The Pythagorean Theorem

As you've never seen it before

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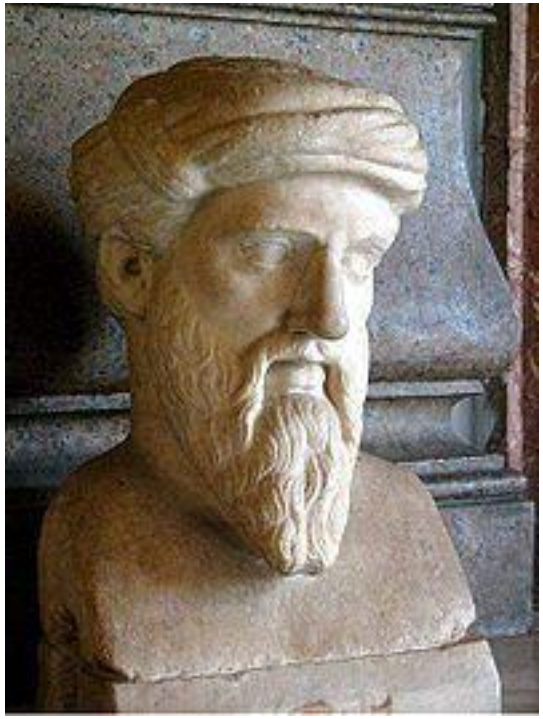
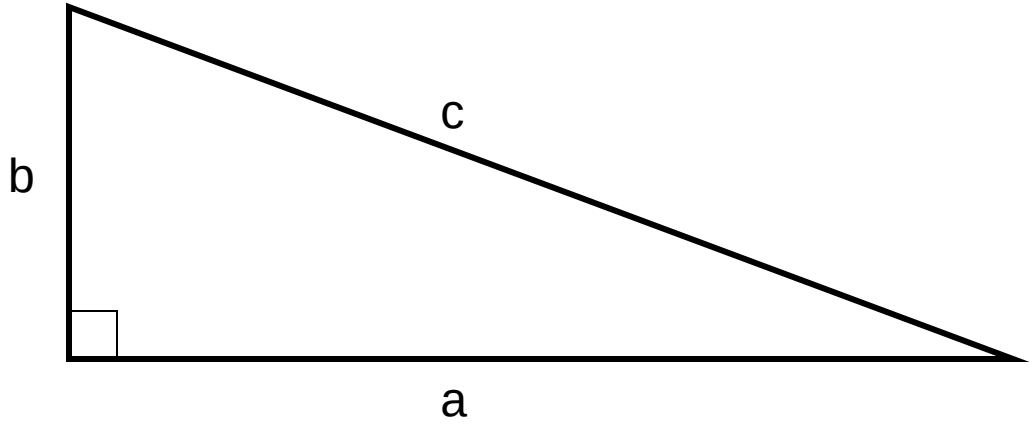
Pythagoras

- 560BCE-480BCE
- Born on the Isle of Samos to a gem cutter named Mnesarchus
- Studied and travelled to Syros and Egypt.
- Founded new society in Croton (Italy)
- After a rebellion moved to Metapontum where he died

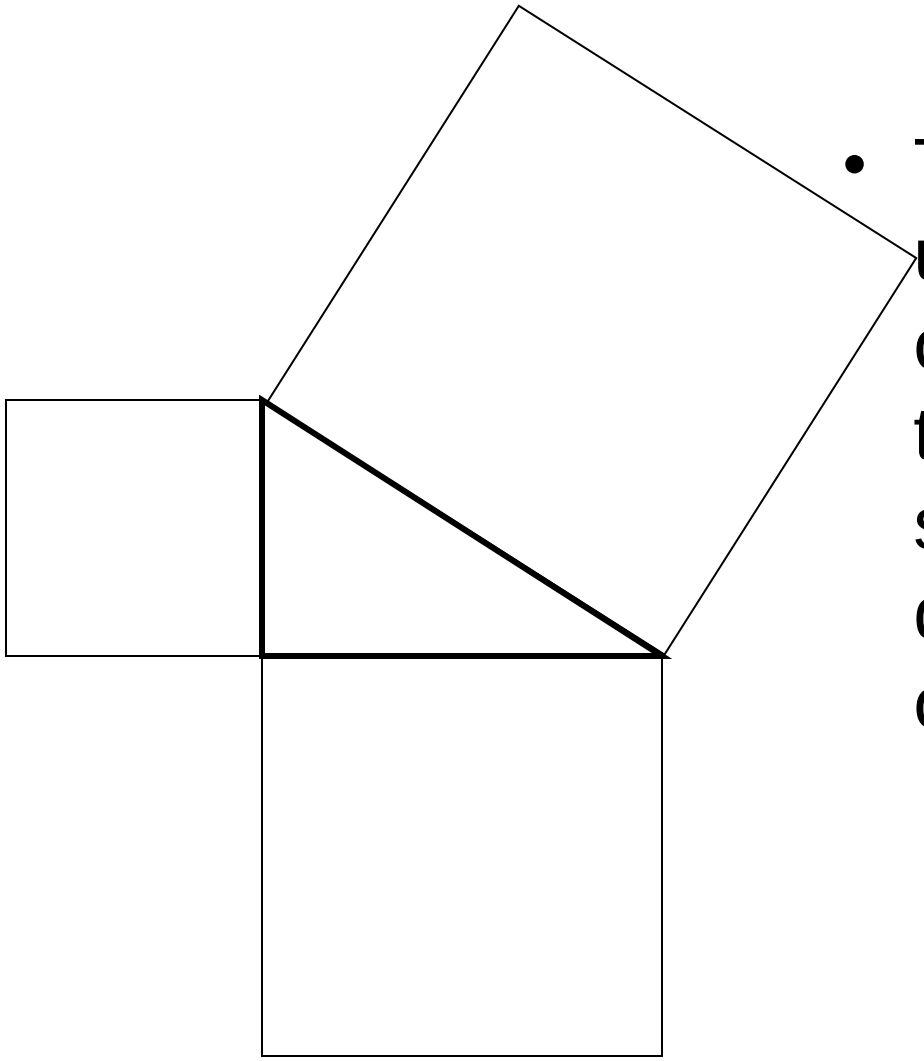


Pythagorean Theorem

- In a right angled triangle with sidelengths a, b and c, where c is opposite the right angle, $a^2+b^2=c^2$.

