ is 4.9729 and $2.24^2$ is 5.0176.
- Compare √2 and √3 by estimating their values, plotting them on a number line, and making comparative statements.
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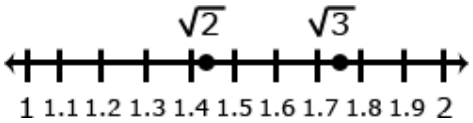


  Solution: Statements for the comparison could include:
  √2 is approximately 0.3 less than √3
  √2 is between the whole numbers 1 and 2
  √3 is between 1.7 and 1.8

**Common Misconceptions:**

See **8.NS.1**.

**Domain: Expressions and Equations (EE)**

Cluster: Work with radicals and integer exponents.

Standard: **8.EE.1.** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections:**

This cluster goes beyond the Grade 8 Critical Areas of Focus to address **Working with irrational numbers, interger exponents, and Scientific notation.** This cluster connects to previous understandings of place value, very large and very small numbers.

**Instructional Strategies**

Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to know and use the properties of exponents and to extend the meaning beyond counting-number exponents. It is no accident that these expectations are simultaneous, because it is the properties of counting-number exponents that provide the rationale for the properties of integer exponents. In other words, s*tudents should not be told these properties but rather should derive them through experience and reason.*

For counting-number exponents (and for nonzero bases), the following properties follow directly from the meaning of exponents.

1. *anam = an+m*
2. (*an*)*m = anm*
3. *anbn* = (*ab*)*n*

Students should have experience simplifying numerical expressions with exponents so that these properties become natural and obvious. For example,

23·25=(2·2·2)·(2·2·2·2·2)=28
(53)4=(5·5·5)·(5·5·5)·(5·5·5)·(5·5·5)=512
(3·7)4=(3·7)·(3·7)·(3·7)·(3·7)=(3·3·3·3)·(7·7·7·7)=34·74

If students reason about these examples with a sense of generality about the numbers, they begin to articulate the properties. For example, "I see that 3 twos is being multiplied by 5 twos, and the results is 8 twos being multiplied together, where the 8 is the sum of 3 and 5, the number of twos in each of the original factors. That would work for a base other than two (as long as the bases are the same)." Note: When talking about the meaning of an exponential expression, it is easy to say (incorrectly) that "35 means 3 multiplied by itself 5 times." But by writing out the meaning, 35=3·3·3·3·3, students can see that there are only 4 multiplications. So a better description is "35 means 5 3s multiplied together." Students also need to realize that these simple descriptions work only for counting-number exponents. When extending the meaning of exponents to include 0 and negative exponents, these descriptions are limiting: Is it sensible to say "30 means 0 3s multiplied together" or that "3-2 means -2 3s multiplied together"?

The motivation for the meanings of 0 and negative exponents is the following principle: *The properties of counting-number exponents should continue to work for integer exponents.*

Example next page

For example, Property 1 can be used to reason what 30 should be. Consider the following expression and simplification:
30·35=30+5=35. This computation shows that the when 30 is multiplied by 35, the result (following Property 1) should be 35, which implies that 30 must be 1. Because this reasoning holds for any base other than 0, we can reason that *a*0 = 1 for any nonzero number *a*.
To make a judgment about the meaning of 3-4, the approach is similar: 3−4·34=3−4+4=30=1. This computation shows that 3-4 should be the reciprocal of 34, because their product is 1. And again, this reasoning holds for any nonzero base. Thus, we can reason that *a*−*n* = 1/*an*.
Putting all of these results together, we now have the properties of integer exponents, shown in the above chart. For mathematical completeness, one might prove that properties 1-3 continue to hold for integer exponents, but that is not necessary at this point.
A supplemental strategy for developing meaning for integer exponents is to make use of patterns, as shown in the chart to the right.
The meanings of 0 and negative-integer exponents can be further explored in a place-value chart:

| *thousands* | *hundreds* | *tens* | *ones* | | *tenths* | *hundredths* | *thousandths* |
|---|---|---|---|---|---|---|---|
| $10^3$ | $10^2$ | $10^1$ | $10^0$ | | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |
| 3 | 2 | 4 | 7 | . | 5 | 6 | 8 |

**Properties of Integer Exponents**
For any nonzero real numbers *a* and *b* and integers *n* and *m*:
1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = 1/a^n$

**Patterns in Exponents**

| | |
|---|---|
| ⋮ | ⋮ |
| $5^4$ | 625 |
| $5^3$ | 125 |
| $5^2$ | 25 |
| $5^1$ | 5 |
| $5^0$ | 1 |
| $5^{-1}$ | 1/5 |
| $5^{-2}$ | 1/25 |
| $5^{-3}$ | 1/125 |
| ⋮ | ⋮ |

As the exponent decreases by 1, the value of the expression is divided by 5, which is the base. Continue that pattern to 0 and negative exponents.

Thus, integer exponents support writing any decimal in expanded form like the following:
3247.568=3·103+2·102+4·101+7·100+5·10−1+6·10−2+8·10−3.
Expanded form and the connection to place value is important for helping students make sense of scientific notation, which allows very large and very small numbers to be written concisely, enabling easy comparison. To develop familiarity, go back and forth between standard notation and scientific notation for numbers near, for example, 1012 or 10-9. Compare numbers, where one is given in scientific notation and the other is given in standard notation. Real-world problems can help students compare quantities and make sense about their relationship.
Provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If then . This flexibility should be experienced symbolically and verbally. 32=9 9=3

Opportunities for conceptually understanding irrational numbers should be developed. One way is for students to draw a square that is one unit by one unit and find the diagonal using the Pythagorean Theorem. The diagonal drawn has an irrational length of √2. Other irrational lengths can be found using the same strategy by finding diagonal lengths of rectangles with various side lengths.

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## Explanations and Examples

**8.EE.1** Integer (positive and negative) exponents are further used to generate equivalent numerical expressions when multiplying, dividing or raising a power to a power. Using numerical bases and the laws of exponents, students generate equivalent expressions.

Examples:

- $\frac{4^3}{5^2} = \frac{64}{25}$
- $\frac{4^3}{4^7} = 4^{3-7} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$

$$\frac{4^{-3}}{5^2} = 4^{-3} \cdot \frac{1}{5^2} = \frac{1}{4^3} \cdot \frac{1}{5^2} = \frac{1}{64} \cdot \frac{1}{25} = \frac{1}{16,000}$$

## Common Misconceptions

Equivalent to or Is $x^2 \oplus x^3$ $x^5$ $x^6$ ? Students may make the relationship that in scientific notation, when a number contains one nonzero digit and a positive exponent, that the number of zeros equals the exponent. This pattern may incorrectly be applied to scientific notation values with negative values or with more than one nonzero digit. Students may mix up the product of powers property and the power of a power property.

Domain: **Expressions and Equations**

Cluster: Work with radicals and integer exponents.

Standard: **8.EE.2**. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

*See* **8.EE.1.**

**Explanations and Examples**

**8.EE.2** Students recognize that squaring a number and taking the square root √ of a number are inverse operations; likewise, cubing a number and taking the cube root $^3$ are inverse operations. This understanding is used to solve equations containing square or cube numbers. Equations may include rational numbers such as $x_2 = 1/4$, $x^2 = 4/9$ or $x^3 = 1/8$ (NOTE: Both the numerator and denominators are perfect squares or perfect cubes.) Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students understand that in geometry a square root is the length of the side of a square and a cube root is the length of the side of a cube. The value of p for square root and cube root equations must be positive.

Examples:

- $3^2 = 9$ and $\sqrt{9} = \pm 3$
- $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$
- Solve $x^2 = 9$
  Solution: $x^2 = 9$
  $\sqrt{x^2} = \pm\sqrt{9}$
  $x = \pm 3$
- Solve $x^3 = 8$
  Solution: $x^3 = 8$
  $\sqrt[3]{x^3} = \sqrt[3]{8}$
  $x = 2$

**Instructional Strategies**

*See* **8.EE.1.**

**Common Misconceptions:**

*See* **8.EE.1.**

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| Domain: **Expressions and Equations** |
| Cluster: Work with radicals and integer exponents. |
| Standard: **8.EE.3**. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as $3\times10^8$ and the population of the world as $7\times10^9$, and determine that the world population is more than 20 times larger.* |
| **Standards for Mathematical Practice (MP):**<br>MP.2. Reason abstractly and quantitatively.<br>MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision. |
| **Connections**:<br>*See* **8.EE.1.** |
| **Explanations and Examples**<br>**8.EE.3** Students express numbers in scientific notation. Students compare and interpret scientific notation quantities in the context of the situation. If the exponent increases by one, the value increases 10 times. Students understand the magnitude of the number being expressed in scientific notation and choose an appropriate corresponding unit. For example, $3 \times 10^8$ is equivalent to 30 million, which represents a large quantity. Therefore, this value will affect the unit chosen.<br><br>**Instructional Strategies**<br><br>*See* **8.EE.1.** |
| **Common Misconceptions:**<br>*See* **8.EE.1.** |



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| **8.EE.3** | | | | | |

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| Domain: **Expressions and Equations** |
| Cluster: Work with radicals and integer exponents. |
| Standard: 8**.EE.4.** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. |
| **Standards for Mathematical Practice (MP):**<br>MP.2. Reason abstractly and quantitatively.<br>MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision. |
| **Connections**:<br>*See* **8.EE.1.** |
| **Explanations and Examples**<br>**8.EE.4** Students use laws of exponents to multiply or divide numbers written in scientific notation. Additionally, students understand scientific notation as generated on various calculators or other technology.<br><br>Students can convert decimal forms to scientific notation and apply rules of exponents to simplify expressions. In working with calculators or spreadsheets, it is important that students recognize scientific notation. Students should recognize that the output of 2.45E+23 is $2.45 \times 10^{23}$ and 3.5E-4 is $3.5 \times 10^{-4}$. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.<br><br>**Instructional Strategies**<br><br>*See* **8.EE.1.** |
| **Common Misconceptions:**<br>*See* **8.EE.1.** |

Domain: **Expressions and Equations (EE)**

Cluster: Understand the connections between proportional relationships, lines, and linear equations

Standard: **8.EE.5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

**Standards for Mathematical Practice (MP):**

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections:**

This cluster is connected to the Grade 8 Critical Area of Focus #1, **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equaitons** and Critical Area of Focus #3, **Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem**.

