“The Magic Money Machine”

Objectives: Understand the relationship between the slope and y-intercept in all three representations of linear functions: the graph, t-chart and equation.

Standards Addressed: This unit addresses the entire “Functions” domain.
8.F.1 *Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.*

8.F.2 *Compare properties of two function each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).*

8.F.3 *Interpret the equation y=mx+b as defining a linear function, whose graph is a straight line;*

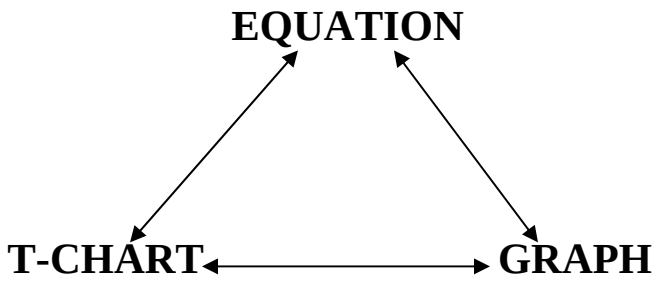
8.F.4 *Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or table of values.*

8.F.5 *Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g where the function is increasing or decreasing, linear or non-linear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.*

Materials:
Graph Paper
Ruler
“What’s My Rule?” Blackline Master
Magic Money Machine Blackline Master
Two different colors of crayons or markers (one for the slope and one for the y-intercept)

Time Line: 3-6 days

Rationale: Students need to understand that there are three representations of linear equations and that they can flexibly go between the representations for different purposes.



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Day 1: T-chart to Equation

Tell the students that you are giving them a magic money machine. It's magic because if you put money into the machine, a different amount might come out. The machine has an LCD screen that shows the machine's settings, but the screen is broken. However, the dial still works so we can change the settings if we want, we just don't know what will happen until we test the machine. The way it works is to put some money into the machine, press the "start" button and see what happens.

SMART Board connection: go to www.harveyshomepage.com and download "Function Man."

Draw one of the t-charts on the board. Tell students that in order to organize what's happening, we will use a t-chart to separate what we put into the machine from what comes out of the machine. Use the suggested scenarios below or make up your own for your students. Use a different t-chart for each new scenario. Be sure to draw students' attention to the repeating pattern in the OUT column (y-values) and document it (this is the slope and we want the students to be able to identify it later). Help students formalize each equation into y = mx + b form. This is explained by telling them "The amount that comes out (y) is equal to the amount that goes in (x) times a certain number (m) plus whatever happened when I didn't put any money into the machine (b) and just pressed the start button." You may need to remind them that the repeated addition in the t-chart is the same thing as multiplication.

Some possible scenarios for the Magic Money Machine:

Scenario 1: *Y = 3X*

I drop in $0 and no money comes out.
I drop in $1 and $3 comes out.
I drop in $2 and $6 comes out.
I drop in $3 and $9 comes out.
How much will come out when I drop in $10? How much will come out when I drop in $100?
What is happening to the money inside the machine?
(help students to generalize it into an equation form on the line below the t-chart)
My friend dropped in some money when I wasn't looking and $33 came out. How much got dropped in? How do you know?

Scenario 2: *Y = 2X + 3*

I drop in $0 and $3 comes out.
I drop in $1 and $5 comes out.
I drop in $2 and $7 comes out.
I drop in $3 and $9 comes out.
How much will come out when I drop in $10? How much will come out when I drop in $100?
What is happening to the money inside the machine?
(help students to generalize it into an equation form on the line below the t-chart)
My friend dropped in some money when I wasn't looking and $43 came out. How much got dropped in? How do you know?

Scenario 3: Y = 5X -2

I drop in $0 and a pair of handcuffs come out and lock me to the machine until I pay it $2!!
I drop in $1 and $3 comes out.
I drop in $2 and $8 comes out.
I drop in $3 and $13 comes out.
How much will come out when I drop in $10? How much will come out when I drop in $100?
What is happening to the money inside the machine?
(help students to generalize it into an equation form on the line below the t-chart)
My friend dropped in some money when I wasn't looking and $173 came out. How much got dropped in? How do you know?

Scenario 4: Y = -3X-2

I drop in $0 and a pair of handcuffs come out and lock me to the machine until I pay it $2!!
I drop in $1 and I owe the machine $5!
I drop in $2 and I owe the machine $8!
I drop in $3 and I owe the machine $11!
What happens if I drop in $10? What happens when I drop in $100?
What is happening to the money inside the machine?
(help students to generalize it into an equation form on the line below the t-chart)
My friend dropped in some money when I wasn't looking and she owed the machine $62!.
How much got dropped in? How do you know?