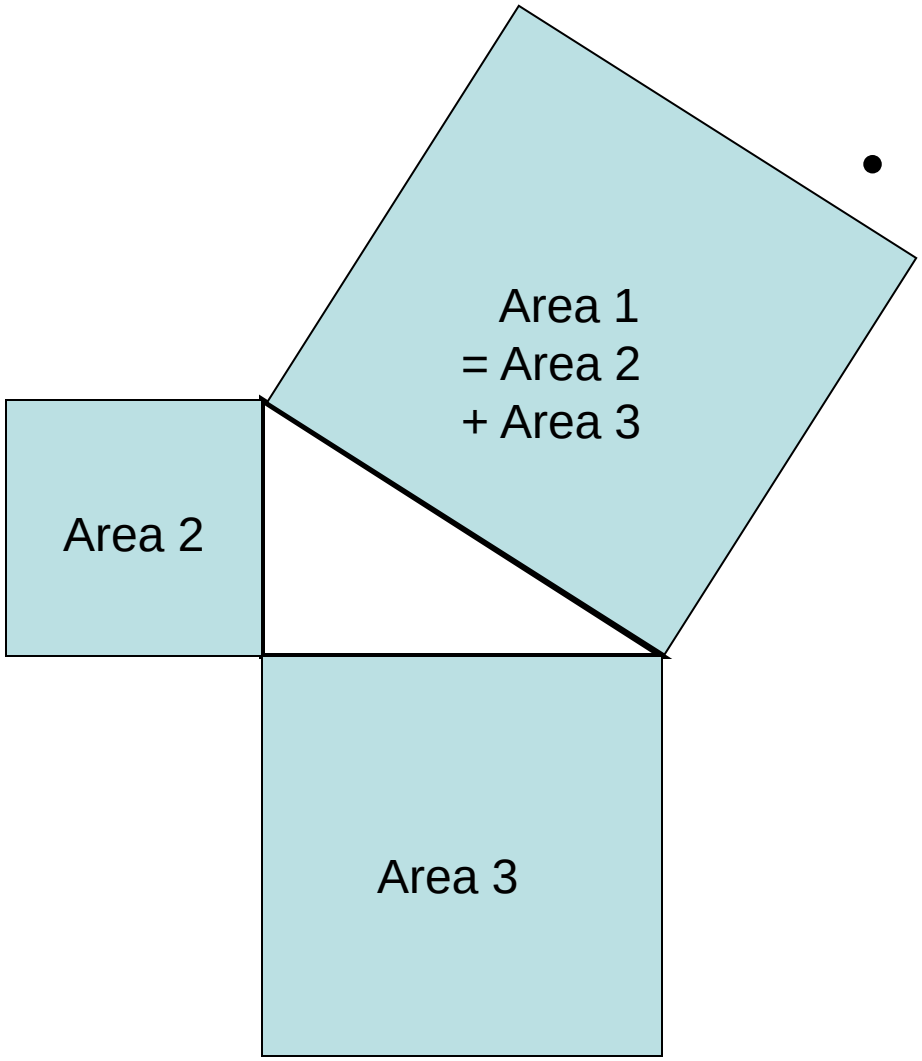


A Nice Theorem from Egypt



- The square described upon the hypotenuse of a right-angled triangle is equal to the sum of the squares described upon the other two sides

A Nice Theorem from Egypt



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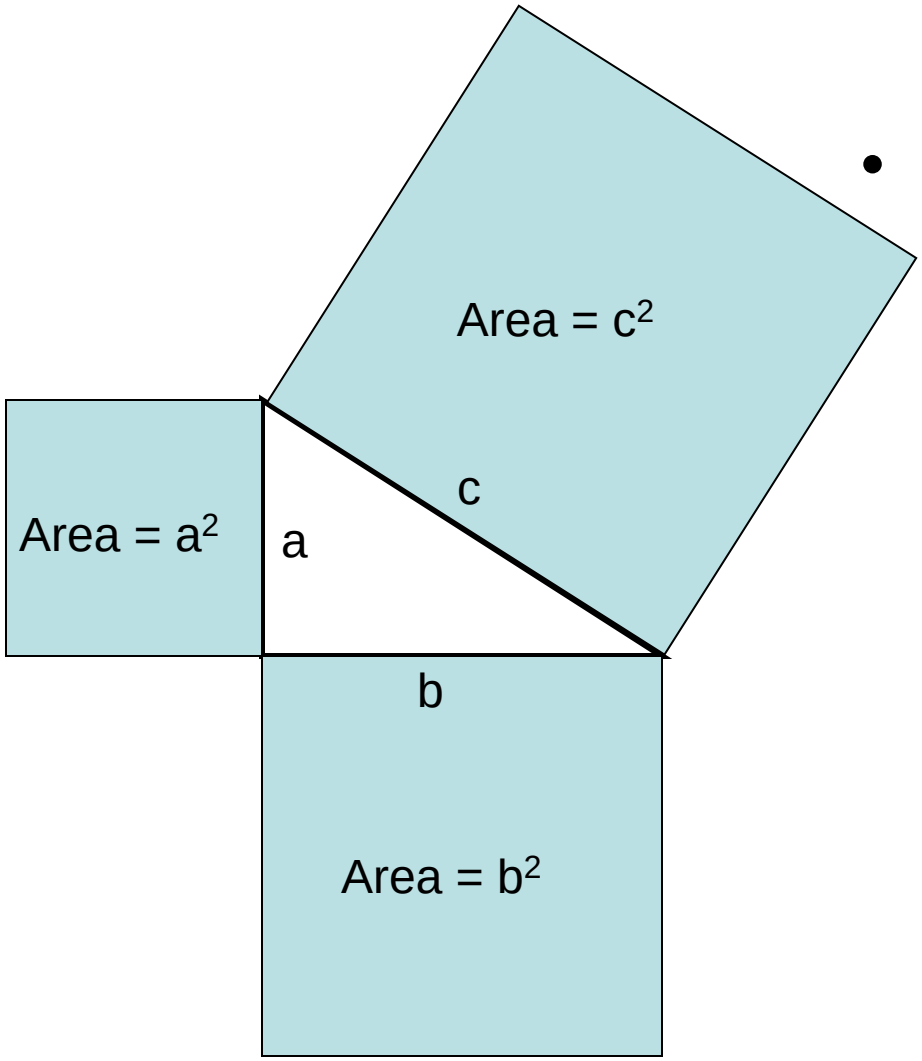
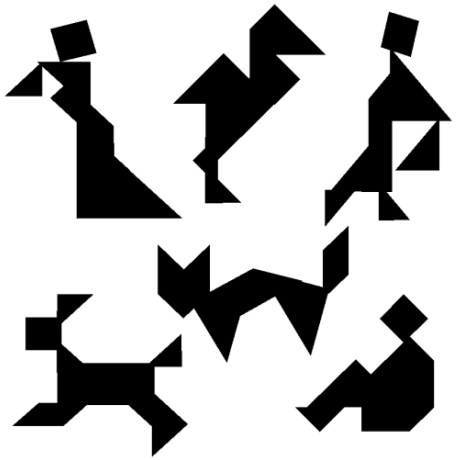
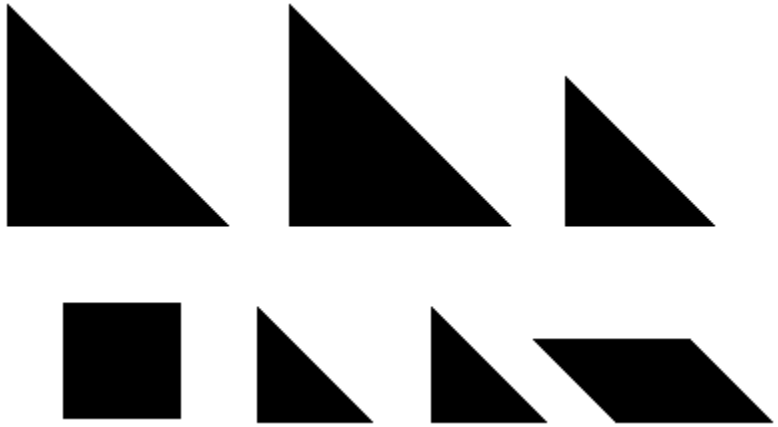