**Explanations and Examples**

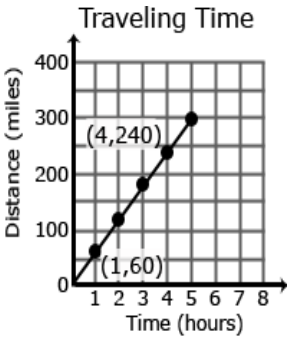
**8.EE.5** Students build on their work with unit rates from 6 grade and proportional relationships in 7 grade to compare graphs, tables and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables and equations to compare two or more proportional relationships.

Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs.

Example:

- Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation.

Scenario 1:




Scenario 2:

$y = 50x$
$x$ is time in hours
$y$ is distance in miles

**Instructional Strategies**

This cluster focuses on extending the understanding of ratios and proportions. Unit rates have been explored in Grade 6 as the comparison of two different quantities with the second unit a unit of one, (unit rate). In seventh grade unit rates were expanded to complex fractions and percents through solving multistep problems such as: discounts, interest, taxes, tips, and percent of increase or decrease. Proportional relationships were applied in scale drawings, and students should have developed an informal understanding that the steepness of the graph is the slope or unit rate. Now unit rates are addressed formally in graphical representations, algebraic equations, and geometry through similar triangles.

Distance time problems are notorious in mathematics. In this cluster, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed and described in different ways: graphically and algebraically. Emphasize the creation of representative graphs and the meaning of various points. Then compare the same information when represented in an equation. By using coordinate grids and various sets of three similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate of change equal. After proving with multiple sets of triangles, students can be led to generalize the slope to $y = mx$ for a line through the origin and $y = mx + b$ for a line through the vertical axis at b.

**Instructional Resources/Tools**

Carnegie Math™
graphing calculators
SMART™ technology with software emulator
National Library of Virtual Manipulatives (NLVM)©,
The National Council of Teachers of Mathematics, Illuminations
Annenberg™ video tutorials, www.nsdl.org

Domain: Expressions and Equations

Cluster: Understand the connections between proportional relationships, lines, and linear equations.

Standard: **8.EE.6.** Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y* = *mx* for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections**:

*See* **8.EE.5.**

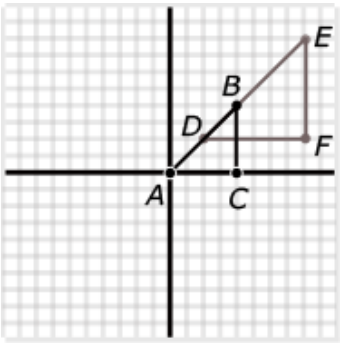
**Explanations and Examples**

**8.EE.6** Triangles are similar when there is a constant rate of proportion between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line.
The triangle between A and B has a vertical height of 2 and a horizontal length of 3. The triangle between B and C has a vertical height of 4 and a horizontal length of 6. The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of 2/3for the line.

Students write equations in the form y = mx for lines going through the origin, recognizing that m represents the slope of the line. Students write equations in the form y = mx + b for lines not passing through the origin, recognizing that m represents the slope and b represents the y-intercept.

Example:

- Explain why $\Delta ACB$ is similar to $\Delta DFE$, and deduce that $\overline{AB}$ has the same slope as $\overline{BE}$. Express each line as an equation.




**Instructional Strategies**

*See* **8.EE.5.**

**Common Misconceptions:**

*See* **8.EE.5.**

**Domain: Expressions and Equations (EE)**

Cluster: Analyze and solve linear equations and pairs of simultaneous linear equations.

Standard: **8.EE.7.** Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).
b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

This cluster is connected to the Grade 8 Critical Area of Focus #1, **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equaitons.** This cluster also builds upon the understandings in Grades 6 and 7 of Expressions and Equations, Ratios and Proportional Relationships, and utilizes the skills developed in the previous grade in The Number System.

**Explanations and Examples**

**8.EE.7** Students solve one-variable equations with the variables being on both sides of the equals sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. Equations shall include rational numbers, distributive property and combining like terms.

Equations have one solution when the variables do not cancel out. For example, $10x - 23 = 29 - 3x$ can be solved to $x = 4$. This means that when the value of x is 4, both sides will be equal. If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be (4, 17).

$$10 \bullet 4 - 23 = 29 - 3 \bullet 4$$
$$40 - 23 = 29 - 12$$
$$17 = 17$$

Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for x that will make the sides equal. For example, the equation $-x + 7 - 6x = 19 - 7x$, can be simplified to $-7x + 7 = 19 - 7x$. If 7x is added to each side, the resulting equation is $7 = 19$, which is not true. No matter what value is substituted for x the final result will be $7 = 19$. If each side of the equation were treated as a linear equation and graphed, the lines would be parallel.

Continued next page

An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation. For example the following equation, when simplified will give the same values on both sides.
−1 /2(36a − 6) = 3
4(4 − 24a)
−18a + 3 = 3 − 18a

If each side of the equation were treated as a linear equation and graphed, the graph would be the same line.

As students transform linear equations in one variable into simpler forms, they discover the equations can have one solution, infinitely many solutions, or no solutions.

When the equation has one solution, the variable has one value that makes the equation true as in 12-4*y*=16. The only value for y that makes this equation true is -1.

When the equation has infinitely many solutions, the equation is true for all real numbers as in 7*x* + 14 = 7 (*x*+2). As this equation is simplified, the variable terms cancel leaving 14 = 14 or 0 = 0. Since the expressions are equivalent, the value for the two sides of the equation will be the same regardless which real number is used for the substitution.

When an equation has no solutions it is also called an inconsistent equation. This is the case when the two expressions are not equivalent as in 5*x* - 2 = 5(*x*+1). When simplifying this equation, students will find that the solution appears to be two numbers that are not equal or -2 = 1. In this case, regardless which real number is used for the substitution, the equation is not true and therefore has no solution.

Examples:

- Solve for x:
  - $-3(x+7)=4$
  - $3x-8=4x-8$
  - $3(x+1)-5=3x-2$

- Solve:
  - $7(m-3)=7$

$$\frac{1}{4}-\frac{2}{3}y=\frac{3}{4}-\frac{1}{3}y$$

**<u>Instructional Strategies</u> 8.EE.7-8**

In Grade 6, students applied the properties of operations to generate equivalent expressions, and identified when two expressions are equivalent. This cluster extends understanding to the process of solving equations and to their solutions, building on the fact that solutions maintain equality, and that equations may have only one solution, many solutions, or no solution at all. Equations with many solutions may be as simple as 3*x* = 3*x*, 3x + 5 = x + 2 + *x* + *x* + 3, or 6*x* + 4*x* = *x*(6 + 4), where both sides of the equation are equivalent once each side is simplified.

Continued next page

Table 3 on page 90 of CCSS generalizes the properties of operations and serves as a reminder for teachers of what these properties are. Eighth graders should be able to describe these relationships with real numbers and justify their reasoning using words and not necessarily with the algebraic language of Table 3. In other words, students should be able to state that 3(-5) = (-5)3 because multiplication is commutative and it can be performed in any order (it is commutative), or that 9(8) + 9(2) = 9(8 + 2) because the distributive property allows us to distribute multiplication over addition, or determine products and add them. Grade 8 is the beginning of using the generalized properties of operations, but this is not something on which students should be assessed.

Pairing contextual situations with equation solving allows students to connect mathematical analysis with real-life events. Students should experience analyzing and representing contextual situations with equations, identify whether there is one, none, or many solutions, and then solve to prove conjectures about the solutions. Through multiple opportunities to analyze and solve equations, students should be able to estimate the number of solutions and possible values(s) of solutions prior to solving. Rich problems, such as computing the number of tiles needed to put a border around a rectangular space or solving proportional problems as in doubling recipes, help ground the abstract symbolism to life.