Scenario 5: $Y = X^2$

I drop in $0 and no money comes out.
I drop in $1 and $1 comes out.
I drop in $2 and $4 comes out.
I drop in $3 and $9 comes out.
I drop in $4 and $16 comes out.
How much will come out when I drop in $10? How much will come out when I drop in $100?
What is happening to the money inside the machine? How was this different than the others?
(help students to generalize it into an equation form on the line below the t-chart)
My friend dropped in some money when I wasn't looking and $400 came out. How much got dropped in? How do you know?

Scenario 6: Y = 1/2X +4

I drop in $0 and $4.
I drop in $1 and $4.50 comes out.
I drop in $2 and $5 comes out.
I drop in $3 and $5.50 comes out.
I drop in $4 and $6 comes out.
How much will come out when I drop in $10? How much will come out when I drop in $100?
What is happening to the money inside the machine? How was this different than the others?
(help students to generalize it into an equation form on the line below the t-chart)
My friend dropped in some money when I wasn't looking and $40 came out. How much got dropped in? How do you know?

Students can also make up their own scenarios to share with friends.

Day 2: T-Chart to Graph

On day 2, show students how to plot the x and y values on a graph. Refer back to each equation and draw their attention to the coefficient in front of X in the equation and ask them where they see it on the graph. Color-code that number with one of your colors on both the equation and the graph. Ask if they see it in the t-chart and color it in the same color as well. Draw their attention to the constant in the equation and ask them where they see that number in the graph. Color code those with the other color and ask students to find it in the t-chart as well.

Help them to formally define slope and y-intercept. Students should note the distinguishing factors of each (document student responses on chart paper and save it for future days of the unit)

Give them practice t-tables to practice writing equations, then graph them. Students should color-code the slope and y-intercepts on each practice problem.

Day 3: Equation to T-Chart to Graph

Explain to students that when we know the equation, we can always substitute in values (x) and solve for y or vice versa. The easiest way to do this is to pick easy dollar amounts to try. Typically, we choose $0, $1, $2 and $3 as our in values (x) and figure out the out amount (y). Give students practice equations and let them practice creating t-charts. It's a great time to have a discussion about the infinite possibilities of amounts we could plot, including negative values, even though they may not make sense with our money machine scenarios. Once they have created t-charts for each equation, have the students color code the slope and y-intercept on both the equation and on the t-chart. After that, students should graph their lines by plotting the points from the t-chart. Finally, they should color-code the graphed line with the slope and y-intercept.

Day 4: Equation to Graph without a T-Chart

Give students a sample equation and ask them to identify the slope and y-intercept with using the color-coding strategy. Ask students where we saw the slope and y-intercept on the graph in previous days lessons. Tell them that a short-cut to making a t-chart is to simply graph the y-intercept then use what I know about slope to create the next few points. Draw a straight line. They can always fall back on the t-chart to check if their points are correct by plugging in x values and determining whether their y-values were correct. Give students examples of equations to graph. Students should continue to color-code the slope and y-intercept. Be sure to address what happens when Y = a number and X = a number. What happens to the lines in these cases? Why?

Day 5: Graph to Equation

Explain to students that we can also go the other way—if we can see the graph, we can usually derive the slope and y-intercept from it. Using the color-coding system, have students identify the slope and y-intercept in each graph. Look back at previous days' equations and ask students what features are always present in those equations (the y, the = sign, the X and usually a + or – sign). Since those are always present, I can fill in the missing pieces which are the repeating stair-step pattern (slope) which we know gets multiplied by x since repeated addition is the same as multiplication, and the point at which the line crosses the y-axis is the y-intercept. Show students where those belong in the equation. Ask them what they might do if they were asked to identify more points on the graph than what is shown. Give students some graphs to practice writing equations from.

Day 6: Graph to T-Chart

Sometimes the equation may not be readily apparent to the student and/or the student may have a question that asks them to identify a certain number of points on the graph but the picture only shows a couple of them. Help students to use the picture to put the information from the graph into the t-chart. Ask them to identify what would happen when x is 0, -1, -2, and -3. Remind them that we did something similar on day 3 when we choose easy numbers for our t-charts. Explain that these are often numbers we use when we are asked to graph the part of a line that doesn't necessarily have a positive x value, which doesn't make sense for our money machine, but is important for other possible scenarios.

Drop the color-coding system as students become more fluent with identifying the slope and y-intercept. Eventually, they should be able to pick these out form any representation without color-coding.

Extensions:

1. What happens to the slope and y-intercept in each representation when the lines are parallel? What about perpendicular? Give an example of a set of parallel lines on a graph, then identify their equations and t-charts. Do the same for a set of perpendicular lines.

2. What happens when I graph two equations on the same piece of graph paper? Explore graphing systems of equations using the same strategies in this unit. You can also use the same strategies for linear inequalities.