- The area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides $c^2=b^2+a^2$

Tangrams

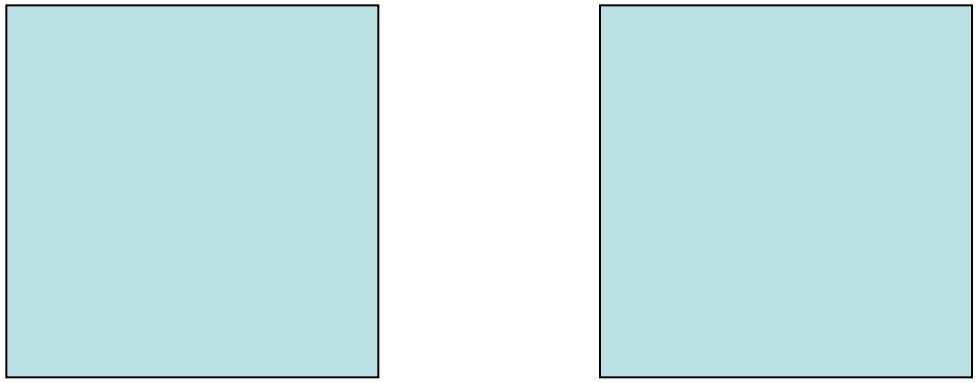




- Chinese Puzzle created about 1800
- First puzzle craze
- Swept the world in the 1820’s

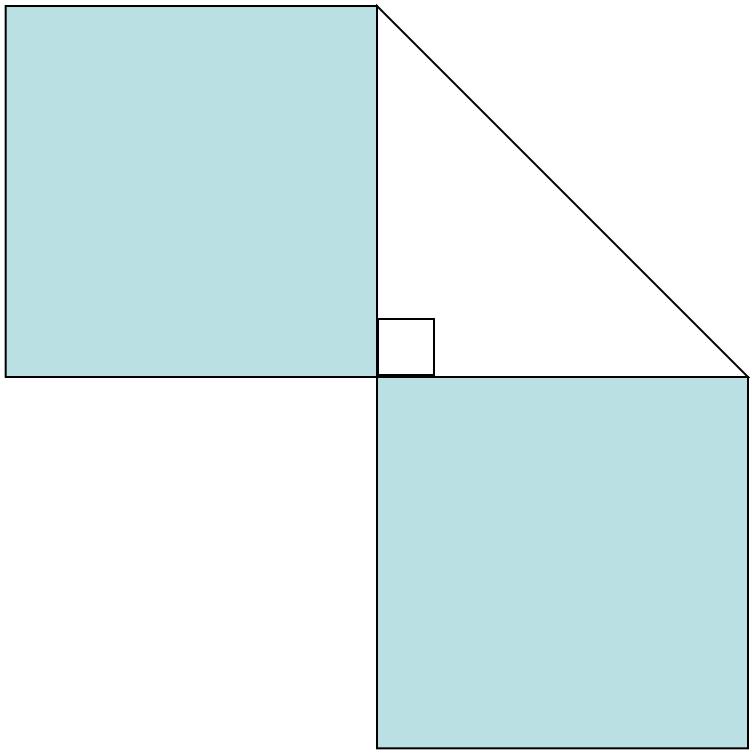
Tangram Proof

- Can you use ALL 7 tangram pieces to make two congruent squares?