Experiences should move through the stages of concrete, conceptual and algebraic/abstract. Utilize experiences with the pan balance model as a visual tool for maintaining equality (balance) first with simple numbers, then with pictures symbolizing relationships, and finally with rational numbers allows understanding to develop as the complexity of the problems increases. Equation-solving in Grade 8 should involve multistep problems that require the use of the distributive property, collecting like terms, and variables on both sides of the equation.

This cluster builds on the informal understanding of slope from graphing unit rates in Grade 6 and graphing proportional relationships in Grade 7 with a stronger, more formal understanding of slope. It extends solving equations to understanding solving systems of equations, or a set of two or more linear equations that contain one or both of the same two variables. Once again the focus is on a solution to the system. Most student experiences should be with numerical and graphical representations of solutions. Beginning work should involve systems of equations with solutions that are ordered pairs of integers, making it easier to locate the point of intersection, simplify the computation and hone in on finding a solution. More complex sytems can be investigated and solve by using graphing technology.

Contextual situations relevant to eighth graders will add meaning to the solution to a system of equations. Students should explore many problems for which they must write and graph pairs of equations leading to the generalization that finding one point of intersection is the single solution to the system of equations. Provide opportunities for students to connect the solutions to an equation of a line, or solution to a system of equations, by graphing, using a table and writing an equation. Students should receive opportunities to compare equations and systems of equations, investigate using graphing calculators or graphing utilities, explain differences verbally and in writing, and use models such as equation balances.

Continued next page

Problems such as, "Determine the number of movies downloaded in a month that would make the costs for two sites the same, when Site A charges $6 per month and $1.25 for each movie and Site B charges $2 for each movie and no monthly fee."
Students write the equations letting y = the total charge and x = the number of movies.
Site A: $y = 1.25x + 6$ Site B: $y = 2x$
Students graph the solutions for each of the equations by finding ordered pairs that are solutions and representing them in a t-chart. Discussion should encompass the realization that the intersection is an ordered pair that satisfies both equations. And finally students should relate the solution to the context of the problem, commenting on the practicality of their solution.
Problems should be structured so that students also experience equations that represent parallel lines and equations that are equivalent. This will help them to begin to understand the relationships between different pairs of equations: When the slope of the two lines is the same, the equations are either different equations representing the same line (thus resulting in many solutions), or the equations are different equations representing two not intersecting, parallel, lines that do not have common solutions.
System-solving in Grade 8 should include estimating solutions graphically, solving using substitution, and solving using elimination. Students again should gain experience by developing conceptual skills using models that develop into abstract skills of formal solving of equations.
Provide opportunities for students to change forms of equations (from a given form to slope-intercept form) in order to compare equations.

**Common Misconceptions 8.EE.7-8**

Students think that only the letters x and y can be used for variables.
Students think that you always need a variable = a constant as a solution.
The variable is always on the left side of the equation.
Equations are not always in the slope intercept form, $y=mx+b$
Students confuse one-variable and two-variable equations.

Domain: **Expressions and Equations**

Cluster: Analyze and solve linear equations and pairs of simultaneous linear equations.

Standard: **8.EE.8.** Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*
c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections**:
*See* **8.EE.7**

**Explanations and Examples:**

Systems of linear equations can also have one solution, infinitely many solutions or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically.

A system of linear equations whose graphs meet at one point (intersecting lines) has only one solution, the ordered pair representing the point of intersection. A system of linear equations whose graphs do not meet (parallel lines) has no solutions and the slopes of these lines are the same. A system of linear equations whose graphs are coincident (the same line) has infinitely many solutions, the set of ordered pairs representing all the points on the line.

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole number and/or decimals/fractions.

Examples:

• Find x and y using elimination and then using substitution.

$$\begin{array}{rl} & 3x + 4y = 7 \\ - & 2x + 8y = 10 \end{array}$$

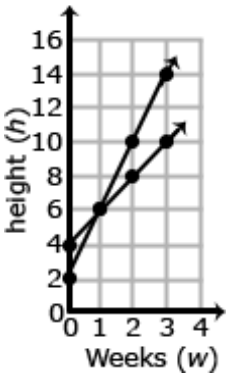
• Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same.

Let W = number of weeks
Let H = height of the plant after W weeks

| Plant A | | |
|---|---|---|
| W | H | |
| 0 | 4 | (0,4) |
| 1 | 6 | (1,6) |
| 2 | 8 | (2,8) |
| 3 | 10 | (3,10) |

| Plant B | | |
|---|---|---|
| W | H | |
| 0 | 2 | (0,1) |
| 1 | 6 | (1,6) |
| 2 | 10 | (2,10) |
| 3 | 14 | (3,14) |

Given each set of coordinates, graph their corresponding lines.
Solution:




Write an equation that represent the growth rate of Plant A and Plant B.

Solution:

Plant A H = 2W + 4
Plant B H = 4W + 2

• At which week will the plants have the same height?

Solution:

The plants have the same height after one week.
Plant A: H = 2W + 4 Plant B: H = 4W + 2
Plant A: H = 2(1) + 4 Plant B: H = 4(1) + 2
Plant A: H = 6 Plant B: H = 6

After one week, the height of Plant A and Plant B are both 6 inches.

## Extended Standards:

**The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "...materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication.**

| 8th Grade Mathematics<br>Expressions and Equations | | |
|---|---|---|
| **Common Core State Standards** | **Essence** | **Extended Common Core** |
| **Understand the connections between proportional relationships, lines, and linear equations.**<br>**Cluster**<br>1. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*<br>2. Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$. | **Proportional relationships** | **Understand the connections between proportional relationships, lines, and linear equations.**<br>**Cluster**<br>1. Make equivalent ratios given the unit rate.<br>2. Graph equivalent ratios in the first quadrant. |

| | | |
|---|---|---|
| **Analyze and solve linear equations and pairs of simultaneous linear equations.**<br>**Cluster**<br>3. Solve linear equations in one variable.<br>a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).<br>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.<br>4. Analyze and solve pairs of simultaneous linear equations.<br>a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.<br>b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*<br>c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* | **Solve equations** | **Analyze and solve linear equations and pairs of simultaneous linear equations.**<br>**Cluster**<br>3. Use equations to solve problems using all operations when a part is unknown. |

Domain: **Functions (F)**

Cluster: Define, evaluate, and compare functions.

Standard: **8.F.1**. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)

**Standards for Mathematical Practice (MP):**

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections**:

**Connections:**

This Cluster is connected to the Grade 8 Critical Area of Focus #2, **Grasping the concept of a function and using functions to describe quantitative relationships.** Expressions and Equations: Linear equations in two variables can be used to define linear functions, and students can use graphs of functions to reason toward solutions to linear equations.
Geometry: Similar triangles are used to show that the slope of a line is constant.
Statistics and Probability: Bivariate data can often be modeled by a linear function.

**Explanations and Examples**

**8.F.1** Students distinguish between functions and non-functions, using equations, graphs, and tables. Non- functions occur when there is more than one y-value is associated with any x-value. Students are not expected to use the function notation f(x) at this level.

For example, the rule that takes $x$ as input and gives $x^2+5x+4$ as output is a function. Using y to stand for the output we can represent this function with the equation $y = x^2+5x+4$, and the graph of the equation is the graph of the function. Students are not yet expected use function notation such as $f(x) = x^2+5x+4$.

**Instructional Strategies**

In grade 6, students plotted points in all four quadrants of the coordinate plane. They also represented and analyzed quantitative relationships between dependent and independent variables. In Grade 7, students decided whether two quantities are in a proportional relationship. In Grade 8, students begin to call relationships functions when each input is assigned to exactly one output. Also, in Grade 8, students learn that proportional relationships are part of a broader group of linear functions, and they are able to identify whether a relationship is linear. Nonlinear functions are included for comparison. Later, in high school, students use function notation and are able to identify types of nonlinear functions.
To determine whether a relationship is a function, students should be expected to reason from a context, a graph, or a table, after first being clear which quantity is considered the input and which is the output. When a relationship is not a function, students should produce a counterexample: an "input value" with at least two "output values." If the relationship is a function, the students should explain how they verified that for each input there was exactly one output. The "vertical line test" should be avoided because (1) it is too easy to apply without thinking, (2) students do not need an efficient strategy at this point, and (3) it creates

misconceptions for later mathematics, when it is useful to think of functions more broadly, such as whether x might be a function of y.
"Function machine" pictures are useful for helping students imagine input and output values, with a rule inside the machine by which the output value is determined from the input.
Notice that the standards explicitly call for exploring functions numerically, graphically, verbally, and algebraically (symbolically, with letters). This is sometimes called the "rule of four." For fluency and flexibility in thinking, students need experiences translating among these. In Grade 8, the focus, of course, is on linear functions, and students begin to recognize a linear function from its form y = mx + b. Students also need experiences with nonlinear functions, including functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule, such as a girl's height as a function of her age.
In the elementary grades, students explore number and shape patterns (sequences), and they use rules for finding the next term in the sequence. At this point, students describe sequences both by rules relating one term to the next and also by rules for finding the *n*th term directly. (In high school, students will call these recursive and explicit formulas.) Students express rules in both words and in symbols. Instruction should focus on additive and multiplicative sequences as well as sequences of square and cubic numbers, considered as areas and volumes of cubes, respectively.
When plotting points and drawing graphs, students should develop the habit of determining, based upon the context, whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting the dots can be misleading. For example, if a function is used to model the height of a stack of *n* paper cups, it does not make sense to have 2.3 cups, and thus there will be no ordered pairs between n = 2 and n = 3.
Provide multiple opportunities to examine the graphs of linear functions and use graphing calculators or computer software to analyze or compare at least two functions at the same time.
Illustrate with a slope triangle where the run is "1" that slope is the "unit rate of change."
Compare this in order to compare two different situations and identify which is increasing/decreasing as a faster rate.
Students can compute the area and perimeter of different-size squares and identify that one relationship is linear while the other is not by either looking at a table of value or a graph in which the side length is the independent variable (input) and the area or perimeter is the dependent variable (output).