3. If students have learned Pythagorean theorem, have them graph lines that will form right triangles then determine the length of each side using the coordinate grid and Pythagorean theorem. For example, "graph these three equations on the same piece of graph paper:

$X = 3$ $\qquad$ $Y = 3$ $\qquad$ $Y = -\frac{1}{2}X + 10$

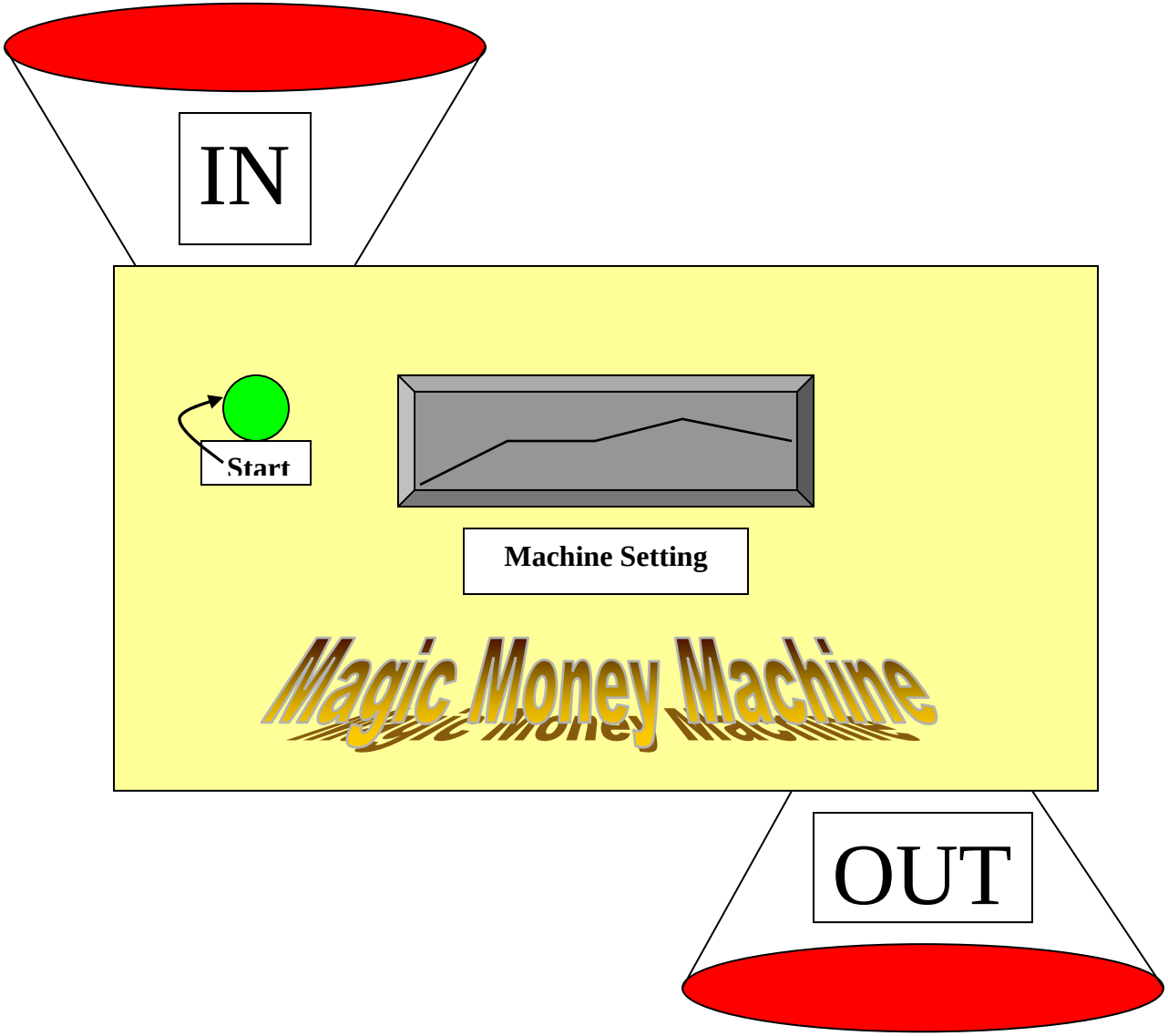
Daily Assessment Idea:

Use the old state tests to find examples of each day's concept. Copy one of these problems per day and give it to the students as an "exit slip" to determine who does and does not get that type of representation.

Resources

Make Your Own Graph Paper Resource:
http://www.incompetech.com/graphpaper/

Resources for Daily Practice Problems:
http://www.kutasoftware.com/free.html
District approved text-books


IN
Start
Machine Setting
Magic Money Machine
OUT

What's My Rule?

Scenario 1:

In	Out

Rule: _____

Scenario 2:

In	Out

Rule: _____

Scenario 3:

In	Out

Rule: _____

Scenario 4:

In	Out

Rule: _____

Scenario 5:

In	Out

Rule: _____

Scenario 6:

In	Out

Rule: _____

Free Plain Graph Paper from http://incompetech.com/graphpaper/plain/