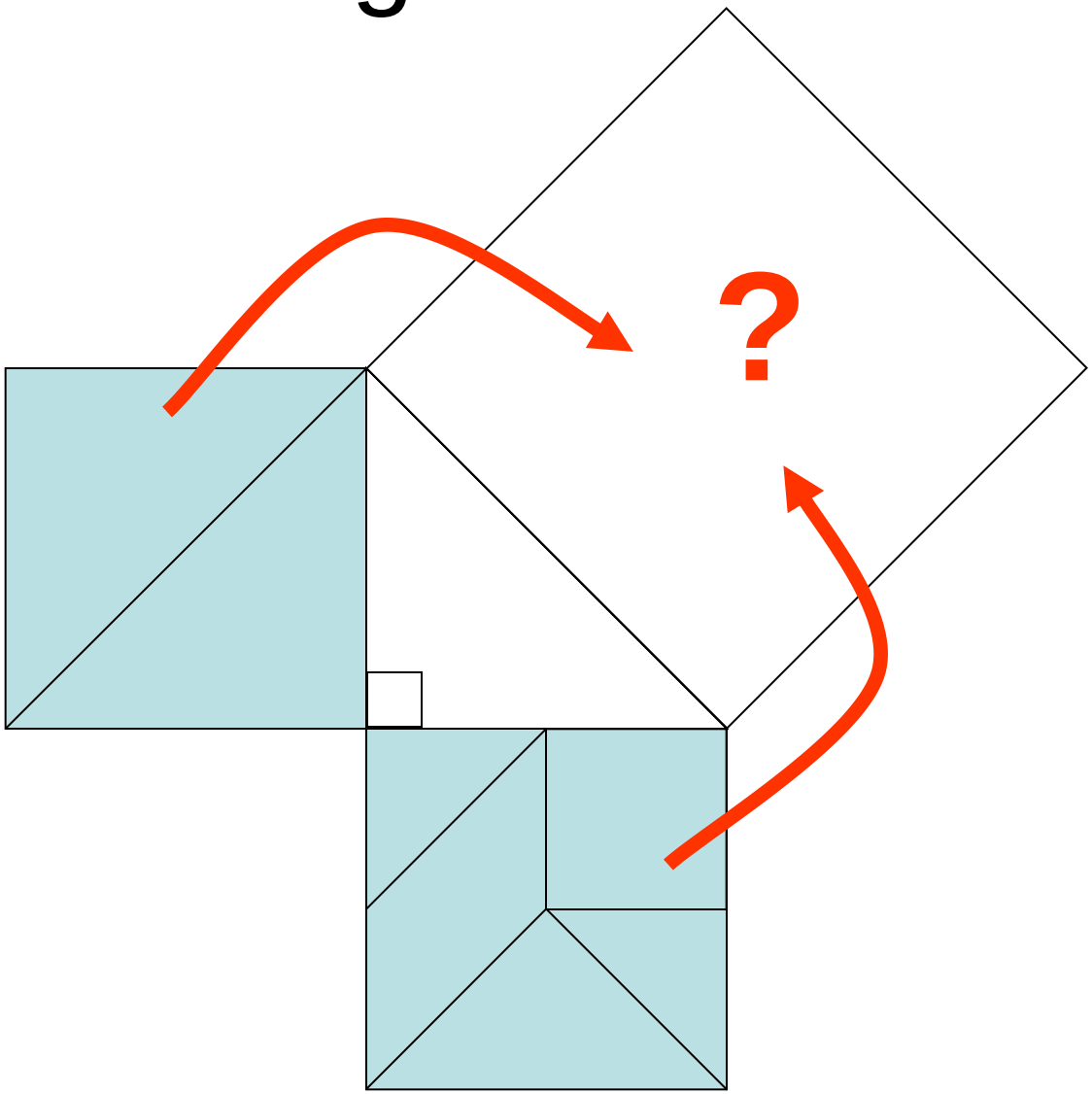


Tangram Proof

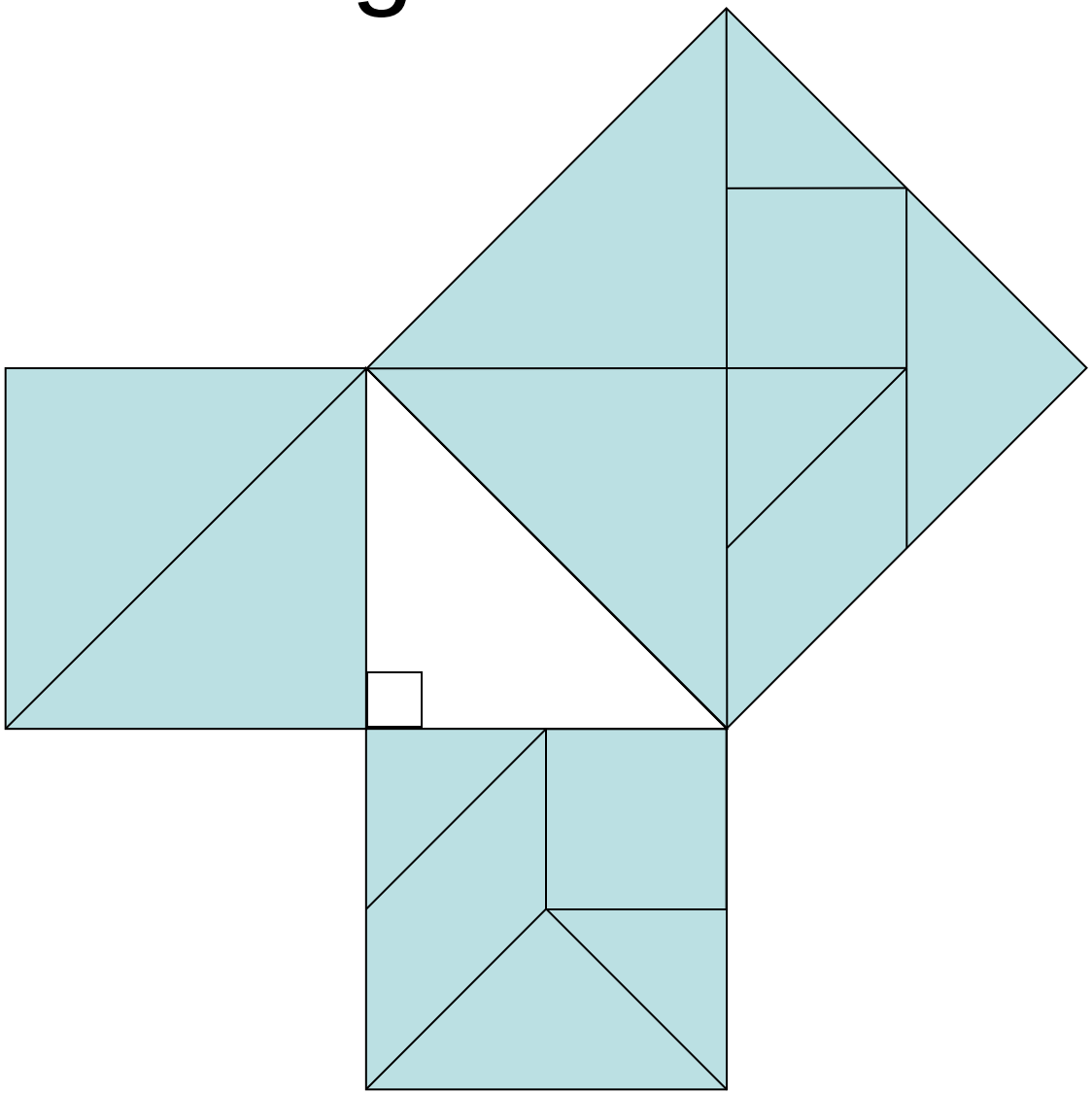
- Now, using the same seven pieces make one square, the square on the hypotenuse.



Tangram Proof

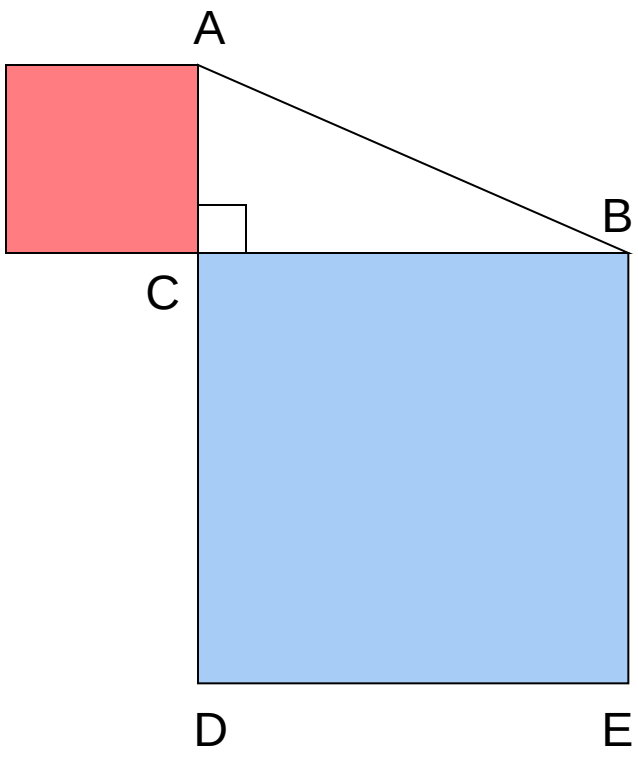


Tangram Proof



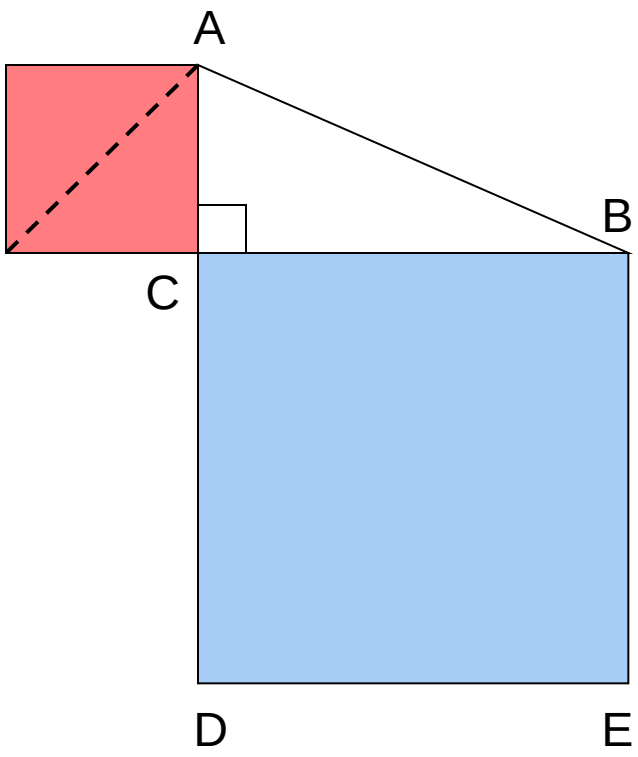
A Dissection Proof Attributed to Liu Hui (300 C.E.)