**Common Misconceptions**

Some students will mistakenly think of a straight line as horizontal or vertical only.
Some students will mix up *x*- and *y*-axes on the coordinate plane, or mix up the ordered pairs. When emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up), point out that this is merely a convention: It could have been otherwise, but it is very useful for people to agree on a standard customary practice.

Domain: **Functions (F)**

Cluster: Define, evaluate, and compare functions.

Standard: **8.F.2.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

**Standards for Mathematical Practice (MP):**

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections**:

*See* **8.F.1**.

**Explanations and Examples**

**8.F.2** Students compare functions from different representations. For example, compare the following functions to determine which has the greater rate of change.

Function 1: $y = 2x + 4$

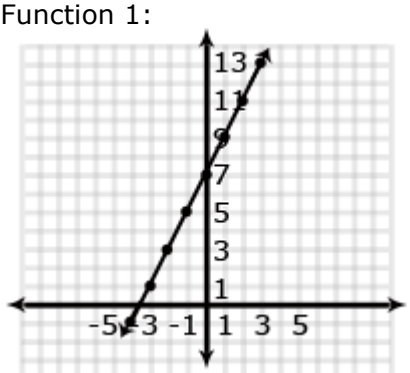
Function 2:

| x | y |
|---|---|
| -1 | -6 |
| 0 | -3 |
| 2 | 3 |

Examples:

- Compare the two linear functions listed below and determine which equation represents a greater rate of change.

Function 1:




Function 2:

The function whose input x and output y are related by

$y = 3x + 7$

- Compare the two linear functions listed below and determine which has a negative slope.

Function 1: Gift Card
Samantha starts with $20 on a gift card for the book store. She spends $3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks, *x*.

| *x* | *y* |
|---|---|
| 0 | 20 |
| 1 | 16.50 |
| 2 | 13.00 |
| 3 | 9.50 |
| 4 | 6.00 |

Function 2:
The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (*m*).

Solution:
Function 1 is an example of a function whose graph has negative slope. Samantha starts with $20 and spends money each week. The amount of money left on the gift card decreases each week. The graph has a negative slope of -3.5, which is the amount the gift card balance decreases with Samantha's weekly magazine purchase. Function 2 is an example of a function whose graph has positive slope. Students pay a yearly nonrefundable fee for renting the calculator and pay $5 for each month they rent the calculator. This function has a positive slope of 5 which is the amount of the monthly rental fee. An equation for Example 2 could be $c = 5m + 10$.

**Instructional Strategies**
*See* **8.F.1**.

**Common Misconceptions:**
*See* **8.F.1**.

Domain: **Functions**

Cluster: Define, evaluate, and compare functions.

Standard: **8.F.3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

*See* **8.F.1**.

**Explanations and Examples**

**8.F.3** Students use equations, graphs and tables to categorize functions as linear or non-linear. Students recognize that points on a straight line will have the same rate of change between any two of the points.

Example:

- Determine which of the functions listed below are linear and which are not linear and explain your reasoning.
  - $y = -2x^2 + 3$ non linear
  - $y = 2x$ linear
  - $A = \pi r^2$ non linear
  - $y = 0.25 + 0.5(x - 2)$ linear

**Instructional Strategies**

*See* **8.F.1**.

**Common Misconceptions:**

*See* **8.F.1**.

Domain: **Functions (F)**

Cluster: Use functions to model relationships between quantities.

Standard: **8.F.4.** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

**Standards for Mathematical Practice (MP):**

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections**:

**Connections:**

This cluster is connected to the Grade 8 Critical Area of Focus #2, **Grasping the concept of a function and using functions to describe quantitative relationships.** Expressions and Equations: Linear equations in two variables can be used to define linear functions, and students can use graphs of functions to reason toward solutions to linear equations.
Geometry: Similar triangles are used to show that the slope of a line is constant.
Statistics and Probability: Bivariate data can often be modeled by a linear function.

**Explanations and Examples**

**8.F.4** Students identify the rate of change (slope) and initial value (y-intercept) from tables, graphs, equations or verbal descriptions.

Students recognize that in a table the y-intercept is the y-value when x is equal to 0. The slope can the determined by finding the ratio *y x* between the change in two y-values and the change between the two corresponding x-values.

The y-intercept in the table below would be (0, 2). The distance between 8 and -1 is 9 in a negative direction is -9; the distance between -2 and 1 is 3 in a positive direction. The slope is the ratio of rise to run or *y* /*x* or 9 /3 = -3.

| x | y |
|---|---|
| -2 | 8 |
| 0 | 2 |
| 1 | -1 |

Using graphs, students identify the y-intercept as the point where the line crosses the y-axis and the slope as the rise. run

In a linear equation the coefficient of x is the slope and the constant is the y-intercept. Students need to be given the equations in formats other than y = mx + b, such as y = ax + b (format from graphing calculator), y = b + mx (often the format from contextual situations), etc. Note that point-slope form and standard forms are not expectations at this level.

In contextual situations, the y-intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem. Rates of change can often occur over years. In these situations it is helpful for the years to be "converted" to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960).

Students use the slope and y-intercepts to write a linear function in the form y = mx +b. Situations may be given as a verbal description, two ordered pairs, a table, a graph, or rate of change and another point on the line. Students interpret slope and y-intercept in the context of the given situation.

Examples:

- The table below shows the cost of renting a car. The company charges $45 a day for the car as well as charging a one-time $25 fee for the car's navigation system (GPS).Write an expression for the cost in dollars, *c,* as a function of the number of days, *d*.

  Students might write the equation $c = 45d + 25$ using the verbal description or by first making a table.

| Days (*d*) | Cost (*c*) in dollars |
|---|---|
| 1 | 70 |
| 2 | 115 |
| 3 | 160 |
| 4 | 205 |

Students should recognize that the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one time fees vs. recurrent fees will help students model contextual situations.

- When scuba divers come back to the surface of the water, they need to be careful not to ascend too quickly. Divers should not come to the surface more quickly than a rate of 0.75 ft per second. If the divers start at a depth of 100 feet, the equation $d = 0.75t - 100$ shows the relationship between the time of the ascent in seconds (*t)* and the distance from the surface in feet (*d*).
  - Will they be at the surface in 5 minutes? How long will it take the divers to surface from their dive?

Make a table of values showing several times and the corresponding distance of the divers from the surface. Explain what your table shows. How do the values in the table relate to your equation?

**<u>Instructional Strategies</u>**

In Grade 8, students focus on linear equations and functions. Nonlinear functions are used for comparison.

Students will need many opportunities and examples to figure out the meaning of $y = mx + b$. What does *m* mean? What does *b* mean? They should be able to "see" m and b in graphs, tables, and formulas or equations, and they need to be able to interpret those values in contexts. For example, if a function is used to model the height of a stack of *n* paper cups, then the rate of change, *m*, which is the slope of the graph, is the height of the "lip" of the cup: the amount each cup sticks above the lower cup in the stack. The "initial value" in this case is not valid in the

context because 0 cups would not have a height, and yet a height of 0 would not fit the equation. Nonetheless, the value of b can be interpreted in the context as the height of the "base" of the cup: the height of the whole cup minus its lip.

Use graphing calculators and web resources to explore linear and non-linear functions. Provide context as much as possible to build understanding of slope and y-intercept in a graph, especially for those patterns that do not start with an initial value of 0.

Give students opportunities to gather their own data or graphs in contexts they understand. Students need to measure, collect data, graph data, and look for patterns, then generalize and symbolically represent the patterns. They also need opportunities to draw graphs (qualitatively, based upon experience) representing real-life situations with which they are familiar. Probe student thinking by asking them to determine which input values make sense in the problem situations.

Provide students with a function in symbolic form and ask them to create a problem situation in words to match the function. Given a graph, have students create a scenario that would fit the graph. Ask students to sort a set of "cards" to match a graphs, tables, equations, and problem situations. Have students explain their reasoning to each other.