- Draw any right triangle and the squares on the legs.
- Ensure the smallest square (red) is to the left and above the other square (blue).



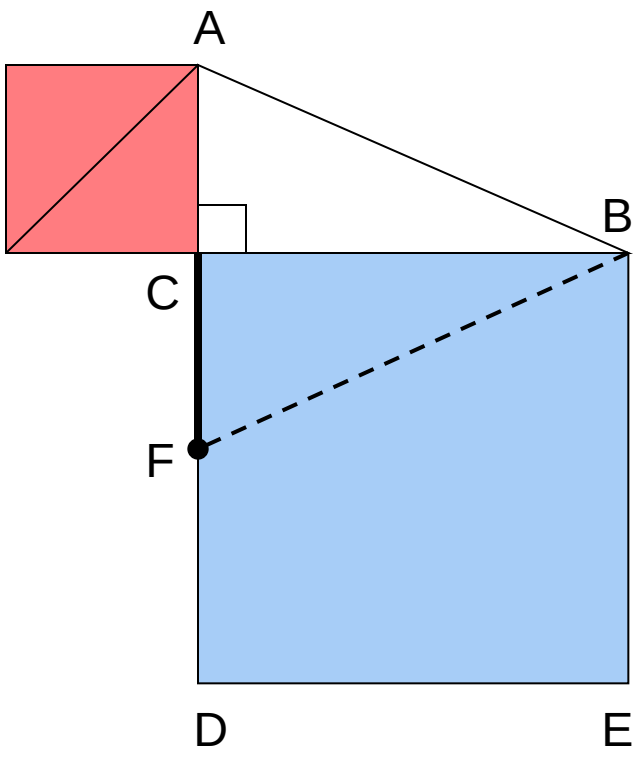
A Dissection Proof Attributed to Liu Hui (300 C.E.)

- Divide the red square along the diagonal.



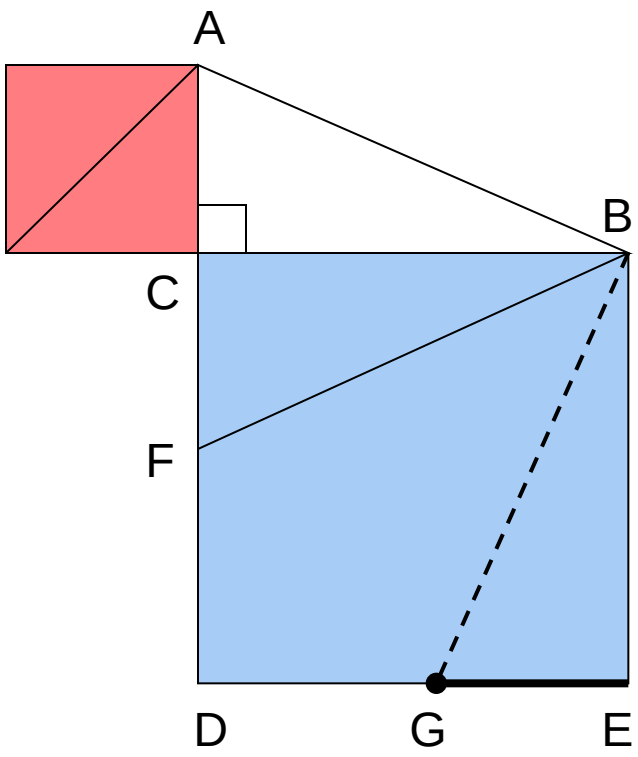
A Dissection Proof Attributed to Liu Hui (300 C.E.)

- Mark off a point F on the left side of the blue square so that AC=FC
- Connect F to B.



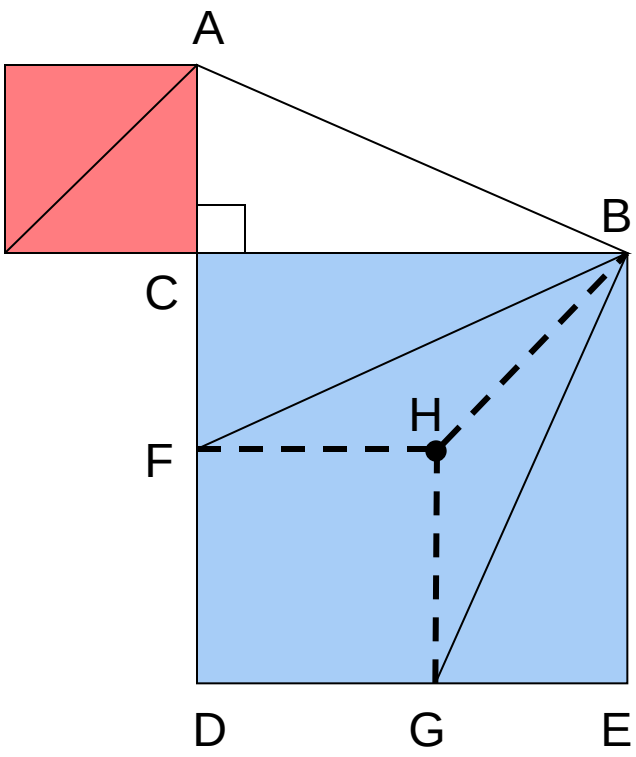
A Dissection Proof Attributed to Liu Hui (300 C.E.)

- Mark off a point G on the bottom of the blue square so that GE=AC
- Connect G to B



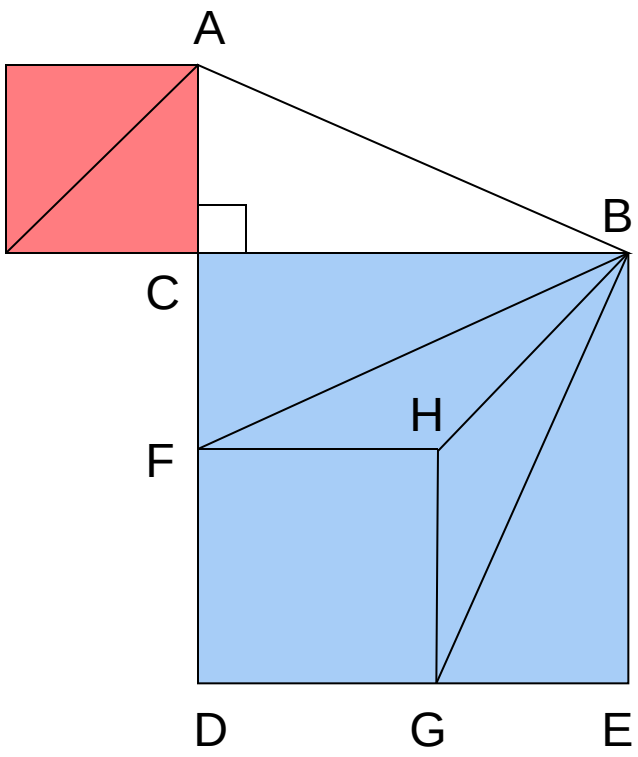
A Dissection Proof Attributed to Liu Hui (300 C.E.)