From a variety of representations of functions, students should be able to classify and describe the function as linear or non-linear, increasing or decreasing. Provide opportunities for students to share their ideas with other students and create their own examples for their classmates.

Use the slope of the graph and similar triangle arguments to call attention to not just the change in x or y, but also to the rate of change, which is a ratio of the two.

Emphasize key vocabulary. Students should be able to explain what key words mean: e.g., model, interpret, initial value, functional relationship, qualitative, linear, non-linear. Use a "word wall" to help reinforce vocabulary.

**Common Misconceptions**

Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning—and both types of formulas—are important for developing proficiency with functions.

When input values are not increasing consecutive integers (e.g., when the input values are decreasing, when some integers are skipped, or when some input values are not integers), some students have more difficulty identifying the pattern and calculating the slope. It is important that all students have experience with such tables, so as to be sure that they do not overgeneralize from the easier examples.

Some students may not pay attention to the scale on a graph, assuming that the scale units are always "one." When making axes for a graph, some students may not using equal intervals to create the scale.

Some students may infer a cause and effect between independent and dependent variables, but this is often not the case.

Some students graph incorrectly because they don't understand that x usually represents the independent variable and y represents the dependent variable. Emphasize that this is a convention that makes it easier to communicate.

Domain: **Functions**

Cluster: Use functions to model relationships between quantities.

Standard: **8.F.5.** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

**See 8.F.4.**

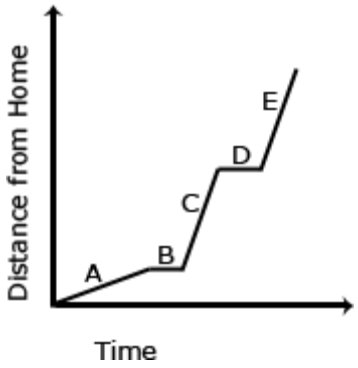
**Explanations and Examples**

**8.F.5** Given a verbal description of a situation, students sketch a graph to model that situation. Given a graph of a situation, students provide a verbal description of the situation.

Example:

- The graph below shows a student's trip to school. This student walks to his friend's house and, together, they ride a bus to school. The bus stops once before arriving at school.

  Describe how each part A-E of the graph relates to the story.




**Instructional Strategies**

**See 8.F.4.**

**Common Misconceptions:**

**See 8.F.4.**

Domain: **Geometry (G)**

Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.

Standard: **8.G.1.** Verify experimentally the properties of rotations, reflections, and translations:
a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
Parallel lines are taken to parallel lines.

**Standards for Mathematical Practice (MP):**
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections**:
This cluster is connected to the Grade 8 Critical Area of Focus #3, **Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.** This cluster builds from Grade 7 Geometry, Ratios and Proportional Relationships, and prepares students for more formal work in high school geometry.

**Explanations and Examples**
**8.G.1** In a translation, every point of the pre-image is moved the same distance and in the same direction to form the image. A reflection is the "flipping" of an object over a line, known as the "line of reflection". A rotation is a transformation that is performed by "spinning" the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise.

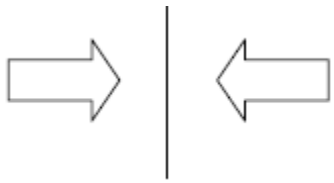
Students use compasses, protractors and ruler or technology to explore figures created from translations, reflections and rotations. Characteristics of figures, such as lengths of line segments, angle measures and parallel lines are explored before the transformation (pre-image) and after the transformation (image). Students understand that these transformations produce images of exactly the same size and shape as the pre-image and are known as rigid transformations.

Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same angle after they have been rotated, reflected, and/or translated.

Students are not expected to work formally with properties of dilations until high school.

**Instructional Strategies**
A major focus in Grade 8 is to use knowledge of angles and distance to analyze two- and three-dimensional figures and space in order to solve problems. This cluster interweaves the relationships of symmetry, transformations, and angle relationships to form understandings of similarity and congruence. Inductive and deductive reasoning are utilized as students forge into the world of proofs. Informal arguments are justifications based on known facts and logical reasoning. Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes). Students are expected to use logical thinking, expressed in words using correct terminology. They are NOT expected to use theorems, axioms, postulates or a formal format of proof as in two-column proofs.
Transformational geometry is about the effects of rigid motions, rotations, reflections and

translations on figures. Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system. For example, when reflecting over a line, each vertex is the same distance from the line as its corresponding vertex. This is easier to visualize when not using regular figures. Time should be allowed for students to cut out and trace the figures for each step in a series of transformations. Discussion should include the description of the relationship between the original figure and its image(s) in regards to their corresponding parts (length of sides and measure of angles) and the descriptionof the movement, including the atributes of transformations (line of symmetry, distance to be moved, center of rotation, angle of rotationand the amount of dilation).The case of distance – preserving transformation leads to the idea of congruence.

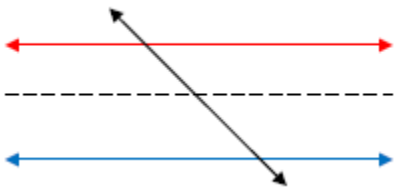


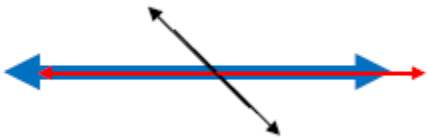
It is these distance-preserving transformations that lead to the idea of congruence.

Work in the coordinate plane should involve the movement of various polygons by addition, subtraction and multiplied changes of the coordinates. For example, add 3 to *x*, subtract 4 from *y*, combinations of changes to *x* and *y*, multiply coordinates by 2 then by 12. Students should observe and discuss such questions as 'What happens to the polygon?' and 'What does making the change to all vertices do?'. Understandings should include generalizations about the changes that maintain size or maintain shape, as well as the changes that create distortions of the polygon (dilations). Example dilations should be analyzed by students to discover the movement from the origin and the subsequent change of edge lengths of the figures. Students should be asked to describe the transformations required to go from an original figure to a transformed figure (image). Provide opportunities for students to discuss the procedure used, whether different procedures can obtain the same results, and if there is a more efficient procedure to obtain the same results. Students need to learn to describe transformations with both words and numbers.

Through understanding symmetry and congruence, conclusions can be made about the relationships of line segments and angles with figures. Students should relate rigid motions to the concept of symmetry and to use them to prove congruence or similarity of two figures. Problem situations should require students to use this knowledge to solve for missing measures or to prove relationships. It is an expectation to be able to describe rigid motions with coordinates. Provide opportunities for students to physically manipulate figures to discover properties of similar and congruent figures, for example, the corresponding angles of similar figures are equal. Additionally use drawings of parallel lines cut by a transversal to investigate the relationship among the angles.

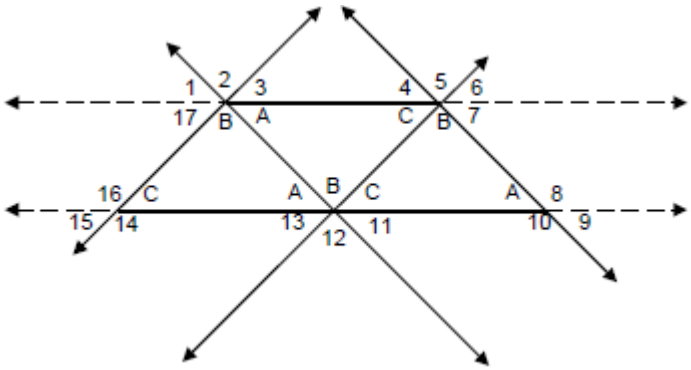
For example, what information can be obtained by cutting between the two intersections and sliding one onto the other?





In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, vertical). Now, the focus is on learning the about the sum of the angles of a triangle and using it to, find the measures of angles formed by transversals (especially with parallel lines), or to find the measures of exterior angles of triangles and to informally prove congruence.
By using three copies of the same triangle labeled and placed so that the three different angles form a straight line, students can:
• explore the relationships of the angles,
• learn the types of angles (interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, same side exterior), and
• explore the parallel lines, triangles and parallelograms formed.

Further examples can be explored to verify these relationships and demonstrate their relevance in real life.




Investigations should also lead to the Angle-Angle criterion for similar triangles. For instance, pairs of students create two different triangles with one given angle measurement, then repeat with two given angle measurements and finally with three given angle measurements. Students observe and describe the relationship of the resulting triangles. As a class, conjectures lead to the generalization of the Angle-Angle criterion.
Students should solve mathematical and real-life problems involving understandings from this cluster. Investigation, discussion, justification of their thinking, and application of their learning will assist in the more formal learning of geometry in high school.

**Common Misconceptions:**
Students often confuse situations that require adding with multiplicative situations in regard to scale factor. Providing experiences with geometric figures and coordinate grids may help students visualize the difference.



| | | | | | |
|---|---|---|---|---|---|
| **8.G.1** | | | | | |

Domain: **Geometry (G)**

Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.

Standard: **8.G.2**. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

*See* **8.G.1.**

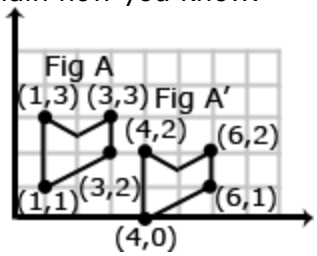
**Explanations and Examples**

**8.G.2** This standard is the students' introduction to congruency. Congruent figures have the same shape and size.
Translations, reflections and rotations are examples of rigid transformations. A rigid transformation is one in which the pre-image and the image both have exactly the same size and shape since the measures of the corresponding angles and corresponding line segments remain equal (are congruent).

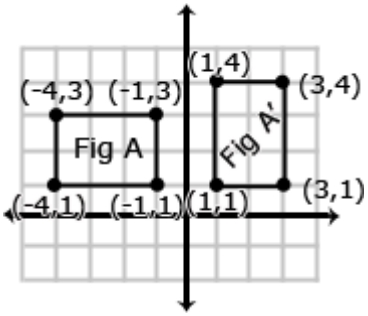
Students examine two figures to determine congruency by identifying the rigid transformation(s) that produced the figures. Students recognize the symbol for congruency (≅) and write statements of congruency.

Examples:

- Is Figure A congruent to Figure A'? Explain how you know.




- Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.




**Instructional Strategies**

*See* **8.G.1.**

Domain: **Geometry**

Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.

Standard: **8.G.3.** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

**Standards for Mathematical Practice (MP):**

MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

*See* **8.G.1.**

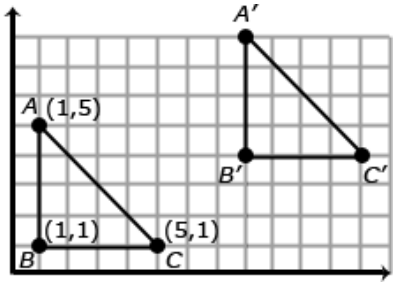
**Explanations and Examples**

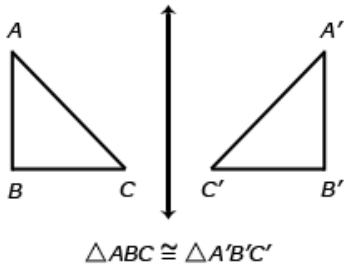
**8.G.3** Students identify resulting coordinates from translations, reflections, and rotations (90°, 180°and 270°both clockwise and counterclockwise), recognizing the relationship between the coordinates and the transformation. For example, a translation of 5 left and 2 up would subtract 5 from the x-coordinate and add 2 to the y-coordinate. D (- 4, -3) à D' (-9, -1). A reflection across the x-axis would change B (6, -8) to B' (6, 8) .

Additionally, students recognize the relationship between the coordinates of the pre-image, the image and the scale factor following a dilation from the origin. Dilations are non-rigid transformations that enlarge (scale factors greater than one) or reduce (scale factors less than one) the size of a figure using a scale factor.

A dilation is a transformation that moves each point along a ray emanating from a fixed center, and multiplies distances from the center by a common scale factor. In dilated figures, the dilated figure is *similar* to its pre-image.

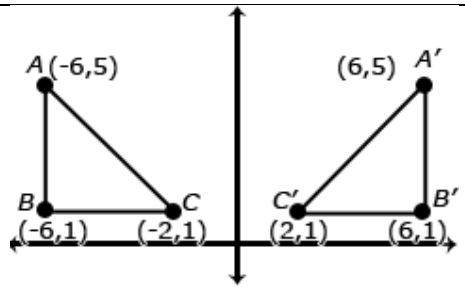
Translation: A translation is a transformation of an object that moves the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image. $\Delta ABC$ has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from $x = 1$ to $x = 8$) and 3 units up (from $y = 5$ to $y = 8$). Points B + C also move in the same direction (7 units to the right and 3 units up).



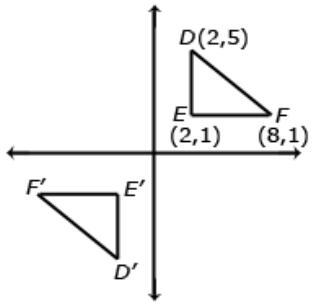

Reflection: A reflection is a transformation that flips an object across a line of reflection (in a coordinate grid the line of reflection may be the x or y axis). In a rotation, the rotated object is *congruent* to its pre-image.

When an object is reflected across the y axis, the reflected x coordinate is the opposite of the pre-image x coordinate.




Rotation: A rotated figure is a figure that has been turned about a fixed point. This is called the center of rotation. A figure can be rotated up to 360°. Rotated figures are congruent to their pre-image figures.

Consider when $\Delta DEF$ is rotated 180° clockwise about the origin. The coordinates of $\Delta DEF$ are D(2,5), E(2,1), and F(8,1). When rotated 180°, $\Delta D'E'F'$ has new coordinates D′(-2,-5), E′(-2,-1) and F′(-8,-1). Each coordinate is the opposite of its pre-image.




**Instructional Strategies**

*See* **8.G.1.**

**Common Misconceptions:**

*See* **8.G.1.**

Domain: **Geometry**

Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.

Standard: **8.G.4.** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

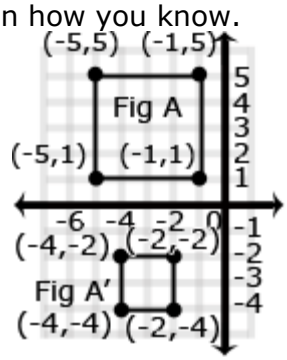
*See* **8.G.1.**

**Explanations and Examples**

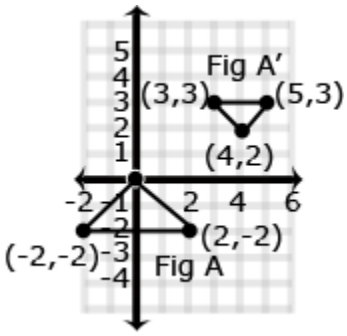
**8.G.4** This is the students' introduction to similarity and similar figures. Students understand similar figures have angles with the same measure and sides that are proportional. Similar figures are produced from dilations. Students describe the sequence that would produce similar figures, including the scale factors. Students understand that a scale factor greater than one will produce an enlargement in the figure, while a scale factor less than one will produce a reduction in size.

Examples:

- Is Figure A similar to Figure A'? Explain how you know.




- Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.




**Instructional Strategies**

*See* **8.G.1.**

Domain: **Geometry**

Cluster: Understand congruence and similarity using physical models, transparencies, or geometry software.

Standard: **8.G.5**. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

**Standards for Mathematical Practice (MP):**

MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

*See* **8.G.1.**

**Explanations and Examples**

**8.G.5** Students use exploration and deductive reasoning to determine relationships that exist between a) angle sums and exterior angle sums of triangles, b) angles created when parallel lines are cut by a transversal, and c) the angle- angle criterion for similarity of triangle.
Students construct various triangles and find the measures of the interior and exterior angles. Students make conjectures about the relationship between the measure of an exterior angle and the other two angles of a triangle. (The measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles) and the sum of the exterior angles (360°). Using these relationships, students use deductive reasoning to find the measure of missing angles.

Students construct parallel lines and a transversal to examine the relationships between the created angles. Students recognize vertical angles, adjacent angles and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles. Using these relationships, students use deductive reasoning to find the measure of missing angles.

Students construct various triangles having line segments of different lengths but with two corresponding congruent angles. Comparing ratios of sides will produce a constant scale factor, meaning the triangles are similar.

- Examples: Students can informally prove relationships with transversals.

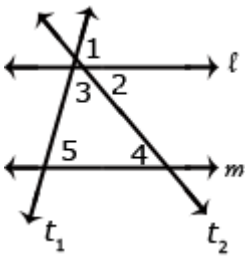
  Show that $m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ$ if l and *m* are parallel lines and $t_1$ & $t_2$ are transversals.

  $\angle 1 + \angle 2 + \angle 3 = 180^\circ$. Angle 1 and Angle 5 are congruent because they are corresponding angles ( $\angle 5 \cong \angle 1$ ). $\angle 1$ can be substituted for $\angle 5$.

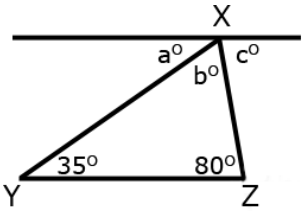
  Continued next page

$\angle 4 \cong \angle 2$ : because alternate interior angles are congruent.
$\angle 4$ can be substituted for $\angle 2$

Therefore m $\angle 3$ + m $\angle 4$ + m $\angle 5$ = $180^\circ$




Students can informally conclude that the sum of a triangle is 180º (the angle-sum theorem) by applying their understanding of lines and alternate interior angles. In the figure below, line x is parallel to line *yz*:




$\overline{X} \parallel \overline{YZ}$

Angle *a* is 35º because it alternates with the angle inside the triangle that measures 35º. Angle *c* is 80º because it alternates with the angle inside the triangle that measures 80º. Because lines have a measure of 180º, and angles *a* + *b* + *c* form a straight line, then angle *b* must be 65 º (180 – 35 + 80 = 65). Therefore, the sum of the angles of the triangle are 35º + 65 º + 80 º

**<u>Instructional Strategies</u>**

*See* **8.G.1.**

**<u>Common Misconceptions:</u>**
*See* **8.G.1.**

Domain: **Geometry (G)**

Cluster: Understand and apply the Pythagorean Theorem.