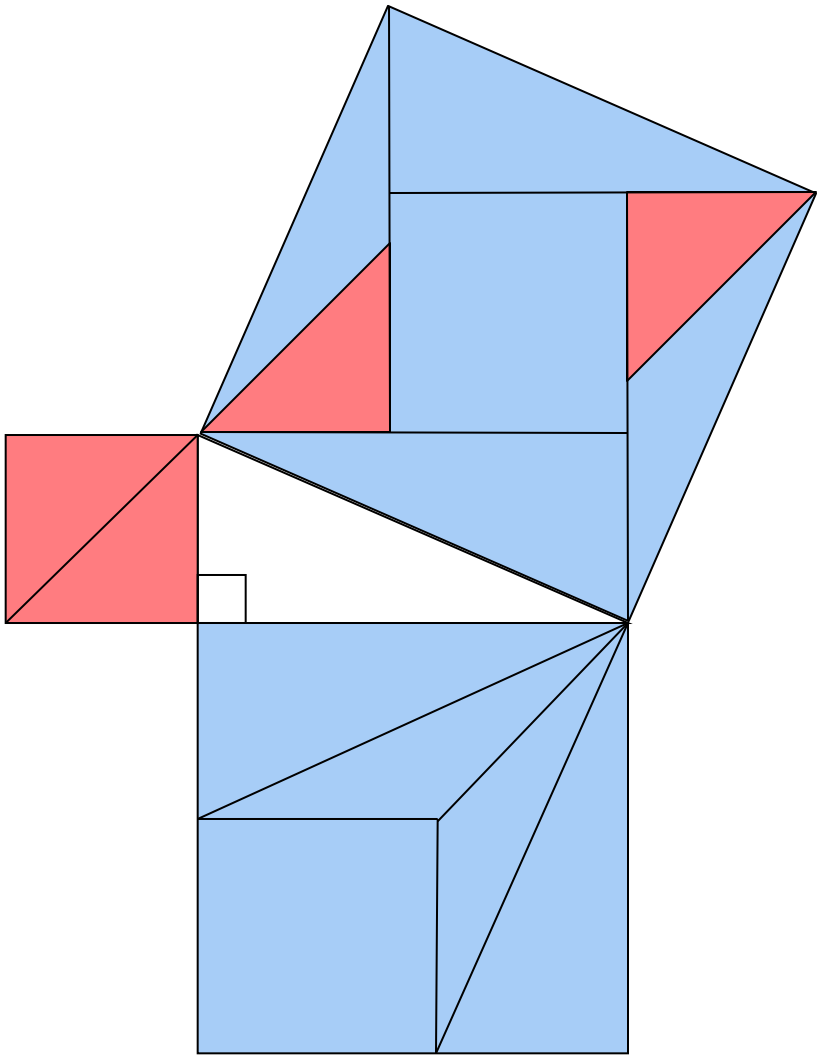
- Construct a point H so that DFGH is a square.
- Connect H to B, F and G.



A Dissection Proof Attributed to Liu Hui (300 C.E.)

- Cut out the red and blue pieces and rearrange them to form the square on the hypotenuse.





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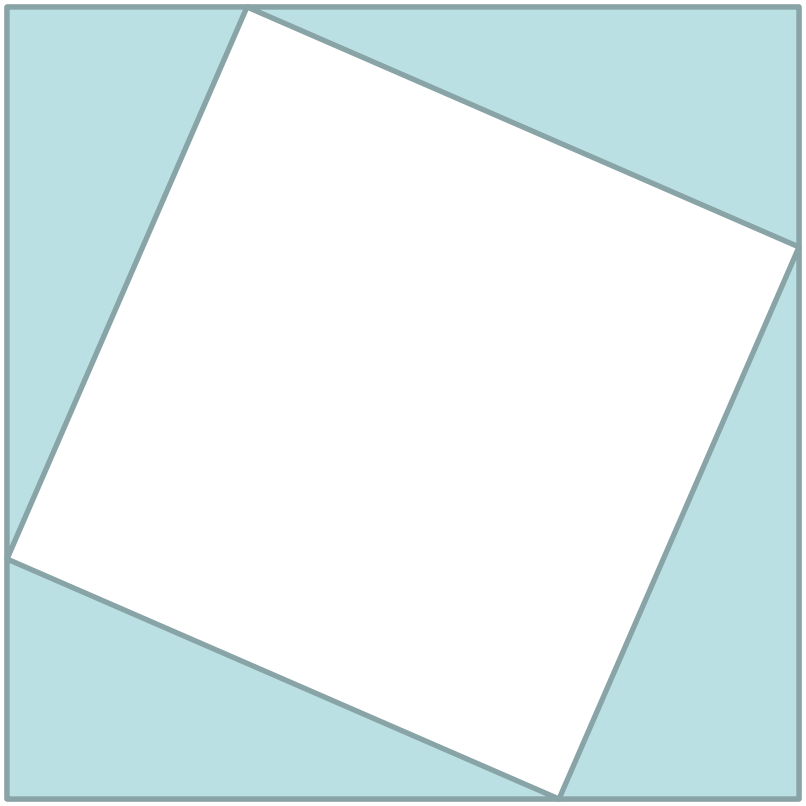
Chou Pei Suan Ching

anonymous, 200 B.C.E.