Standard: **8.G.6.** Explain a proof of the Pythagorean Theorem and its converse.

**Standards for Mathematical Practice (MP):**
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:
This cluster is connected to the Grade 8 Critical Area of Focus #3, **Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.**

**Explanations and Examples**
**8.G.6** Students explain the Pythagorean Theorem as it relates to the area of squares coming off of all sides of a right triangle. Students also understand that given three side lengths with this relationship forms a right triangle.

Students should verify, using a model, that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students should also understand that if the sum of the squares of the 2 smaller legs of a triangle is equal to the square of the third leg, then the triangle is a right triangle.

**Instructional Strategies**
Previous understanding of triangles, such as the sum of two side measures is greater than the third side measure, angles sum, and area of squares, is furthered by the introduction of unique qualities of right triangles. Students should be given the opportunity to explore right triangles to determine the relationships between the measures of the legs and the measure of the hypotenuse. Experiences should involve using grid paper to draw right triangles from given measures and representing and computing the areas of the squares on each side. Data should be recorded in a chart such as the one below, allowing for students to conjecture about the relationship among the areas within each triangle.

| Triangle | Measure of Leg 1 | Measure of Leg 2 | Area of Square on Leg 1 | Area of Square on Leg 2 | Area of Square on Hypotenuse |
|---|---|---|---|---|---|
| 1 | | | | | |

Students should then test out their conjectures, then explain and discuss their findings. Finally, the Pythagorean Theorem should be introduced and explained as the pattern they have explored. Time should be spent analyzing several proofs of the Pythagorean Theorem to develop a beginning sense of the process of deductive reasoning, the significance of a theorem, and the purpose of a proof. Students should be able to justify a simple proof of the Pythagorean Theorem or its converse.

Previously, students have discovered that not every combination of side lengths will create a triangle. Now they need situations that explore using the Pythagorean Theorem to test whether or not side lengths represent right triangles. (Recording could include Side length *a*, Side length *b*, Sum of $a^2 + b^2$, $c^2$, $a^2 + b^2 = c^{2,}$ Right triangle? Through these opportunities, students should realize that there are Pythagorean (triangular) triples such as (3, 4, 5), (5, 12, 13), (7, 24, 25), (9, 40, 41) that always create right triangles, and that their multiples also form right triangles. Students should see how similar triangles can be used to find additional triples. Students should be able to explain why a triangle is or is not a right triangle using the Pythagorean Theorem.
The Pythagorean Thereom should be applied to finding the lengths of segments on a coordinate grid, especially those segments that do not follow the vertical or horizontal lines, as a means of discussing the determination of distances between points. Contextual situations, created by both the students and the teacher, that apply the Pythagorean theorem and its converse should be provided. For example, apply the concept of similarity to determine the height of a tree using the ratio between the student's height and the length of the student's shadow. From that, determine the distance from the tip of the tree to the end of its shadow and verify by comparing to the computed distance from the top of the student's head to the end of the student's shadow, using the ratio calculated previously. Challenge students to identify additional ways that the Pythagorean Theorem is or can be used in real world situations or mathematical problems, such as finding the height of something that is difficult to physically measure, or the diagonal of a prism.

| |
|---|
| Domain: **Geometry (G)** |
| Cluster: Understand and apply the Pythagorean Theorem. |
| Standard: **8.G.7.** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. |
| **Standards for Mathematical Practice (MP):**<br>MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.4. Model with mathematics.<br>MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure. |
| **Connections**:<br>*See* **8.G.6.** |
| **Explanations and Examples**<br>**8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.<br><br>Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean triplets.<br><br>**Instructional Strategies**<br>*See* **8.G.6.** |

| |
|---|
| Domain: **Geometry** |
| Cluster: Understand and apply the Pythagorean Theorem. |
| Standard: **8.G.8.** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. |
| **Standards for Mathematical Practice (MP):**<br>MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.4. Model with mathematics.<br>MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure. |
| **Connections**:<br>*See* **8.G.6.** |
| **Explanations and Examples**<br>**8.G.8** One application of the Pythagorean Theorem is finding the distance between two points on the coordinate plane. Students build on work from 6th grade (finding vertical and horizontal distances on the coordinate plane) to determine the lengths of the legs of the right triangle drawn connecting the points. Students understand that the line segment between the two points is the length of the hypotenuse. The use of the distance formula is not an expectation.<br><br>Example:<br>• Students will create a right triangle from the two points given (as shown in the diagram below) and then use the Pythagorean Theorem to find the distance between the two given points.<br><br>**Instructional Strategies**<br><br>*See* **8.G.6.** |

Domain: **Geometry (G)**

Cluster: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres

Standard: **8.G.9.** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

**Standards for Mathematical Practice (MP):**

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

**Connections**:

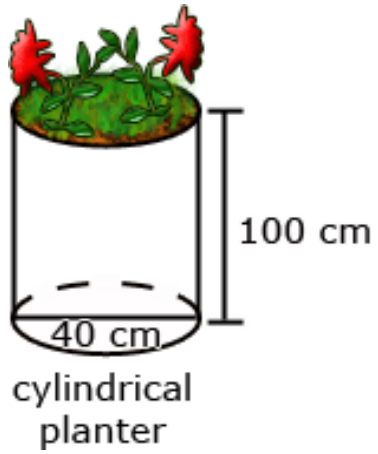
This cluster is connected to the Grade 8 Critical Area of Focus #3, **Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.**

**Explanations and Examples**

**8.G.9** Students build on understandings of circles and volume from 7th grade to find the volume of cylinders, cones and spheres. Students understand the relationship between the volume of a) cylinders and cones and b) cylinders and spheres to the corresponding formulas.

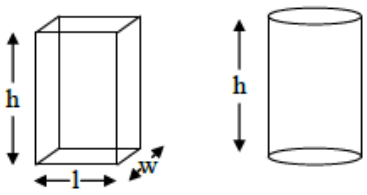
Example:

- James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter's volume.




**Instructional Strategies**

Begin by recalling the formula, and its meaning, for the volume of a right rectangular prism: $V = l \times w \times h$. Then ask students to consider how this might be used to make a conjecture about the volume formula for a cylinder:

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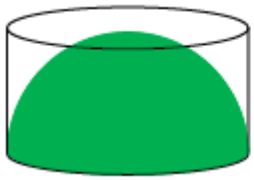
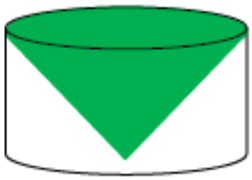
Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a "base" times the height, and so because the area of the base of a cylinder is pi $r^2$ the volume of a cylinder is $Vc = \pi r^2 h$.
To motivate the formula for the volume of a cone, use cylinders and cones with the same base and height. Fill the cone with rice or water and pour into the cylinder. Students will discover/experience that 3 cones full are needed to fill the cylinder. This non-mathematical derivation of the formula for the volume of a cone, $V = 1/3\ \pi r^2 h$, will help most students remember the formula.
In a drawing of a cone inside a cylinder, students might see that that the triangular cross-section of a cone is 1/2 the rectangular cross-section of the cylinder. Ask them to reason why the volume (three dimensions) turns out to be *less* than 1/2 the volume of the cylinder. It turns out to be 1/3.





For the volume of a sphere, it may help to have students visualize a hemisphere "inside" a cylinder with the same height and "base." The radius of the circular base of the cylinder is also the radius of the sphere and the hemisphere. The area of the "base" of the cylinder and the area of the section created by the division of the sphere into a hemisphere is $\pi r^2$. The height of the cylinder is also r so the volume of the cylinder is $\pi r^3$. Students can see that the volume of the hemisphere is less than the volume of the cylinder and more than half the volume of the cylinder. Illustrating this with concrete materials and rice or water will help students see the relative difference in the volumes. At this point, students can reasonably accept that the volume of the hemisphere of radius r is $2/3\ \pi r^3$ and therefore volume of a sphere with radius r is twice that or $4/3\ \pi r^3$. There are several websites with explanations for students who wish to pursue the reasons in more detail. (Note that in the pictures above, the hemisphere and the cone together fill the cylinder.)
Students should experience many types of real-world applications using these formulas. They should be expected to explain and justify their solutions.

**Common Misconceptions:**
A common misconception among middle grade students is that "volume" is a "number" that results from "substituting" other numbers into a formula. For these students there is no recognition that "volume" is a measure – related to the amount of space occupied. If a teacher discovers that students do not have an understanding of volume as a measure of space, it is important to provide opportunities for hands on experiences where students "fill" three dimensional objects. Begin with right"- rectangular prisms and fill them with cubes will help students understand why the units for volume are cubed. See Cubes http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

## Extended Standards:

**The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "...materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication.**

| 8th Grade Mathematics<br>Geometry | | |
|---|---|---|
| **Common Core State Standards** | **Essence** | **Extended Common Core** |
| **Understand congruence and similarity using physical models, transparencies, or geometry software.**<br>**Cluster**<br>1. Verify experimentally the properties of rotations, reflections, and translations:<br>a. Lines are taken to lines, and line segments to line segments of the same length.<br>b. Angles are taken to angles of the same measure.<br>c. Parallel lines are taken to parallel lines.<br>2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.<br>3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.<br>4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.<br>5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* | **Congruency** | **Understand congruence using physical models.**<br>**Cluster**<br>1. Describe the attributes of figures: number of faces or edges, equal sizes of sides and number angles.<br>2. Understand congruence in polygons with different orientations (proximity, position, directions and turns). |
| **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**<br>**Cluster**<br>6. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. | **Measure volume** | **Solve real-world and mathematical problems involving volume of right rectangular prisms.**<br>**Cluster**<br>3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.<br>a. A cube with side length 1 unit called a "unit cube" is said to have "one cubic unit" of volume, and can be used to measure volume.<br>b. Understand volume is the number of cubes used to fill a solid figure without gaps and overlaps.<br>4. Measure volumes of right rectangular figures by counting unit cubes. |

**Domain: Statistics and Probability (SP)**

Cluster: Investigate patterns of association in bivariate data.

Standard: **8.SP.1.** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

This Cluster is connected to the grade 8 Critical Area of Focus #1, **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations.**

**Explanations and Examples**

**8.SP.1** Bivariate data refers to two variable data, one to be graphed on the x-axis and the other on the y-axis. Students represent measurement (numerical) data on a scatter plot, recognizing patterns of association. These patterns may be linear (positive, negative or no association) or non-linear

Students build on their previous knowledge of scatter plots examine relationships between variables. They analyze scatterplots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets. (http://nces.ed.gov/nceskids/createagraph/default.aspx)

Examples:

- Data for 10 students' Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Math | 64 | 50 | 85 | 34 | 56 | 24 | 72 | 63 | 42 | 93 |
| Science | 68 | 70 | 83 | 33 | 60 | 27 | 74 | 63 | 40 | 96 |

- Data for 10 students' Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Math score | 64 | 50 | 85 | 34 | 56 | 24 | 72 | 63 | 42 | 93 |
| Dist from school (miles) | 0.5 | 1.8 | 1 | 2.3 | 3.4 | 0.2 | 2.5 | 1.6 | 0.8 | 2.5 |

- Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

| Number of staff | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| Average time to fill order (seconds) | 180 | 138 | 120 | 108 | 96 | 84 |

- The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

| Date | 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 | 2005 |
|---|---|---|---|---|---|---|---|---|
| Life Expectancy (in years) | 70.8 | 72.6 | 73.7 | 74.7 | 75.4 | 75.8 | 76.8 | 77.4 |

**Instructional Strategies**

Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. Students will extend their descriptions and understanding of variation to the graphical displays of bivariate data.
Scatter plots are the most common form of displaying bivariate data in Grade 8. Provide scatter plots and have students practice informally finding the *line of best fit*. Students should create and interpret scatter plots, focusing on outliers, positive or negative association, linearity or curvature. By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. Have students use a graphing calculator to determine a linear regression and discuss how this relates to the graph. Students should informally draw a line of best fit for a scatter plot and informally measure the strength of fit. Discussion should include "What does it mean to be above the line, below the line?"
The study of the line of best fit ties directly to the algebraic study of slope and intercept. Students should interpret the slope and intercept of the line of best fit in the context of the data. Then students can make predictions based on the line of best fit.

**Common Misconceptions:**
Students may believe:
Bivariate data is only displayed in scatter plots. 8.SP.4 in this cluster provides the opportunity to display bivariate, categorical data in a table.
In general, students think there is only one correct answer in mathematics. Students may mistakenly think their lines of best fit for the same set of data will be exactly the same. Because students are informally drawing lines of best fit, the lines will vary slightly. To obtain the exact line of best fit, students would use technology to find the line of regression.

Domain: **Statistics and Probability**

Cluster: Investigate patterns of association in bivariate data.

Standard: **8.SP.2.** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

*See* **8.SP.1.**

**Explanations and Examples**

**8.SP.2** Students understand that a straight line can represent a scatter plot with linear association. The most appropriate linear model is the line that comes closest to most data points. The use of linear regression is not expected.

Examples:

- The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

| Miles Traveled | 0 | 75 | 120 | 160 | 250 | 300 |
|---|---|---|---|---|---|---|
| Gallons Used | 0 | 2.3 | 4.5 | 5.7 | 9.7 | 10.7 |

**Instructional Strategies**

*See* **8.SP.1.**

Domain: **Statistics and Probability**

Cluster: Investigate patterns of association in bivariate data.

Standard: **8.SP.3.** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height*.

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

*See* **8.SP.1.**

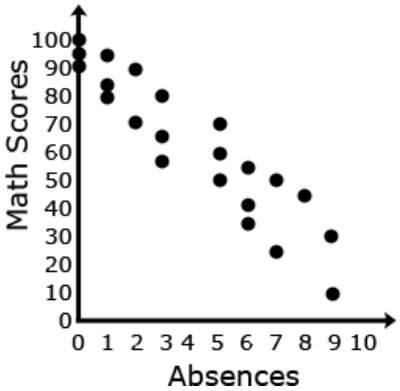
**Explanations and Examples**

**8.SP.3** Linear models can be represented with a linear equation. Students interpret the slope and y-intercept of the line in the context of the problem.

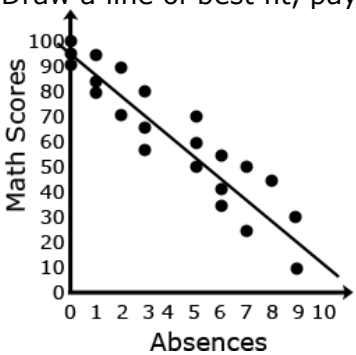
Examples:

- 1. Given data from students' math scores and absences, make a scatterplot.

| Absences | Math Scores |
|---|---|
| 3 | 65 |
| 5 | 50 |
| 1 | 95 |
| 1 | 85 |
| 3 | 80 |
| 6 | 34 |
| 5 | 70 |
| 3 | 56 |
| 0 | 100 |
| 7 | 24 |
| 8 | 45 |
| 2 | 71 |
| 9 | 30 |
| 0 | 95 |
| 6 | 55 |
| 6 | 42 |
| 2 | 90 |
| 0 | 92 |
| 5 | 60 |
| 7 | 50 |
| 9 | 10 |
| 1 | 80 |




2. Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.




- 3. From the line of best fit, determine an approximate linear equation that models the given data (about y = $-\frac{25}{3}x+95$)
- 4. Students should recognize that 95 represents the y intercept and $-\frac{25}{3}$ represents the slope of the line.

5. Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4 absences should expect to receive a math score of about 62. They can then compare this value to their line.

**Instructional Strategies**

*See* **8.SP.1.**

Domain: **Statistics and Probability**

Cluster: Investigate patterns of association in bivariate data.

Standard: **8.SP.4**. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

**Standards for Mathematical Practice (MP):**

MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

**Connections**:

*See* **8.SP.1.**

**Explanations and Examples**

**8.SP.4** Students recognize that categorical data can also be described numerically through the use of a two-way table. A two-way table is a table that shows categorical data classified in two different ways. The frequency of the occurrences are used to identify possible associations between the variables. For example, a survey was conducted to determine if boys eat breakfast more often than girls. The following table shows the results:

| | Male | Female |
|---|---|---|
| Eat breakfast on a regular basis | 190 | 110 |
| Do not eat breakfast on a regular basis | 130 | 165 |

Students can use the information from the table to compare the probabilities of males eating breakfast (190 of the 320 males à 59%) and females eating breakfast (110 of the 375 females à29%) to answer the question. From this data, it can be determined that males do eat breakfast more regularly than females.

Example:

- The table illustrates the results when 100 students were asked the survey questions: Do you have a curfew? and Do you have assigned chores? Is there evidence that those who have a curfew also tend to have chores?

| | | Curfew | |
|---|---|---|---|
| | | Yes | No |
| Chores | Yes | 40 | 10 |
| | No | 10 | 40 |

Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores.



| | | | | | |
|---|---|---|---|---|---|
| **8.SP.4** | | | | | |

## Extended Standards:

**The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities Non-Regulatory Guidance* states, "…materials should show a clear link to the content standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication.**

| 8th Grade Mathematics<br>Statistics and Probability | | | | |
|---|---|---|---|---|
| **Common Core State Standards** | | **Essence** | **Extended Common Core** | |
| **Investigate patterns of association in bivariate data.** | | **Patterns of association in bivariate data** | **Investigate patterns of association in bivariate data.** | |
| **Cluster** | 1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.<br>2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.<br>3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*<br>4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* | | **Cluster** | 1. Describe trends such as positive, negative or no association given a scatter plot. |