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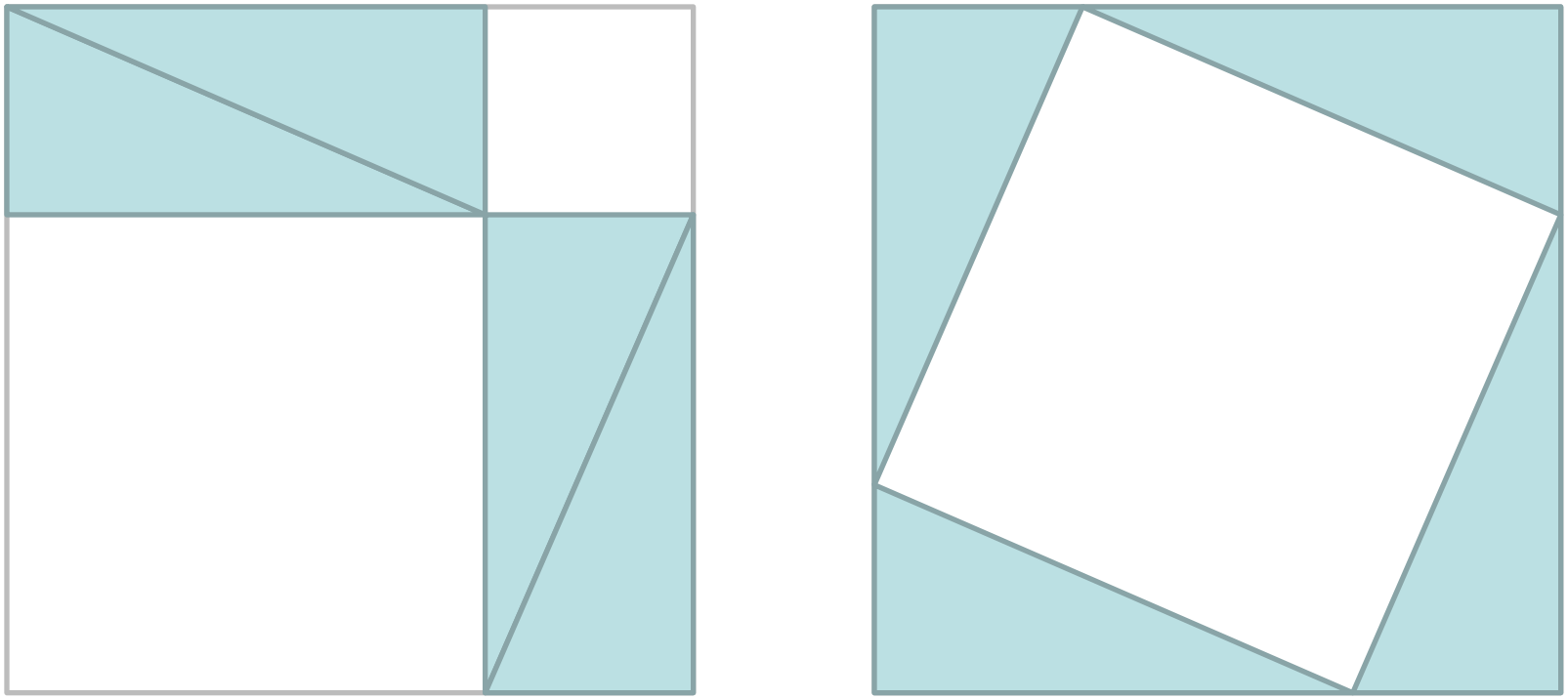

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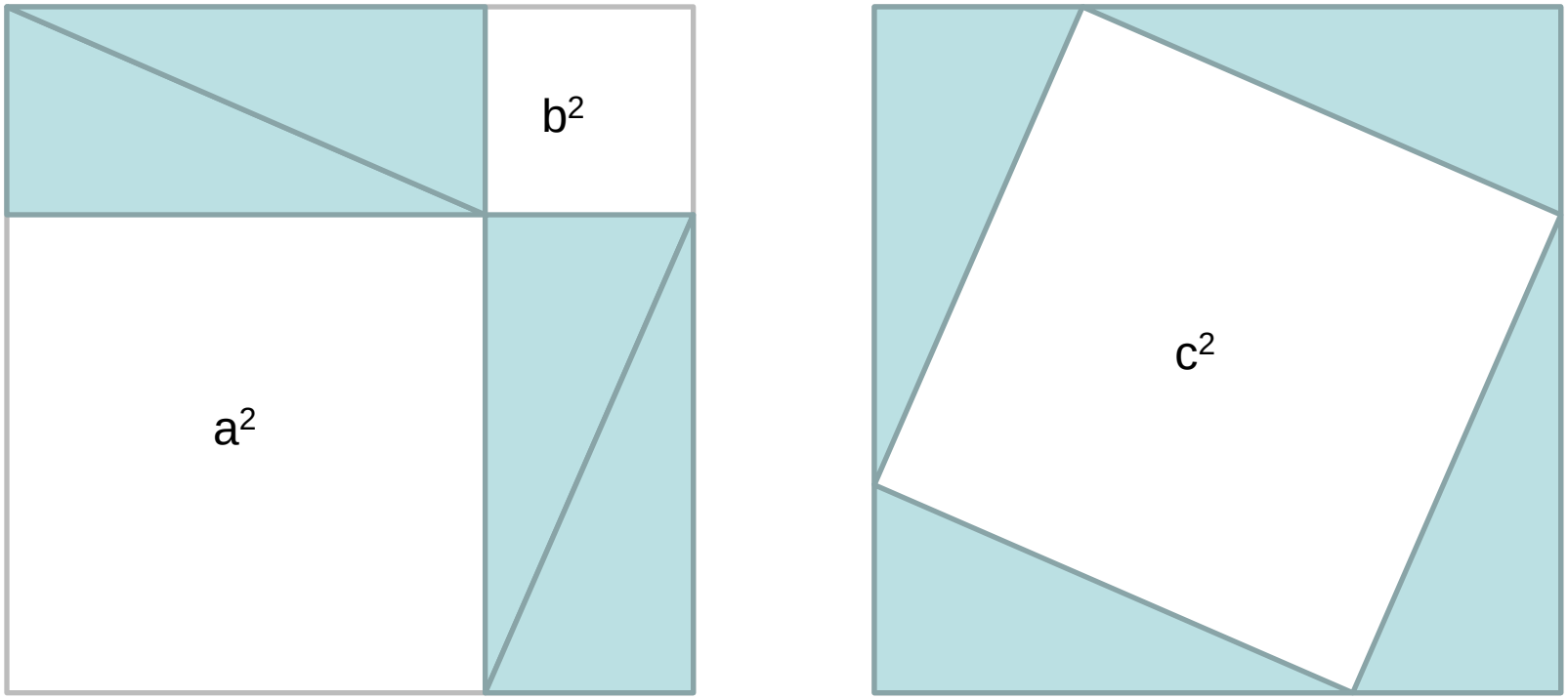
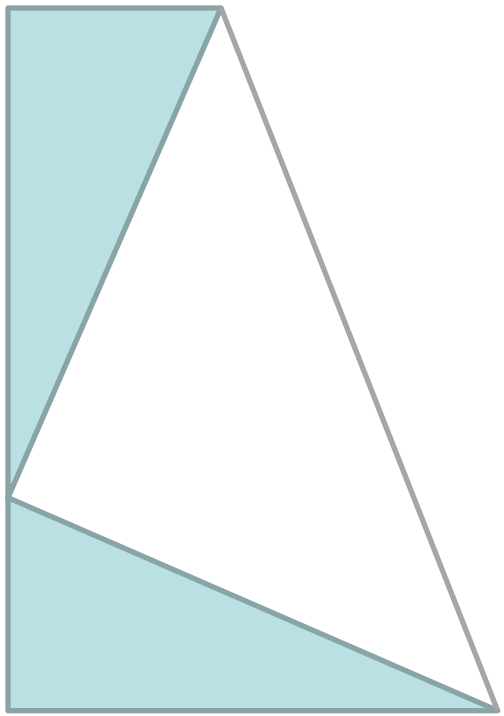
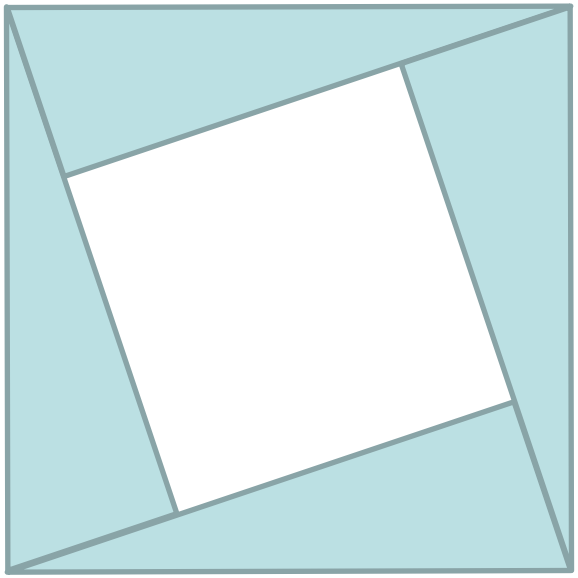
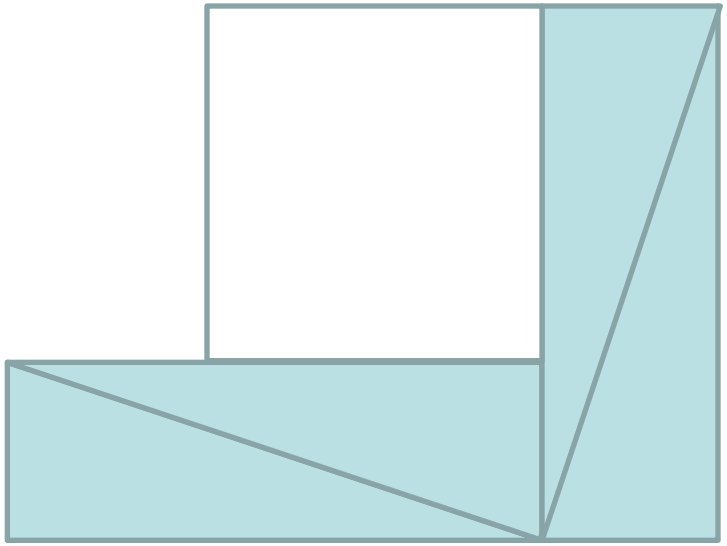


Garfield’s Proof



Bhaskara – BEHOLD!





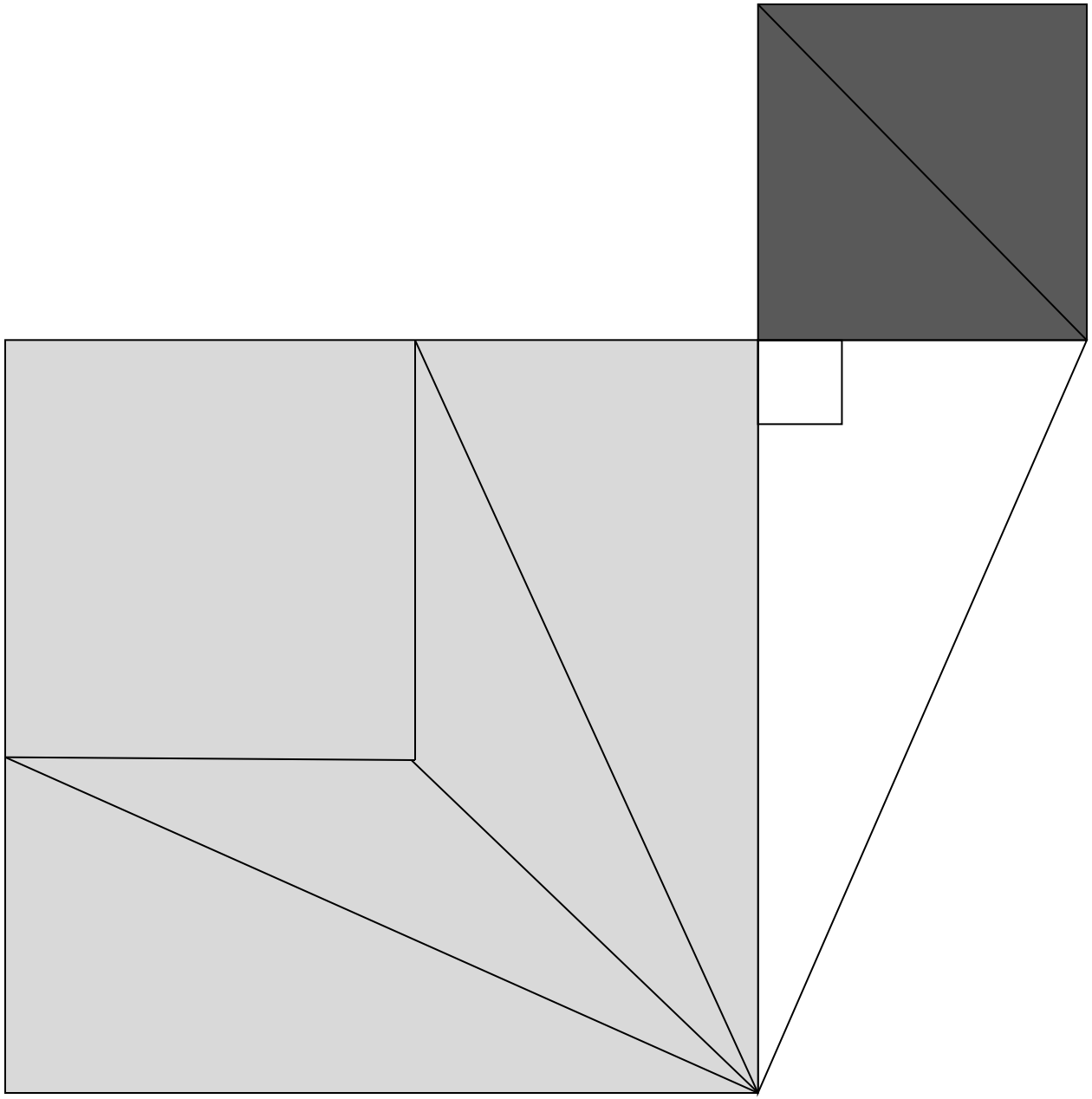
Famous Story

Three Indian women are sitting around in their tipi, just shooting the breeze. The first woman points to the hide she's sitting on and says, "See this hide? My son killed this bear with his own two hands. It's worth quite a bit." The second woman nods, and points to the hide she's sitting on. "This is a bear hide, too, and MY son killed it with his own hands as well. I imagine it's worth at least as much as yours."

Finally, the third, and oldest, woman speaks up. "See this hide?" she cackles. "This is a hippopatamus hide! I killed this hippo with MY own two hands, and it's worth easily what both those bear hides are worth together."

Moral of the Story

- *The squaw on the hippopotamus is equal to the sons of the squaws on the other two hides.*

References

- Meet the Philosophers of Ancient Greece by Patrica O’Grady
- Proof Without Words, Roger B. Nelsen, MAA.
- Proof Without Words II, Roger B. Nelsen, MAA.
- www.cut-the-knot.org/pythagoras